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# Rents, dissipation and lost treasures: comment

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**Abstract**

In an interesting recent paper, Dari-Mattiacci and Parisi (2005) extended Tullock's (1980) rent-seeking game with an entry decision. The mixed strategies identified by Dari-Mattiacci and Parisi for the case of increasing returns in the contest success function ($r > 2$) do not constitute an equilibrium of the game they study. However, these strategies are an equilibrium if the strategy space of the game is restricted by a minimum expenditure requirement, and this minimum expenditure requirement is an element of a specific interval.

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# 1 Introduction

In an interesting recent paper, Giuseppe Dari-Mattiacci and Francesco Parisi (2005) reconsidered Gordon Tullock's (1980) rent-seeking game.[1] They introduce an exit option and propose a mixed strategy equilibrium for the case of increasing returns in the contest success function ($r > 2$). Each player mixes between two strategies: nonparticipation, and participation with one uniquely defined positive level of investment in rent-seeking activities.

This note shows that these strategies do not constitute an equilibrium of the game specified by Dari-Mattiacci and Parisi (2005). However, these strategies do constitute an equilibrium if the strategy space is restricted by a minimum expenditure requirement, and this minimum expenditure requirement is an element of a specific interval.

Minimum expenditure requirements in rent-seeking contests have been studied by Hillman and Samet (1987), Yang (1993), and Schoonbeeck and Kooreman (1997). Hillman and Samet (1987) study a perfectly discriminating contest, where the player who chooses the highest investment in rent-seeking wins with certainty. Yang (1993) studies a Dollar-Auction between two players with alternating moves. Schoonbeeck and Kooreman (1997) consider Tullock's rent-seeking game with two players and $r = 1$.

The assumption of a minimum expenditure requirement captures the fact that, in reality, it is often necessary to invest at least a certain minimum amount in rent-seeking activities in order to have some impact at all. For example, Yang (1993) argues that a lobby group that wants to influence a government has to articulate its aims to the public to some extend, for otherwise the government will not take the lobby group seriously. Another example is given by Schoonbeeck and Kooreman (1997), who point out that, in the Netherlands, political parties have to pay an entry fee before they can participate in an election contest for the parliament or the municipal councils.

[1] This game has received a considerable amount of scholarly attention, see Lockard and Tullock (2000) for a collection of important papers.

## 2 The Unrestricted Tullock Contest With an Entry Decision

This section lays out the game introduced by Dari-Mattiacci and Parisi (2005), which I call the Unrestricted Tullock Contest With an Entry Decision. The word "unrestricted" refers to the assumption that the strategy space is *not* restricted by a minimum expenditure requirement. Two risk neutral players ($a$ and $b$) compete for a rent. The size of the rent is normalized to one. Let $x_i$ denote the strategy of player $i$. Usually, the strategy space in Tullock's rent seeking game is the set of nonnegative real numbers. Dari-Mattiacci and Parisi (2005) allow players an exit option: players simultaneously decide whether to participate or not, and in case of participation, how much to invest in rent-seeking activities. Thus, I take it that the strategy space is $[0, \infty) \cup \{N\}$, with the interpretation that $x_i = N$ means that $i$ does not participate, and $x_i = x \in [0, \infty)$ means that $i$ participates with an investment equal to $x$.

The payoff of player $i = a, b$ is as follows:

$$S_i = \begin{cases} 0, & \text{if } x_i = N, \\ \frac{x_i^r}{x_i^r + x_j{}^r} - x_i, & \text{if } x_i, x_j \in [0, \infty) \text{ and } x_i^r + x_j{}^r \neq 0, \\ \frac{1}{2}, & \text{if } x_i = x_j = 0, \\ 1 - x_i, & \text{if } x_i \in [0, \infty) \text{ and } x_j = N. \end{cases} \tag{1}$$

The first line in equation (1) says that a player who does not participate gets a payoff of zero. The second line describes the case where both players participate, and at least one of the investments in rent-seeking activities is strictly positive. Player $a$ gets the share $x_a^r / (x_a{}^r + x_b{}^r)$ of the rent, and $b$ gets the remainder; both players have to bear the cost of their investment in rent-seeking activities. I will concentrate on the case where $r > 2$. The third line takes care of the situation where both players participate with an investment of zero: in that case, players share the rent equally.[2] Finally, the fourth line says that if only one player participates, he

[2] This assumption is often used in the literature, see e. g. Baye, Kovenock, and de Vries (1994). Alternatively one could assume that no one gets the rent if $x_a = x_b = 0$. This is also sometimes assumed in the literature, see e. g. Yang (1994). Dari-Mattiacci and Parisi (2005) do not state what they assume for the case that both players participate with an investment of

gets all the rent, but has to pay the cost of his investment.[3]

The game is a simultaneous move one-shot game. This is in contrast to Higgins, Shughart, and Tollison (1985) who considered a two stage rent-seeking game with endogenous participation where players announce their participation decision in a first stage, and those who participate decide on their investment in rent-seeking after observing the number of active competitors in a second stage.[4]

# 3 The Restricted Tullock Contest With an Entry Decision

In order to highlight the role of a minimum expenditure requirement, I now introduce the Restricted Tullock Contest With an Entry Decision. In this game the strategy space is restricted by a minimum expenditure requirement: a player has to invest at least $z > 0$ if he participates. Thus, the strategy space is $[z, \infty) \cup \{N\}$.[5] Otherwise, the specification of the game is as in the Unrestricted Tullock Contest With an Entry Decision.

# 4 The proposed strategies

Dari-Mattiacci and Parisi (2005) propose a mixed strategy equilibrium where each player mixes between two strategies: nonparticipation, and participation with one uniquely defined positive level of investment in rent-seeking activities. To analyze this, I follow their notation: $\pi_A$ ($\pi_B$) denotes the probability that player $a$ ($b$) participates, and $A$ ($B$) denotes the investment of player $a$ ($b$) in case of participation.

zero. However, my results do not depend on which assumption is taken - in fact the proofs go through without modification under both assumptions.

[3]The formulation above differentiates between nonparticipation and an investment of zero. An alternative interpretation of Dari-Mattiacci and Parisi (2005) would be to take an investment of zero as equivalent to nonparticipation. In that case, the strategy space is $[0, \infty)$. The payoff of player $i$ is $\frac{x_i^r}{x_i^r + x_j{}^r} - x_i$ if $x_i > 0$, and zero if $x_i = 0$. My comment does not depend on which interpretation is adopted - I take care to formulate all the results and proofs such that they go through under both interpretations.

[4]See also Corcoran (1984) and Corcoran and Karels (1985).

[5]Under the alternative interpretation of Dari-Mattiacci and Parisi (2005) described in footnote 3, the strategy space of the restricted game is $[z, \infty) \cup \{0\}$.

Dari-Mattiacci and Parisi (2005: 417) propose the following mixed strategies: players participate with probability

$$\pi_A^* = \pi_B^* = \frac{4}{2+r}, \tag{2}$$

and, in case of participation, investments in rent-seeking equal

$$A^* = B^* = \frac{r}{2+r}. \tag{3}$$

To see the logic behind this, consider the expected payoff of player $a$, given that he participates and $b$ mixes between nonparticipation and investing $B$ (Dari-Mattiacci and Parisi 2005: 416):

$$S_A(A\,|\pi_B, B) = \pi_B\left(\frac{A^r}{A^r+B^r} - A\right) + (1-\pi_B)(1-A). \tag{4}$$

Inserting $\pi_B^*$ and $B^*$,

$$S_A(A\,|\pi_B^*, B^*) = \frac{4}{2+r}\left(\frac{A^r}{A^r+\left(\frac{r}{2+r}\right)^r} - A\right) + \left(1-\frac{4}{2+r}\right)(1-A). \tag{5}$$

It is straightforward to calculate that $S_A(A^*\,|\pi_B^*, B^*) = 0$. That is, participating and investing $A^*$ results in an expected payoff of zero. Nonparticipation also results in a payoff of zero. Hence player $a$ is indifferent between nonparticipation and participation with an investment of $A^*$.

Moreover, $S_A(A\,|\pi_B^*, B^*)$ has a local maximum at $A^*$. To see this, differentiate line (4):

$$\frac{\partial S_A(A\,|\pi_B, B)}{\partial A} = \pi_B \frac{rB^rA^{r-1}}{(A^r+B^r)^2} - 1, \tag{6}$$

$$\frac{\partial^2 S_A(A\,|\pi_B, B)}{\partial A^2} = \frac{\pi_B r B^r A^{r-2}}{(A^r+B^r)^3}\left((A^r+B^r)(r-1) - 2rA^r\right). \tag{7}$$

Inserting $A^*$, $B^*$, and $\pi_B^*$ leads to

$$\begin{aligned}\frac{\partial S_A\left(A^* \left| \pi_B^*, B^* \right.\right)}{\partial A} &= 0, \\ \frac{\partial^2 S_A\left(A^* \left| \pi_B^*, B^* \right.\right)}{\partial A^2} &< 0.\end{aligned}$$

Thus, at $A^*$ the relevant local first- and second-order conditions hold.

However, given $r > 2$, it follows from equation (7) that $S_A\left(A \left| \pi_B^*, B^* \right.\right)$ is strictly *convex* in $A$ iff

$$A < A_0 := B^* \left(\frac{r-1}{r+1}\right)^{\frac{1}{r}}, \tag{8}$$

and strictly concave iff $A > A_0$. Thus, local first- and second order conditions may not be sufficient to characterize the global maximum.[6]

In fact, this is the case in the Unrestricted Tullock Contest With an Entry Decision. Without a minimum expenditure requirement, a player could participate and invest a very small amount. In this way, his costs are negligible. He still gets the rent in case that the opponent stays out, which happens with strictly positive probability. Thus, if player $a$ participates with a sufficiently small investment, he gets a strictly positive payoff.

**Remark 1** *The strategies described in equations (2) and (3), where the players mix between nonparticipation on the one hand, and investing a specific amount in rent-seeking activities on the other, do not constitute a Nash equilibrium of the Unrestricted Tullock Contest With an Entry Decision.*

**Proof.** Suppose player $b$ behaves according to equations (2) and (3). If $a$ also follows (2) and (3), he gets a payoff of zero. However, $a$ 's payoff $S_A\left(A \left| \pi_B^*, B^* \right.\right)$ gets arbitrarily close to $1 - \pi_B^* > 0$ by choosing a small enough $A > 0$. Hence participating with a sufficiently small investment is strictly better than staying out, or investing $A^*$. ■

As an example, Figure 1 plots $S_A\left(A \left| \pi_B^*, B^* \right.\right)$ (given in equation (5) above) as a function of $A$, assuming $r = 3$.

[6] This point is similar to Baye, Kovenock and de Vries (1994, p. 367).

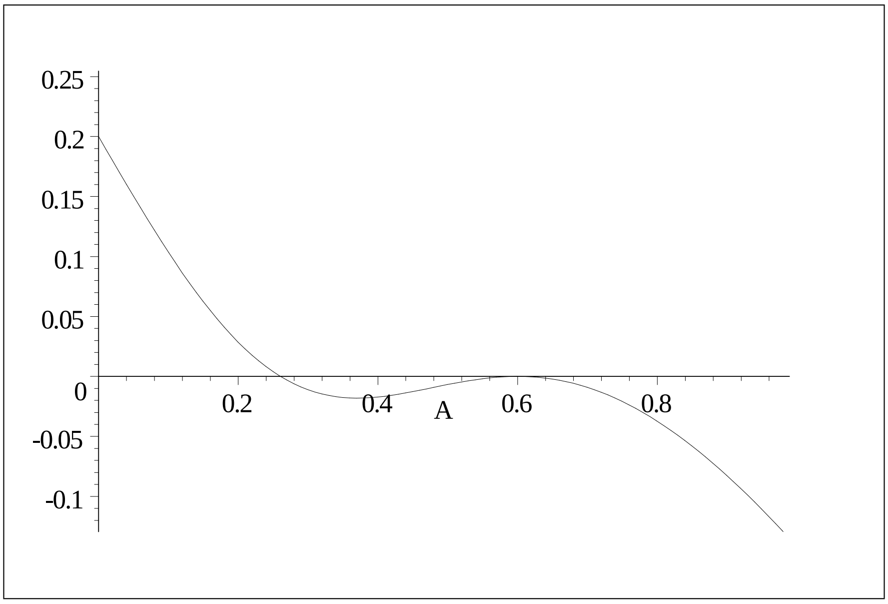

Figure 1: $S_A\left(A\left|\pi_B^*, B^*\right.\right)$ as a function of $A$, assuming $r = 3$. Here, $A^* = r/(2+r) = 0.6$ and $1 - \pi_B^* = (r-2)/(r+2) = 0.2$

# 5 The role of the minimum expenditure requirement

For the case $r = 3$, Figure 1 clearly shows that the strategies (2) and (3) are not an equilibrium of the Unrestricted Tullock Contest with an Entry Decision. However, Figure 1 also indicates that these strategies are an equilibrium if there is a sufficiently high minimum expenditure requirement. For example, $z = 0.3$ will do.

The main result of this note is that these points generalize for all $r > 2$. Lemma 1 defines the appropriate critical level of the minimum expenditure requirement, and Proposition 1 states the result formally.

**Lemma 1** *There exists a unique* $\hat{A} \in (0, r/(2+r))$ *such that*

$$S_A\left(\hat{A}\left|\pi_B^*, B^*\right.\right) = 0. \tag{9}$$

*Moreover,* $S_A(A|\pi_B^*, B^*) > 0$ *for all* $A \in \left(0, \hat{A}\right)$, *and* $S_A(A|\pi_B^*, B^*) < 0$ *for all* $A \in \left(\hat{A}, \infty\right) \setminus \{r/(2+r)\}$.

**Proof.** It has been shown in Section 4 that $S_A(A|\pi_B^*, B^*)$ has a local maximum at $A^* = r/(2+r)$, and $S_A(A^*|\pi_B^*, B^*) = 0$. Thus there exists an $\varepsilon > 0$ such that $S_A(A|\pi_B^*, B^*) < 0$ for all $A \in (A^* - \varepsilon, A^*)$. On the other hand, $\lim_{A \downarrow 0} S_A(A|\pi_B^*, B^*) = (r-2)/(2+r) > 0$. Since $S_A(A|\pi_B^*, B^*)$ is continuous, it follows that there is at least one $\hat{A} \in (0, A^*)$ such that (9) holds.

Next, I show that $\hat{A}$ is unique. Since $S_A(A|\pi_B^*, B^*)$ is strictly concave for all $A > A_0$, and $S_A(A^*|\pi_B^*, B^*) = 0$, it follows that $S_A(A|\pi_B^*, B^*) < 0$ for all $A \in [A_0, A^*)$. In addition, $S_A(A|\pi_B^*, B^*)$ is strictly convex for $A \in (0, A_0)$. Hence $\hat{A}$ is unique. It follows that $S_A(A|\pi_B^*, B^*) > 0$ for all $A \in \left(0, \hat{A}\right)$, and $S_A(A|\pi_B^*, B^*) < 0$ for all $A \in \left(\hat{A}, \infty\right) \setminus \{r/(2+r)\}$. ■

**Proposition 1** *Consider the Restricted Tullock Contest With an Entry Decision, with two contestants and* $r > 2$. *The strategies described in equations (2) and (3), where the players mix between nonparticipation on the one hand, and investing a specific amount in rent-seeking activities on the other, constitute a Nash equilibrium if and only if the minimum expenditure requirement* $z$ *satisfies*

$$z \in \left[\hat{A}, \frac{r}{2+r}\right].$$

**Proof.** If $z < \hat{A}$, then participating and investing $A = z$ gives player $a$ a strictly positive payoff by Lemma 1, contradicting equilibrium. If $z \in \left[\hat{A}, r/(2+r)\right]$, it follows from Lemma 1 that no profitable deviation from the strategies (2) and (3) exists. A similar argument shows that player $b$ has no incentive to deviate, either. Finally, if $z > r/(2+r)$, investing $A^* = r/(2+r)$ is not feasible. ■

# 6 Conclusion

Dari-Mattiacci and Parisi (2005) propose a mixed strategy equilibrium for Tullock's rent-seeking contest with an exit option and $r > 2$. Each player mixes between two strategies: nonparticipation, and participation with one uniquely defined positive level of investment in rent-seeking activities. This note shows that these strategies do not constitute an equilibrium of the game studied by Dari-Mattiacci and Parisi (2005). However, they do constitute an equilibrium if the strategy space is restricted by a minimum expenditure requirement, and this minimum expenditure requirement lies in a specific interval.

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