



**GOVERNANCE AND THE EFFICIENCY
OF ECONOMIC SYSTEMS
GESY**

Discussion Paper No. 177

**Bailouts in a common market:
a strategic approach**

Ela Glowicka*

October 2005

*Ela Glowicka, Wissenschaftszentrum Berlin, Reichpietschufer 50, 10785 Berlin, Germany.
glowicka@wz-berlin.de

Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

Sonderforschungsbereich/Transregio 15 · www.gesy.uni-mannheim.de
Universität Mannheim · Freie Universität Berlin · Humboldt-Universität zu Berlin · Ludwig-Maximilians-Universität München
Rheinische Friedrich-Wilhelms-Universität Bonn · Zentrum für Europäische Wirtschaftsforschung Mannheim

Speaker: Prof. Konrad Stahl, Ph.D. · Department of Economics · University of Mannheim · D-68131 Mannheim,
Phone: +49(0621)1812786 · Fax: +49(0621)1812785

Bailouts in a common market:
a strategic approach *

Ela Głowicka
glowicka/at/wz-berlin.de

Social Science Research Center Berlin (WZB)
Reichpietschufer 50, 10785 Berlin

October 20, 2005

Abstract

Governments in the EU grant *Rescue and Restructure Subsidies* to bail out ailing firms. In an international asymmetric Cournot duopoly we study effects of such subsidies on market structure and welfare. We adopt a common market setting, where consumers from the two countries form one market. We show that the subsidy is positive also when it fails to prevent the exit. The reason is a strategic effect, which forces the more efficient firm to make additional cost-reducing effort. When the exit is prevented, allocative and productive efficiencies are lower and the only gaining player is the rescued firm.

Keywords: subsidies, asymmetric oligopoly, exit, European Union

JEL Classification: F13, L13, L52.

*I am very grateful to Jos Jansen for his comments on an earlier version of this paper. I also thank Paul Heidhues, J. Peter Neary, Lars-Hendrik Röller, Oz Shy and participant of ETSG 2005 conference for helpful discussions. All remaining mistakes are mine.

1 Introduction

Bailouts by governments in the EU are strictly regulated. Each time they must be approved by the EC Commission and the approval is conditional on a set of criteria gathered in the "Community Guidelines on State Aid for Rescuing and Restructuring Firms in Difficulty"¹. The guidelines focus on limiting distortions of competition to the minimum and give strict conditions for approvals of bailouts, e.g. one time -last time principle or a requirement of a proof of limited distortions in trade between member states caused by such a subsidy. Compensation measures are often imposed, such as selling a part of the market share to competitors of the beneficiary. The reason for these strict conditions is that R&R subsidies are particularly prone to distort competition among member states, as they may be acting against competitive forces which caused the exit. Such subsidies disadvantage competitors, who have to restructure with their own resources.

Strategic trade theory confirms this concern: literature started by Brander and Spencer (1983) shows that if countries grant subsidies strategically, they can improve the position of the subsidized national champion at the expense of the foreign firm. To the best of my knowledge, however, none of these models allows for a subsidy to bail out an exiting firm, since only interior solutions in the output game are analyzed. To analyze R&R subsidies we need to account for endogenous exit, which is our contribution to the literature.

In addition, strategic trade theory typically assumes that home and foreign firms compete in a third country, so the governments do not consider consumer surplus in their decisions about aid (an exception is Collie (2000) and (2002)). Still, consumer surplus is an important aspect in the debate on R&R, since monitoring of state aid in the EU is said to take place in order to protect consumers' interests.² Therefore, we incorporate the common market structure into the analysis: the subsidizing government maximizes not only profits of firms in her country, but also country's share in consumer surplus. Countries in the EU are very different in size, therefore asymmetry in this share is also accounted for.

Lahiri and Ono (1988) and Neary (1994) recommend social planners to tax high-cost

¹Official Journal of the European Union, 2004/C 244/02

²EU competition policy and the consumer, *Luxembourg: Office for Official Publications of the European Communities*, 2004.

and subsidize low-cost firms. Such policy creates a higher concentration in the market, but it improves welfare, since the firms with the increased market share are also more efficient. From this perspective, R&R subsidy shifts resources in the "wrong" direction, because it is meant to increase market share of the inefficient firm. The problem is that individual governments maximize only a part of total EU welfare, so a bailout can be harmful for the EU, but still profitable for one country. We assess these externalities of R&R subsidies on welfare in the EU in total and for member states of different sizes separately.

We consider a duopoly with two firms, each located in a different country, but selling in a common market. The firms have ex ante asymmetric unit production costs. They restructure in order to decrease these costs and then compete by setting quantities. Exit results from production inefficiency in one of the firms. In such a case the government can rescue the firm by subsidizing some additional restructuring and the bill is paid by consumers from the country of the aid beneficiary. When making this decision, the government maximizes welfare defined as profit of the firm and consumer surplus in the country. We are interested in two issues: the benefit from keeping the inefficient firm in the market and externalities of R&R subsidies on all players in the model. We also try to find a rationale for strict control of R&R subsidies in the EU.

We have two main results. First, an R&R subsidy fulfils two different roles. If cost differences are low and the subsidizing country small enough, the subsidy rescues the inefficient firm (we call this a *successful rescue*), increases welfare of the intervening country by increasing the profits of the aid beneficiary and decreasing the surplus of all consumers. If the cost asymmetry is higher, the subsidy is given, but it does not prevent the firm from exit (which we call a *failed rescue*). The subsidy is however only an apparent waste, since it provides a threat of no-exit to the efficient firm and forces this firm to cut its costs more than an unconstrained monopoly would. This strategic effect improves both productive and allocative efficiencies. The government is willing to grant such subsidy, because consumer surplus increases as the result of this intervention.

Second, externalities of the subsidy differ depending on whether the rescue succeeds or not. In a successful rescue, consumers and the competitor of the beneficiary are worse off, while the subsidized firm gains. In a failed rescue, consumers are better off and firm 2 loses. Externalities on welfare depend on the country's share in consumer surplus (we

call it *country's size*). The size determines the amount of aid paid out and the way gains from the subsidy are distributed between member states. If the country is small, the government cares relatively more about the profits than about the price and therefore the subsidy is successful for a large set of initial cost asymmetries. Welfare of the other country and total welfare are reduced. When the subsidizing country is big enough, only failed rescues take place. Externality on welfare of the other country is negative, but total welfare increases thanks to higher consumer surplus.

If consumer surplus or total welfare were her objective, the Commission should ban R&R subsidies which are likely to prevent exit, since they result in a price rise and a loss in productive efficiency. Crucial assumptions for this result were asymmetries across countries and across firms. This is an explanation alternative to the argument by Collie (2000) and (2002) that distortionary taxation in a symmetric model may be the reason for aid prohibition. In this way we want to contribute to the discussion on state aid policy in the EU, which is at the top of the agenda of the current Competition Commissioner Neelie Kroes³.

2 Model Setup

There are two countries. Each country has one government and one firm. Firms produce a homogenous good, which is sold on a common market without barriers in trade. Governments maximize welfare defined as profit of the firm in their country and the country's share in consumer surplus. Let country 1's share in consumer surplus be α and country 2's share $1 - \alpha$. Denoting welfare in country i as W_i we have:

$$W_1 = \pi_1 + \alpha CS \tag{1}$$

$$W_2 = \pi_2 + (1 - \alpha)CS \tag{2}$$

$$W = W_1 + W_2 = \pi_1 + \pi_2 + CS. \tag{3}$$

Firms maximize profits. Production cost functions are linear with asymmetric marginal costs denoted by c_1 and c_2 and we assume $c_1 > c_2$. This asymmetry is exogenous. Inverse

³Introductory Statement at EMAC Open Meeting of Coordinators, Feb. 3rd, 2005.

demand is $P = P(x)$, where $x = x_1 + x_2$ with $P'(x) < 0$. The game has three stages. First, the government with the less efficient firm commits to subsidize restructuring if the firm has to exit without help. Then firms restructure on their own and finally firms compete by setting quantities. Restructuring is modelled as process-R&D: it means cutting marginal cost by e_i at the cost of $\frac{d}{2}e_i^2$, where $d > 0$. It is important to note that the cost of restructuring is the same for both firms and for the government.⁴ We solve for the subgame-perfect Nash equilibrium by backward induction.

If there is no exit, there is no subsidy and firms' profits are

$$\pi_i = P(x)x_i - (c_i - e_i)x_i - \frac{d}{2}e_i^2 \quad i = 1, 2. \quad (4)$$

As typically happens in asymmetric Cournot games, if the initial marginal cost difference is high enough, the less efficient firm exits and a monopoly of the more efficient one emerges in the equilibrium. Government 1 can perfectly anticipate when firm 1 will exit in the equilibrium. In such a case, the government can subsidize further restructuring in firm 1 by $\frac{d}{2}k_1^2$ and reduce marginal costs additionally by k_1 . Two payoffs are changed (with subscript s):

$$\pi_{1s} = P(x)x_1 - (c_1 - e_1 - k_1)x_1 - \frac{d}{2}e_1^2 \quad (5)$$

$$W_{1s} = \pi_{1s} - \frac{d}{2}k_1^2 + \alpha CS. \quad (6)$$

This definition of subsidy is similar to definitions used in the literature in the sense that it affects marginal costs directly.⁵ On the other hand, R&R subsidy differs from commonly used subsidy definitions, because with the subsidy the government gives her own restructuring technology to the firm. She behaves as an entrepreneur and actually makes some decisions, what gives country 1 a cost advantage since $\frac{d}{2}e_1^2 + \frac{d}{2}k_1^2 < \frac{d}{2}(e_1 + k_1)^2$. In reality, transfer of technology often happens in R&R cases, since governments and public agencies actively participate in designing a restructuring plan or negotiating with unions.

⁴One could argue that restructuring is more expensive for the low-cost firm since it has no "slack" or X-inefficiency, so that for example parameter d for this firm should be higher. It is a valid concern, since Aghion and Schankerman (2004) allow for such slack and find that the equilibrium can have totally different properties than in the symmetric case.

⁵Mollgaard (2004) is an exception, defining aid as a reduction in the cost of capital.

Sometimes a government hires a consulting firm to do the job of preventing exit. Generally we can say that when bankruptcy is in sight, governments take unusual measures to save the firm and these unusual measures are the new restructuring technology.

Government 2 cannot subsidize, which is another asymmetry in the model and we introduce it in order to reflect the rules in the EU: only otherwise exiting firms can be subsidized and firm 2 in the equilibrium of our benchmark no-subsidy model never exits.

Finally, a comment on technical issues. Our problem is by nature asymmetric. Since asymmetric general models are difficult to solve, we make a compromise by choosing specific functional forms of both cost functions (linear and quadratic), leaving demand general and using linear demand case as an example. Welfare effects are presented only for the example of linear demand. This model structure is motivated by the fact that properties of the demand function seem to be crucial for optimal R&D subsidies. For example, Lahiri and Ono (2004) show that, when firms are symmetric, the sign of the subsidy is the same as the sign of the second derivative of demand. We limit our attention to pure strategy equilibria.

3 Benchmark Equilibrium: no subsidy

As a benchmark for the rescue subsidy game we consider a two-stage game, where both governments are passive and their payoffs result only from actions of firms. Firms simultaneously choose restructuring level and then decide about output level. Since we do not exclude exit, which is an unusual aspect of this model, we will look at this game in some detail.

In the second stage of the game, first order conditions for duopoly equilibrium are

$$P'(x)x_i + P(x) - c_i + e_i = 0 \quad i = 1, 2. \quad (7)$$

which implicitly define $x_i(e_1, e_2)$. We assume that second order conditions $P''(x)x_i + 2P'(x) < 0$, and stability conditions $P''(x)x_i + P'(x) < 0$ and $\frac{\partial^2 \pi_i}{\partial x_i^2} > \frac{\partial^2 \pi_i}{\partial x_i \partial x_j}$ hold. Totally

differentiating (7) and using the assumptions, we obtain comparative statics results:

$$\frac{dx_i}{de_i} > 0 \quad \frac{dx_j}{de_i} < 0 \quad \frac{dx}{de_i} > 0, \quad (8)$$

which indicate that restructuring expands own and total output, while reducing competitor's output. That implies that competitor's market share is decreased. In the first stage, first order conditions⁶ are

$$P'(x)x_i \frac{\partial x}{\partial e_i} + (P(x) - c_i + e_i) \frac{\partial x_i}{\partial e_i} + x_i - de_i = 0 \quad i = 1, 2 \quad (9)$$

and using (7) we can simplify them to

$$x_i + x_i P'(x) \frac{\partial x_j}{\partial e_i} = de_i \quad i = 1, 2. \quad (10)$$

The marginal cost of restructuring (right-hand side) equals the marginal revenue (left-hand side), which is composed of two effects. The direct effect is just a decrease of the production cost, represented here by x_i . The second expression represents the strategic effect restructuring has on the competitor's output. Since both derivatives in this expression are negative, their product is positive, so the strategic effect increases marginal revenue of restructuring.

Equations (7) and (10) implicitly define the equilibrium $x_i^D(c_1, c_2)$ and $e_i^D(c_1, c_2)$ (D stands for duopoly).

Comparing duopoly restructuring levels e_1^D and e_2^D , Lahiri and Ono (2004) prove⁷ the following

Proposition 1 *The firm with lower initial marginal cost invests more in restructuring.*

Low-cost firm reduces marginal cost by more than the rival, so that the initial cost difference is magnified. This observation goes back to the hypothesis of Schumpeter that big firms are better innovators (in terms of production process) than high-cost firms, since they are able to exploit cost-cuts on a larger scale.

For $c_1 - c_2$ sufficiently high, a monopoly of firm 2 is the equilibrium. There can be two

⁶We assume second order conditions to hold.

⁷The proof can be found on page 24 of the book.

qualitatively different outcomes: blockaded monopoly, which operates unconstrained by the potential entry (here rather non-exit) of firm 1, and entry-detering monopoly, where the competitor is strategically excluded from the market. First we consider a blockaded monopoly. First order condition in the output stage is

$$P'(x_2)x_2 + P(x_2) - c_2 + e_2 = 0, \quad (11)$$

which implicitly defines $x_2(e_2)$. We assume second order condition $P'' + 2P' < 0$ to hold. Differentiating (11) with respect to e_2 and using the second order condition we find that

$$\frac{dx_2}{de_2} > 0. \quad (12)$$

In the first stage, the first order condition⁸ is

$$P'(x_2)x_2 \frac{\partial x_2}{\partial e_2} + (P(x_2) - c_2 + e_2) \frac{\partial x_2}{\partial e_2} + x_2 - de_2 = 0 \quad (13)$$

and using (11) it boils down to a simple equation

$$x_2 = de_2. \quad (14)$$

In case of monopoly there is no strategic effect of e_2 on x_1 , so the effect of restructuring is just the reduction of production cost. Equations (11) and (14) together define $x_2^M(c_2)$ and $e_2^M(c_2)$ (M stands for monopoly).

When entry-deterrence takes place, firm 2 increases its output above monopoly level to exclude the competitor. In such a situation, x_2 is at the level which nullifies $x_1(e_1, e_2)$ calculated from (7), that is x_2 such that

$$[P'(x_1 + x_2)x_1 + P(x_1 + x_2) - c_1 + e_1]_{x_1=0} = 0, \quad (15)$$

from which we get

$$P(x_2) = c_1 - e_1. \quad (16)$$

⁸We assume second order conditions to hold.

Optimal output of firm 1 is then 0, and therefore also restructuring level is zero: $x_1^E = 0$ and $e_1^E = 0$ (E stands for entry-deterrence). Going back to equation (16) we get

$$P(x_2^E) = c_1. \quad (17)$$

Firm 2 produces as much as is necessary to price the good at the marginal cost of the competitor and in this way enforces its exit. The optimal restructuring level leading to this output follows from equation (14):

$$x_2^E = de_2^E. \quad (18)$$

3.1 Linear Demand Example

If inverse demand is linear, e.g. $p = 1 - x_1 - x_2$, the equilibrium can be explicitly calculated. To simplify complex calculations, we fix the parameters d and c_2 . We choose $d = 5$ so that all profit functions are concave and $c_2 = 0.4$ so that we avoid a number of corner solutions and focus exclusively on the role of the asymmetry in initial cost for the market structure in the equilibrium⁹. Let us define $\underline{c}_1 = 0.485749$, $\tilde{c}_1 = 0.532987$ and $\bar{c}_1 = 0.6667$.

Proposition 2 *For $c_1 \in (0.4, \underline{c}_1)$ duopoly emerges in the equilibrium with optimal strategies:*

$$e_1^D = 4 \frac{17 - 26c_1}{451} \quad x_1^D = \frac{15}{4} e_1^D \quad (19)$$

$$e_2^D = 4 \frac{0.6 + 15c_1}{451} \quad x_2^D = \frac{15}{4} e_2^D. \quad (20)$$

Entry-detering monopoly of firm 2 emerges when $c_1 \in (\tilde{c}_1, \bar{c}_1)$. Optimal strategies are

$$e_2^E = \frac{1 - c_1}{5} \quad x_2^E = 5e_2^E. \quad (21)$$

For $c_1 \in (\bar{c}_1, 1)$ equilibrium market structure is blockaded monopoly of firm 2 with optimal

⁹Qualitatively, we get the same results choosing any $d \in (4, \infty)$ and $c_2 \in (0.2, 0.5)$. For $c_2 < 0.2$ there are more and for $c_2 > 0.5$ less corner solutions to consider, but the mechanism is the same.

strategies

$$e_2^M = \frac{1}{15} \quad x_2^M = 5e_2^M. \quad (22)$$

The proof can be found in the Appendix, but in order to build some intuition we present reaction functions of the three equilibrium market structures in the first stage of the game:

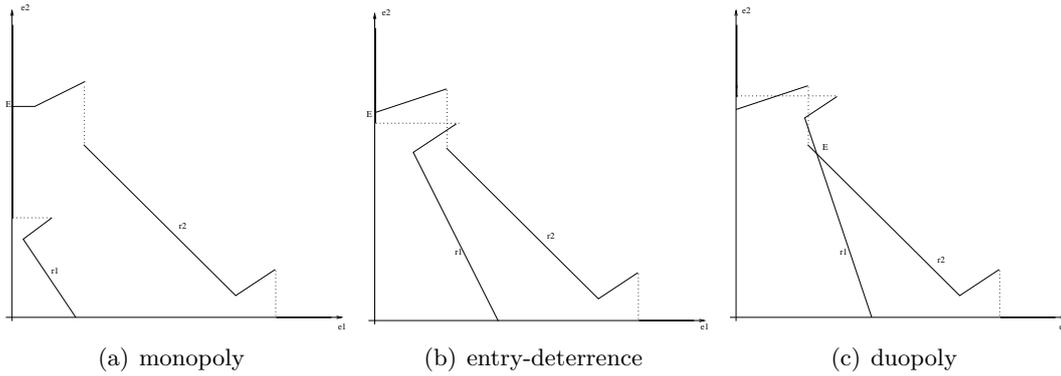


Figure 1: In the equilibrium of the benchmark game there are three possible market structures.

Figure 1(a) shows the case when firm 2 has a very high efficiency advantage. Its reaction function is composed of 5 pieces. For e_1 very low, best-reply of firm 2 is a blockaded monopoly with restructuring constant in e_1 . When e_1 is higher, firm 2 deters entry of firm 1 by increasing its restructuring level. This is the second, increasing part of the reaction function. For higher e_1 , best reply is accommodation, so that the outcome is duopoly. In that case the reaction function is decreasing. The fourth piece is where duopoly as a corner solution is the best-reply. Finally, if e_1 is too high, best reply is not to restructure and exit, so the reaction function coincides with e_1 axis. Firm 1's reaction function has only 3 pieces, since firm 1 has a higher initial production cost and neither monopoly nor entry-deterrence are achievable, but the logic is the same. Both reaction functions cut in point E, where firm 1 chooses to exit and firm 2 restructures at the monopoly level.

Figure 1(b) shows the case when firm 2 has a moderate efficiency advantage. Its reaction function is pushed to the left, so that monopoly part is not possible for positive

e_1 . The equilibrium is entry-deterrence by firm 2, since firm 2's best-reply to $e_2 = 0$ is entry-deterrence restructuring level, which is higher than in monopoly. Finally, figure 1(c) shows duopoly equilibrium, where downward-sloping parts of the reaction functions cut and both firms end up with positive restructuring and production.

Looking at the functional forms of the duopoly restructuring levels (19) and (20), we notice that the result exemplifies Proposition 1: for $c_1 > 0.4$ we have $e_1^D < e_2^D$, and the final cost difference is higher than initially. This result is robust to the definition of competition: Aghion and Schankerman (2004) got it for Salop model, Lahiri and Ono (2004) for Cournot duopoly with general demand, Escriva-Villar (2004) for n-firm Cournot competition with linear demand, Röller and Sinclair-Desgagne (1996) for two-market Cournot duopoly. In entry-deterrence outcome (16), however, e_2^E decreases with c_1 , since the bigger the initial cost difference, the bigger efficiency advantage of firm 2, so that less aggressive predatory behavior is sufficient to deter entry of firm 1. This finding will be crucial in further analysis.

4 Subsidy Equilibrium

If the initial cost asymmetry is too big, firm 1 exits (in the example it happens for $c_1 > \tilde{c}_1$). In the EU, government 1 could then subsidize some additional restructuring in firm 1. We will introduce this possibility to the game by extending the benchmark model by a stage, where government 1 can choose to do it.

The new game has three stages. In the first stage, government 1 commits to subsidize firm 1's restructuring process with $s = \frac{d}{2}k_1^2$ in order to lower its marginal cost by k_1 if without help the firm exits. Government 2 is not allowed to subsidize. In the second stage firms choose their restructuring level and the subsidy is paid. In the third stage they compete á la Cournot. Such formulation of the game reflects the rules of the EU common market, where governments can subsidize restructuring of ailing firms and they usually get involved into restructuring process. The outcomes are denoted by a subscript s .

In the third stage, when duopoly emerges, first order conditions are

$$P'(x)x_1 + P(x) - c_1 + e_1 + k_1 = 0 \tag{23}$$

$$P'(x)x_2 + P(x) - c_2 + e_2 = 0. \tag{24}$$

We assume that second order conditions $P''(x)x_i + 2P'(x) < 0$ and stability conditions hold. Comparative statics is done analogously to the benchmark game and we conclude that

$$\frac{dx_1}{dk_1} > 0 \quad \frac{dx_2}{dk_1} < 0 \quad \frac{dx}{dk_1} > 0. \quad (25)$$

The subsidy works exactly like restructuring by firm 1. It increases the output of the beneficiary at the cost of its more efficient competitor and the total output is also increased. This is the traditional profit-shifting effect of subsidies in strategic trade subsidy games.

In the second stage of duopoly, first order conditions are just like in the benchmark case:

$$x_i + x_i P'(x) \frac{\partial x_j}{\partial e_i} = de_i \quad i = 1, 2. \quad (26)$$

Both first order conditions together implicitly define equilibrium outputs and restructuring as functions of k_1 . Just as before, restructuring has a direct and a strategic impact on the marginal revenue.

In entry-deterrence, if the subsidy is positive, the increase of output in firm 2 must be such that

$$P(x_2^{Es}) = c_1 - k_1. \quad (27)$$

This equation shows the strategic effect of the subsidy. The government changes the initial conditions of the game between the firms: firm 2 has to adapt its strategy to the lower initial cost of firm 1. Since demand is decreasing, we have $x_2^E < x_2^{Es}$. Firm 2 produces more output to deter firm 1, because firm 1 is more efficient thanks to the subsidy.

In unconstrained monopoly, output and restructuring in firm 2 are the same as in the benchmark.

In the first stage, government 1 optimally chooses k_1 . In case of duopoly welfare in country 1 is

$$W_{1s}(k_1) = P(x)x_1 - (c_1 - e_1 - k_1)x_1 - \frac{d}{2}e_1^2 - \frac{d}{2}k_1^2 + \alpha \int_0^{x^D} (P(x) - P(x^D))dx \quad (28)$$

and we further assume that it is concave. Optimal k_1 is the one which nullifies derivative of W_{1s} with respect to k_1 :

$$x_1 + x_1 \frac{\partial e_1}{\partial k_1} + x_1 P'(x) \frac{\partial x_2}{\partial k_1} - \alpha x P'(x) \frac{\partial x}{\partial k_1} = d \left(\frac{\partial e_1}{\partial k_1} e_1 + k_1 \right). \quad (29)$$

The above equation shows all effects of the subsidy on welfare in country 1. The first term is the direct effect on the output in firm 1: marginal cost of production decreases so each unit of output brings higher revenue. The second term is the strategic effect on firm 1, due to which firm 1 restructures more by its own. This is a consequence of Proposition 1., since when the subsidy is positive, firm 1 has lower initial marginal cost. The third term is the strategic effect on the other firm (selection or profit-shifting effect). In reaction to a positive subsidy, firm 2's output decreases and firm 1 grabs a bigger market share. The fourth term on the left-hand side of the equation reflects the price drop as the effect of the subsidy (competitive effect). The higher k_1 , the lower the price, because total output increases. Summing up, as the result of the subsidy allocative efficiency is improved, but productive efficiency may be reduced due to the selection effect. The term on the right-hand side is the marginal cost of restructuring and it consists of the subsidy cost which consumers have to pay, as well as the cost of restructuring paid by the firm which is higher than in the benchmark due to the positive subsidy.

If entry-deterrence is the equilibrium, government 1 chooses k_1 such that

$$W_{1s} = -\frac{d}{2} k_1^2 + \alpha \int_0^{x_2^E} (P(x) - P(x_2^E)) dx \quad (30)$$

is maximized. Again we assume that this function is concave with respect to k_1 . The first order condition is

$$\frac{\partial W_{1s}}{\partial k_1} = -\alpha x_2 P'(x_2) \frac{\partial x_2}{\partial k_1} - dk_1 = 0. \quad (31)$$

We assumed that $P'(x)$ is negative, we also know from (27) that $\frac{\partial x_2}{\partial k_1}$ is positive. Therefore, the derivative $\frac{\partial W_{1s}}{\partial k_1}$ evaluated in $k_1 = 0$ is positive. Since W_{1s} is concave, k_1 satisfying (31) must be positive. We can conclude that in entry-deterrence the subsidy is positive. The government tries to bail the firm out, but the firm exits.

Proposition 3 (Failed Rescue) *If the equilibrium is entry-deterring monopoly of the more efficient firm, the R&R subsidy to the exiting firm is positive.*

Even when the less efficient firm exits for sure and the subsidy is nothing more than burning consumers' money, government 1 has a good reason to do it: the market is imperfectly competitive and the subsidy has a strategic effect on the efficient firm. This firm decreases the price so much, that even having paid for the subsidy, consumers in country 1 are better off. Consumers in country 2 gain even more, since they don't pay for the costly policy of government 1.

4.1 Linear Demand Example

For the sake of brevity, we define the following terms:

$$f_1(\alpha) = 0.33359 + 0.0366975\alpha - 0.024098\sqrt{(\alpha - 28.168411)(\alpha + 0.50916)} \quad (32)$$

$$f_2(\alpha) = 0.5829 - 0.109912\alpha \quad (33)$$

$$f_3(\alpha) = 0.485749 + 0.10285\alpha + 0.062697\sqrt{(\alpha - 5)(\alpha - 0.35357)} \quad \text{for } \alpha \in (0, 0.3535) \quad (34)$$

$$f_4(\alpha) = 1 - 0.14907\sqrt{5 - \alpha} \quad (35)$$

$$f_5(\alpha) = 0.56271 + 0.02856\alpha - 1.38407\sqrt{\frac{(\alpha - 28.1684)^2(\alpha - 0.27334)}{153377 - 5445\alpha}} \quad \text{for } \alpha \in (0.27334, 1) \quad (36)$$

$$f_6(\alpha) = 0.47224 + 0.0467\alpha \quad (37)$$

$$m_4 = 0.52709 + 0.02982\alpha \quad m_5 = 0.532987 + 0.0934\alpha \quad (38)$$

and in the Appendix we prove

Proposition 4 *Six different equilibrium outcomes emerge, depending on the parameters vector (α, c_1) .*

1. *If condition $0.4 < c_1 < \min[f_1(\alpha), f_6(\alpha)]$ is fulfilled, entry-deterrence by firm 1*

emerges in the equilibrium with strategies

$$k_1^{E1} = c_1 - 0.229143 \quad e_1^{E1} = 0.12 \quad x_1^{E1} = 5e_1^{E1}. \quad (39)$$

2. When $f_1(\alpha) < c_1 < \min[m_4(\alpha), f_5(\alpha)]$, duopoly emerges with strategies

$$e_1^D = 4 \frac{17 - 26(c_1 - k_1^D)}{451} \quad x_1^D = \frac{15}{4} e_1^D \quad (40)$$

$$e_2^D = 4 \frac{0.6 + 15(c_1 - k_1^D)}{451} \quad x_2^D = \frac{15}{4} e_2^D \quad (41)$$

$$k_1^D = \frac{-0.804028 + 1.22969c_1 - 0.214158\alpha + 0.133849c_1\alpha}{0.133849\alpha - 3.77031}. \quad (42)$$

3. If $m_4(\alpha) < c_1 < \min[f_2(\alpha), f_3(\alpha)]$, duopoly emerges as well, but the corner subsidy

$$k_1^{Dc} = c_1 - 0.485749 \quad (43)$$

is chosen instead of k_1^D .

4. For $\max[f_2(\alpha), f_5(\alpha), f_6(\alpha)] < c_1 < m_5(\alpha)$, entry-deterrence by firm 2 emerges:

$$k_1^{E2c} = c_1 - 0.532987 \quad e_2^{E2} = \frac{1}{5}(1 - c_1 + k_1^{E2c}) \quad x_2^{E2} = 5e_2^{E2}. \quad (44)$$

5. If $\max[f_3(\alpha), m_5(\alpha)] < c_1 < f_4(\alpha)$, entry-deterrence by firm 2 emerges with strategies

$$k_1^{E2} = \frac{\alpha(1 - c_1)}{5 - \alpha} \quad e_2^{E2} = \frac{1}{5}(1 - c_1 + k_1^{E2}) \quad x_2^{E2} = 5e_2^{E2}. \quad (45)$$

6. Finally, for $c_1 > f_4(\alpha)$, firm 2 is a blockaded monopolist and the optimal subsidy is zero.

How does the subsidy actually work? Figure 2 helps to develop some intuition. Reaction functions of the benchmark game are depicted in black and those of the subsidy game in red.

Figure 2(a) shows a failed rescue. The benchmark equilibrium is entry-detering monopoly of firm 2, denoted by E. Due to the subsidy, equilibrium is moved to point Es, which is also the entry-detering monopoly of firm 2, but with a higher restructuring

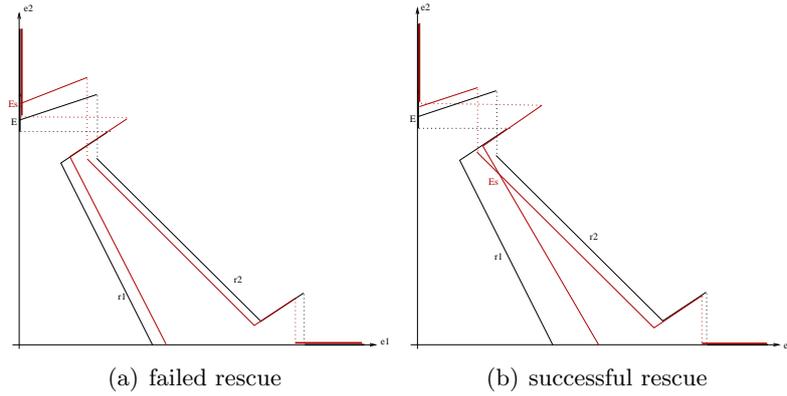


Figure 2: The subsidy game has two very different outcomes.

level. Figure 2(b) illustrates a successful rescue: entry-deterrence equilibrium E changes into duopoly equilibrium E_s , firm 1 is successfully rescued.

When $c_1 > \tilde{c}_1$, firm 1 exits in the equilibrium of the benchmark model and only then the subsidy is legal. Therefore, we analyze the subsidy equilibrium for $c_1 > \tilde{c}_1$ ¹⁰. Figure 3 shows the location of each market structure in the (α, c_1) space.

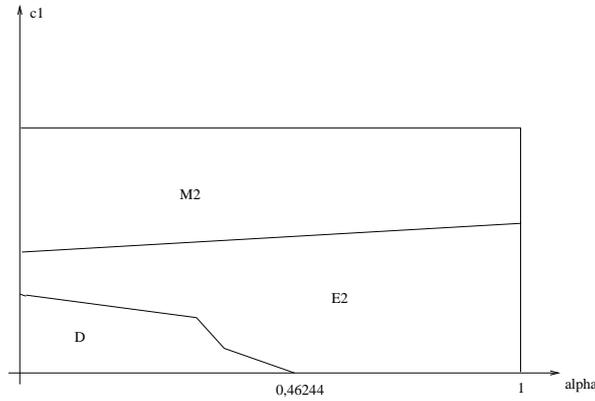


Figure 3: The subsidy game results in one of three market structures: duopoly (D), entry-deterrence by firm 2 (E2) or blockaded monopoly of firm 2 (M2).

Comparing this outcome with the equilibrium in the benchmark case allows us to make a few interesting observations. The first observation is a confirmation of Proposition 3: k_1^{E2c} and k_1^{E2} are positive. In entry-deterrence of firm 2, government 1 still subsidizes firm

¹⁰We skip the set $c_1 \in (\underline{c}_1, \tilde{c}_1)$ since they have no equilibrium in the benchmark.

1 in order to achieve the desired strategic effect on firm 2. This effect consists of making firm 1 a little more efficient, so that firm 2 has to be more aggressive in cost-cutting to deter entry of firm 1. The result is a lower production cost and a lower price, which is beneficial for consumers in both countries.

The second observation is that duopoly is now more common: more parameter vectors result in duopoly. These vectors, which in the benchmark ended up with monopoly and with the subsidy lead to duopoly, guarantee a successful rescue. In these cases the original purpose of the subsidy is achieved: the less efficient firm is rescued from exit. Note that the lower α , the larger the set of c_1 which leads to successful rescue, while for $\alpha > 0.462443$ duopoly is not possible at all. That is because a big country has bigger incentives to subsidize an unsuccessful rescue: there are more consumers who gain from the decline of the price and are ready to pay for the subsidy. That implies that the strategic effect on the competing firm is stronger. From this we can draw a testable conclusion that small countries should experience more successful rescues, while big countries more failed rescues.

In the subsidy equilibrium there is less monopoly than in the benchmark. Entry-detering monopoly of firm 2 is more beneficial to welfare in country 1 than blockaded monopoly of firm 2, so government 1 grants a subsidy in order to force firm 2 to deter entry.

The final remark is on the fact, that the government's intervention allows the market to achieve equilibrium also for parameter vectors, for which there is no pure-strategy equilibrium in the benchmark.

5 Welfare analysis

In this section we study externalities of R&R subsidy on the welfare distribution under the assumption of linear demand. The subsidy is positive in case of successful rescue, which happens for initial parameter vectors (α, c_1) such that $\tilde{c}_1 < c_1 < h(c_1)$, and for failed rescue which happens for initial parameter vectors (α, c_1) such that $h(c_1) < c_1 < f_4$,

where

$$h(c_1) = \begin{cases} f_3 & \text{for } \alpha \in (0, 0.24562), \\ f_2 & \text{for } \alpha \in (0.24562, 0.399563), \\ f_5 & \text{for } \alpha \in (0.399563, 0.462443), \\ \tilde{c}_1 & \text{for } \alpha \in (0.462443, 1). \end{cases} \quad (46)$$

The first comparison we make across prices.

Corollary 5 *Compared with the benchmark equilibrium price, the subsidy equilibrium price increases in successful rescue, but decreases in failed rescue.*

In case of successful rescue, duopoly is the outcome. The high-cost firm also produces, which drives the average production cost up. As the result, the price is higher compared with the benchmark and all consumers are harmed. Consumers in country 1 lose even more than those in country 2, because they additionally have to pay for the subsidy. The gain on firm 1's profit, however, is high enough, so that country 1's welfare is increased. In failed rescue, entry-deterrence of firm 2 is the outcome. The price is then $P(x_2^{Es}) = c_1 - k_1$ and since the subsidy is positive, it is lower than c_1 , which is the price in entry-deterrence in the benchmark. Naturally such $P(x_2^{Es})$ is also lower than monopoly price, which is the highest of all prices in our model.

Comparing profits leads us to the following

Corollary 6 *Industry profits decrease in both successful and in failed rescue.*

If the rescue fails, firm 2 has to restructure more aggressively in order to keep firm 1 out of the market. Therefore, its profits are lower than in the benchmark case. In successful rescue, total profit and profit of firm 2 decrease, since firm 2 restructures less than in the benchmark and additionally inefficient firm 1 also produces, so production costs are higher and even the higher price is not able to compensate it.

An example of welfare changes is depicted in Figures 4 and 5. We plot all parts of welfare as a function of c_1 , holding α fixed at two different levels. Dashed lines represent the benchmark model, while continuous lines illustrate the subsidy game.

We can observe the following regularities. Country 1 is always better off. In successful rescue it gains on profit, in a failed rescue it gains on consumer surplus. This is a conse-

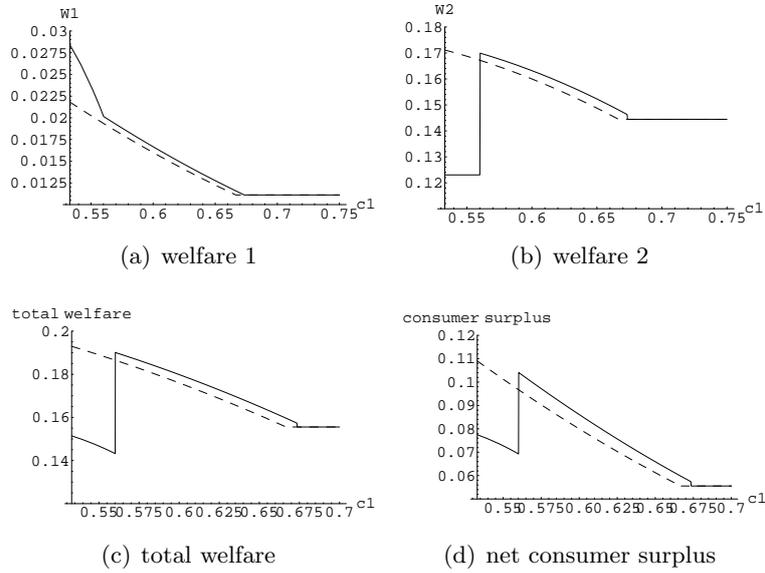


Figure 4: Effects of the subsidy for $\alpha = 0.2$.

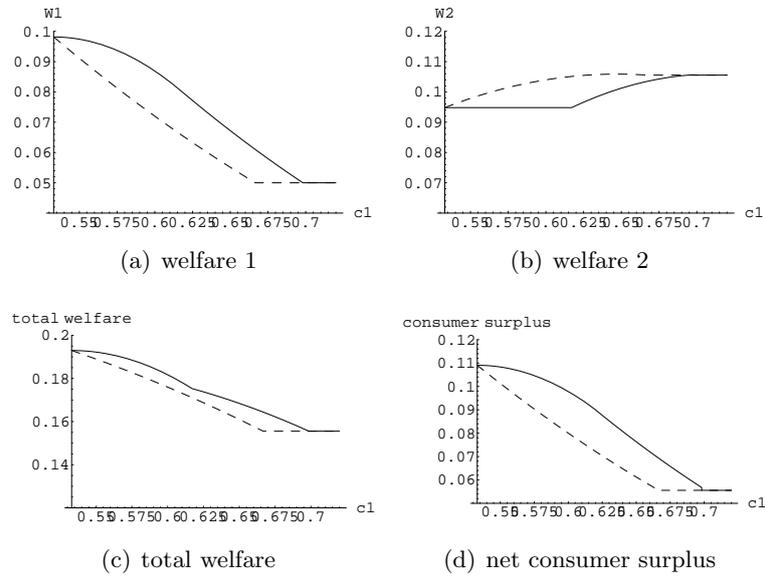


Figure 5: Effects of the subsidy for $\alpha = 0.85$.

quence of the game construction: government 1 acts as the first one and by setting $k_1 = 0$ she can always get at least the benchmark welfare. Therefore in the subsidy game she can do better.

Welfare in country 2 increases when consumer surplus increases a lot and country 2's share in consumer surplus is very high. That is the case for failed rescue and low α . In a successful rescue consumers are harmed and firm 2 loses market share, so welfare 2 in that case decreases compared with the benchmark. Also when α is high enough, welfare 2 declines since entry-detering profits are lower and a big part of consumer gain belongs to country 1.

The change in total welfare is a sum of the above two. In case of a successful rescue total welfare drops and this drop is caused mainly by smaller consumer surplus and higher total production cost. In a failed rescue total welfare increases, since the subsidy enforces a lower price and the cost of the subsidy is low enough to make such rescue profitable for consumers.

6 Conclusions

Summing up, we showed that depending on the initial cost asymmetry between firms and the asymmetry in countries' share in consumer surplus, rescue and restructure subsidy has one of the following effects: it can save the high-cost firm from the exit and in this way reduce productive and allocative efficiencies in the common market, or it can have a strategic impact on the entry-detering monopolist and in this way increase allocative efficiency, without preventing exit of the beneficiary.

We find that the role of asymmetry in size is very important: countries with equally inefficient firms, but different shares in consumer surplus, choose very different policies. Roughly speaking, big countries do not prevent exit, while small countries do.

In the airlines industry, governments of several small countries tried to prevent exit of their national champions, e.g. the Greek government granted rescue aid to Olympic Airways and very recently the Commission approved a bailout of Cyprus Airways. These rescues were successful in the bailout, but were likely to keep prices high. The situation

could be different in case of the Italian government's subsidy for Alitalia. It is still not sure if the exit was prevented, but if Alitalia exits, welfare in both Italy and the whole EC may increase due to lower prices, which come about because other airlines restructure more in the presence of a subsidy, than they would in its absence.

Different behavior of asymmetric firms and countries can be a rationale for the EC control of subsidies. In our model, subsidies which actually succeed in the rescue decrease welfare of all consumers and total welfare. This can be a reason to ban them. On the other hand, subsidies which fail to rescue are welfare improving. Their aim is purely to distort free market, but to the advantage of consumers.

The model of course has its limitations. One of them is lack of dynamics, so that smaller dynamic incentives to invest caused by the subsidy cannot be observed. This effect (called soft budget constraints in the literature) will most likely show up in a dynamic setting and lead to further welfare reduction in case of successful rescues. Another limitation is that governments most likely have other objectives than welfare when rescuing a firm, e.g. jobs, reelection or bribes. These are just two promising ideas for the future work on rescue subsidies.

Appendix

Proof of Proposition 2.

2nd stage: production

In the second stage, levels of restructuring are already chosen and firms choose their preferred level of production. In a Nash equilibrium, each firm optimizes its profits assuming that output of the other firm is constant.

There are five potential market structures: firm 1 or 2 can be a blockaded monopoly, each can deter entry or finally both firms can produce in duopoly.

Suppose first that firm 2 is more efficient: $0.4 - e_2 < c_1 - e_1$. Monopoly of firm 2 emerges if the best-reply of firm 1 is not to enter, that is when the monopoly price is lower than marginal cost in firm 1:

$$1 - x_2^M < c_1 - e_1 \quad (47)$$

Since $x_2^M = \frac{1-(0.4-e_2)}{2}$, (47) reduces to

$$e_2 > 1.4 - 2(c_1 - e_1) = l_1(e_1) \quad (48)$$

Duopoly is the equilibrium if firm 2's duopoly profits are higher than entry-deterrence profits, which reduces to

$$e_2 < 0.65 - \frac{5}{4}(c_1 - e_1) = l_2(e_1) \quad (49)$$

Otherwise, entry-deterrence by firm 2 is the equilibrium.

Suppose now that firm 1 is more efficient: $e_2 < 0.4 - c_1 + e_1 = l_3(e_1)$. By symmetric reasoning, duopoly will emerge if

$$e_2 > 0.2 - \frac{4}{5}(c_1 - e_1) = l_4(e_1) \quad (50)$$

and monopoly if

$$e_2 < -0.1 - \frac{1}{2}(c_1 - e_1) = l_5(e_1) \quad (51)$$

However, l_5 is never binding since it is located outside of the rectangle $\mathcal{C} = (0, c_1) \times (0, c_2)$, in which restructuring efforts must be according to our assumptions. It is a consequence of fixing c_2 low enough.

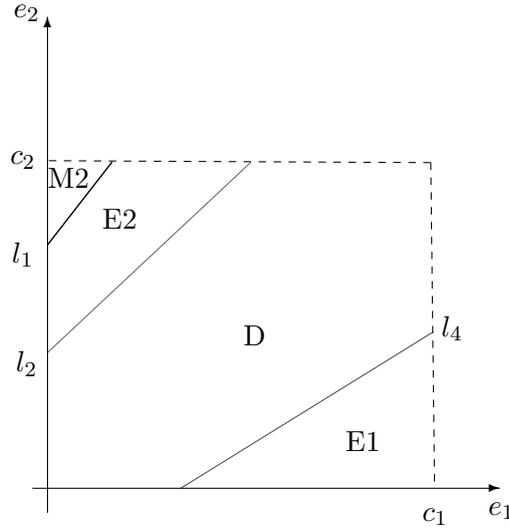


Figure 6: There are four market structures in the equilibrium of the second stage of the benchmark game.

In the M2 area firm 2 is the monopolist, in the E2 area firm 2 deters entry of firm 1, in the D area both firms compete and in the E1 area firm 1 deters entry of firm 2.

1st stage: restructuring

Knowing the equilibrium outcome of the second stage, firms choose restructuring levels: $e_i \in (0, c_i)$. Profits of firms are

$$\Pi_2 = \begin{cases} 0 & \text{if } (e_1, e_2) \in E1 \\ \frac{1}{9}[1 - 2(0.4 - e_2) + (c_1 - e_1)]^2 - \frac{5}{2}e_2^2 & \text{if } (e_1, e_2) \in D \\ [(c_1 - e_1) - (0.4 - e_2)](a - c_1 + e_1) - \frac{5}{2}e_2^2 & \text{if } (e_1, e_2) \in E2 \\ \frac{1}{4}[1 - (0.4 - e_2)]^2 - \frac{5}{2}e_2^2 & \text{if } (e_1, e_2) \in M2 \end{cases} \quad (52)$$

and symmetrically

$$\Pi_1 = \begin{cases} 0 & \text{if } (e_1, e_2) \in M2 \text{ or } E2 \\ \frac{1}{9}[1 - 2(c_1 - e_1) + (0.4 - e_2)]^2 - \frac{5}{2}e_1^2 & \text{if } (e_1, e_2) \in D \\ [(0.4 - e_2) - (c_1 - e_1)](1 - (0.4 - e_2)) - \frac{5}{2}e_1^2 & \text{if } (e_1, e_2) \in E1 \end{cases} \quad (53)$$

In order to find a Nash equilibrium, we first find reaction functions. Their location depends on the value of $c_1 \in (0.4, 1)$. We start with the locating best-reply in each market structure separately by solving first order conditions. This is sufficient, since for $d = 5$ all profit functions Π_i are concave in e_i . Then we compare profits across market structures for a given e_j , taking into account possible corner solutions, and in this way we identify reaction functions.

In monopoly, the best-reply function is independent of the rival's restructuring:

$$e_i^M = \frac{1 - c_i}{9} \quad \Pi_i^M = \frac{5(1 - c_i)^2}{18} \quad (54)$$

Monopoly of firm 2 is possible when area M2 overlaps the rectangle \mathcal{C} , like in the Figure 6. That happens when l_1 intersects $e_2 = 0.4$ for positive e_1 , which is equivalent to $c_1 > 0.5$. Best-reply e_2^M overlaps area M2 for $e_1 \in (0, c_1 - 0.6667)$. Otherwise, e_2^M is located below l_1 , so the best-reply in monopoly is a corner solution which for a given e_1 is closest to e_2^M and that is $e_2 = l_1(e_1)$.

In entry-deterrence, the best-reply function does depend on the rival's restructuring:

$$e_i^E = \frac{1 - (c_j - e_j)}{5} \quad \Pi_i^E = \frac{1 - (c_j - e_j)}{10} [1 - 10c_i + 9(c_j - e_j)] \quad (55)$$

e_2^E always cuts l_1 in the same point as e_2^M and it cuts l_2 for $e_1 = c_1 - 0.428571$. Therefore, for $e_1 > c_1 - 0.428571$, the highest profit in entry-deterrence brings a corner solution $e_2 = l_2$ and for $e_1 < c_1 - 0.66667$ a corner solution $e_2 = l_1$.

Finally, the best-reply function in duopoly:

$$e_i^D = \frac{4}{37} [1 - 2c_i + c_j - e_j] \quad \Pi_i^D = \frac{5}{37} [1 - 2c_i + c_j - e_j]^2 \quad (56)$$

e_2^D cuts l_2 for $e_1 = c_1 - 0.462687$ and for lower e_1 the highest duopoly profit available can be achieved by $e_2 = l_2$. e_2^D cuts l_4 for $e_1 = c_1 - 0.196429$ and for higher e_1 the highest duopoly profit can be achieved by $e_2 = l_4$.

When the rival deters entry or is the monopolist, best-reply is not to restructure at all, since it generates no revenue, but costs.

Finally we can find the reaction function of firm 2. We choose the best-reply which brings the highest payoff to firm 2 for a given e_1 . For $e_1 < c_1 - 0.6667$, which in figure 7 is denoted by letter A, it is necessarily e_2^M . Next, we compare π_2^E with monopoly profit on l_1 and we conclude that firm 2 always prefers to deter entry. Next, we compare optimal entry-deterrence profit π_2^E with optimal duopoly profit π_2^D . Duopoly is more profitable for $e_1 > c_1 - 0.446986$, denoted below as B. For $e_1 = c_1 - 0.196429$ (point C in figure 7), e_2^D cuts l_4 . For higher e_1 the highest duopoly profit for firm 2 is for $e_2 = l_4$. It is the best-reply if it is positive and that happens for e_1 between points C and D. Firm 2's reaction function is depicted in red in figure 7.

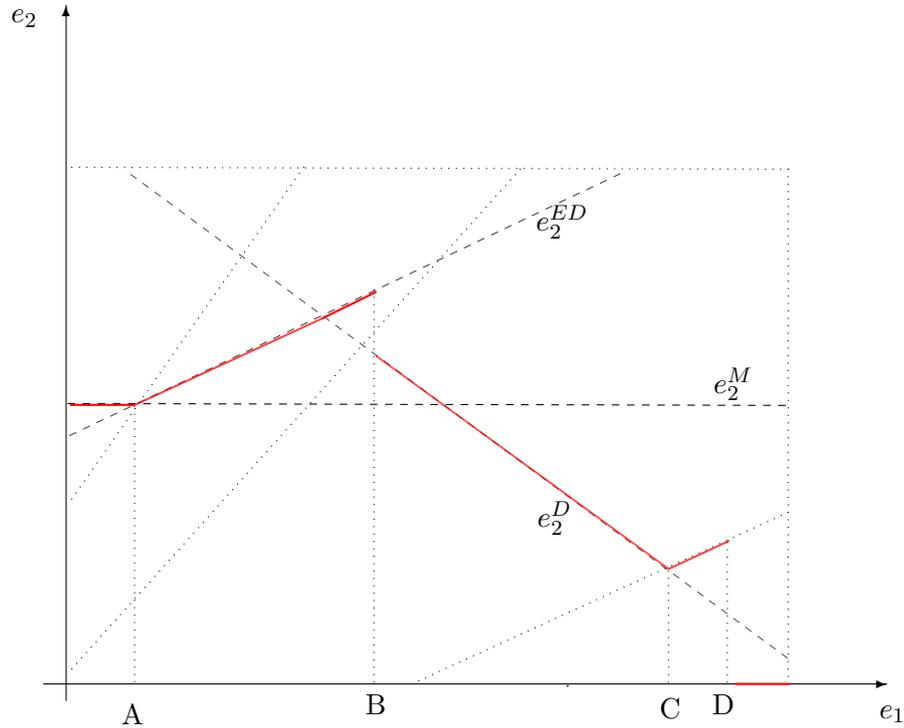


Figure 7: Reaction function of firm 2 is discontinuous.

We find firm 1's reaction function in a similar way. Monopoly is never possible here. e_1^E

never intersects area E1, so the best-reply in entry-deterrence is always a corner solution $e_1 = l_4^{-1}(e_2)$. In duopoly, the best-reply is e_1^D for $e_2 < 0.739286 - 1.33929c_1 = G$ and $l_2^{-1}(e_1)$ otherwise. E is positive when $c_1 < 0.552$.

In order to find firm 1's final reaction function we first compare entry-deterrence profit in the corner solution $e_1 = l_4^{-1}$ with optimal duopoly profits and we find that duopoly is always better. And finally, we check when duopoly profit is positive on l_2^{-1} . That happens for $e_2 < 0.88476 - 1.48476c_1 = H$, with H positive if $c_1 < 0.5958$. Otherwise, best-reply of firm 1 is $e_1 = 0$. The reaction function is depicted in red in Figure 8.

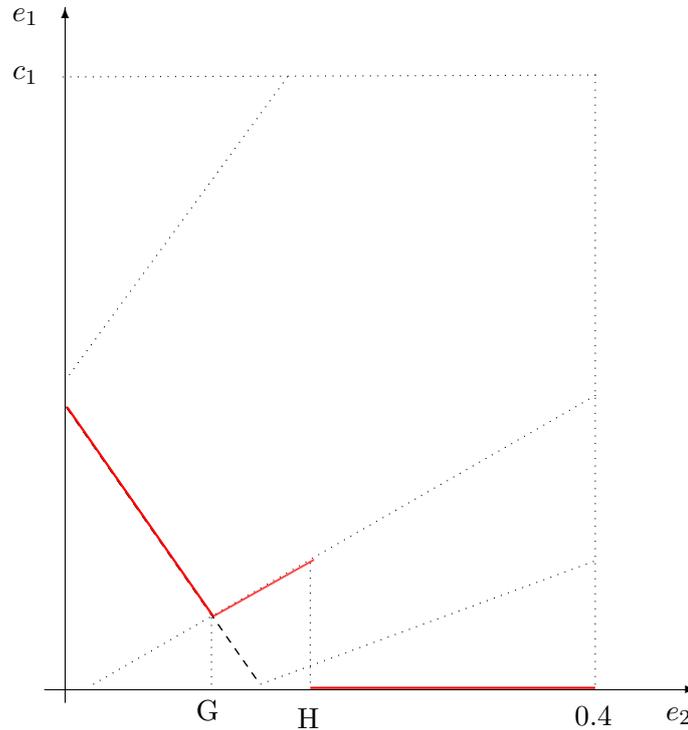


Figure 8: Reaction function of firm 1.

Since we now have the reaction functions, we can look for an equilibrium. If $c_1 > 0.5958$, H is negative and firm 1 always prefers not to restructure. If additionally $c_1 > 0.6667 = \bar{c}_1$, firm 2 is a monopolist, and for $c_1 \in (0.5958, 0.6667)$ it deters entry of firm 1.

For $c_1 \in (0.55, 0.5958)$ G is negative, but H positive. Firm 1 undertakes suboptimal duopoly restructuring or no restructuring. No restructuring reaction always cuts e_2^{ED} in this c_1 range, so the outcome here is also entry-deterrence. Down to $c_1 = 0.532987 = \tilde{c}_1$ the situation is the same, since then $H < e_2^{ED}(e_1 = 0)$ For $c_1 < 0.485749 = \underline{c}_1$, duopoly

best-replies intersect in D, so the outcome is duopoly. For $c_1 \in (\underline{c}_1, \tilde{c}_1)$ reaction functions do not intersect (intersect in the point of discontinuity, perhaps mixed strategy equilibrium there).

Proof of Proposition 4.

We first note that the last two stages of the game are the same as in the benchmark case, with the only difference that firm 1's initial marginal cost is $c_1 - k_1$. A positive k_1 moves reaction function e_2^D down, while pushing e_2^E and e_1^D up in the (e_1, e_2) space. The lines l_1 , l_2 and l_4 dividing the \mathcal{C} area are all pushed upwards, too:

$$l_{1s} = 1.4 - 2(c_1 - k_1 - e_1) \quad (57)$$

$$l_{2s}(e_1) = 0.65 - \frac{5}{4}(c_1 - k_1 - e_1) \quad (58)$$

$$l_{4s}(e_1) = 0.2 - \frac{4}{5}(c_1 - k_1 - e_1) \quad (59)$$

l_{5s} is still located outside of \mathcal{C} .

Best-reply of firm 2 does not change except for replacing c_1 with $c_1 - k_1$, compared with Proposition 2. Best-reply of firm 1 becomes more interesting, because now marginal cost $c_1 - k_1$ may be also lower than 0.4.

Monopoly of firm 1 is excluded by the low choice of c_2 , but entry-detering reaction of firm 1 is now possible. Using the technique from the previous proof, we find that e_1^{Es} is best-reply for firm 1 when $e_2 < 0.32169 - 0.92169(c_1 - k_1)$. For $0.32169 - 0.92169(c_1 - k_1) < e_2 < 0.739286 - 1.33929(c_1 - k_1)$ best-reply is e_1^{Ds} . For $0.739286 - 1.33929(c_1 - k_1) < e_2 < 0.884761 - 1.48476(c_1 - k_1)$, the best-reply is a part of the l_2 line, and for even higher e_2 it is best for firm 1 not to restructure at all.

The next step in the proof is to find the equilibrium in the second stage of the game, which is where the two best-replies cross. Analogously to the previous proof, we get monopoly of firm 2 when $c_1 - k_1 > 0.6667$ and entry-monopoly of firm 2 when $0.6667 > c_1 - k_1 > 0.532987$. Duopoly is the equilibrium when both duopoly reaction functions cross in the D area and this time e_1^{Ds} is longer than in the benchmark, so we get duopoly for $0.485749 > c_1 - k_1 > 0.299953$. Finally, we get entry-deterrence by firm 1 if $c_1 - k_1 < 0.229143$.

1st stage: subsidy

The first stage of the game is the choice of the optimal subsidy. Government 1 understands perfectly what will happen in the later two stages. Welfare in country 1 is a piecewise-defined function, depending on the value of k_1 . For some values of c_1 in the benchmark model pure strategy equilibrium does not exist and then we assume that welfare in country 1 is equal to zero. First we calculate welfare for each market structure separately.

When

$$c_1 - 0.229143 < k_1 < c_1 \quad (60)$$

reaction functions cut each other where firm 1 deters entry and firm 2 exits. Welfare in country 1 is

$$W_{1s}^{E1} = \pi_1^{E1} + \alpha CS^{E1} - \frac{5}{2}k_1^2 = 0.275999 - 0.6c_1 + 0.6k_1 - 2.5k_1^2 + 0.18\alpha \quad (61)$$

and it is maximized for

$$k_1^{E1} = 0.12 \quad (62)$$

which is independent of any parameter and fulfils condition (60) when

$$m_1 = 0.12 < c_1 < 0.349143 = m_2. \quad (63)$$

Since we assumed that $c_1 > 0.4$, we always have $k_1^{E1} < c_1 - 0.229143$. Therefore the government will always choose a corner solution $k_1^{E1c} = c_1 - 0.229143$, with welfare denoted by a star

$$W_1^{E1*} = 0.0072479 + 1.145715c_1 - 2.5c_1^2 + 0.18\alpha. \quad (64)$$

Duopoly is the outcome if

$$c_1 - \underline{c}_1 < k_1 < c_1 - 0.299953. \quad (65)$$

Welfare of country 1 is then equal to

$$\begin{aligned}
W_{1s}^D &= \pi_1^D + \alpha CS^D - \frac{5}{2}k_1^2 = & (66) \\
&= k_1^2(-1.885155 + 0.06692\alpha) + k_1[c_1(-1.229689 - 0.133849\alpha) + (0.80402 + 0.21416\alpha)] \\
&\quad + 0.171326\alpha + 0.262855 + c_1(-0.804027 - 0.214158\alpha) + c_1^2(0.614844 + 0.0669\alpha)
\end{aligned}$$

Optimal subsidy in this case is

$$k_1^D = \frac{-0.804028 + 1.22969c_1 - 0.214158\alpha + 0.133849c_1\alpha}{0.133849\alpha - 3.77031} \quad (67)$$

and it fulfils condition (65) when

$$m_3 = 0.38698 + 0.034802\alpha < c_1 < 0.52709 + 0.029828\alpha = m_4 \quad (68)$$

In this range the government can choose the peak of the welfare parabola and otherwise we have corner solutions. For $\alpha < 0.373931$ we have $m_3 < 0.4$, so is this range of α the lower boundary on c_1 is 0.4.

Entry-detering monopoly of firm 2 emerges when

$$c_1 - \bar{c}_1 < k_1 < c_1 - \tilde{c}_1. \quad (69)$$

Welfare in country 1 is then

$$W_{1s}^{E2} = -\frac{5}{2}k_1^2 + \frac{1}{2}\alpha(1 - c_1 + k_1)^2 \quad (70)$$

and optimal k_1 -the one which fulfils first order condition- is here

$$k_1^{E2} = \frac{\alpha(1 - c_1)}{5 - \alpha} \quad (71)$$

This optimal subsidy fulfils condition (69) when

$$m_5 = 0.532987 + 0.0934\alpha < c_1 < 0.6667 + 0.06666\alpha = m_6 \quad (72)$$

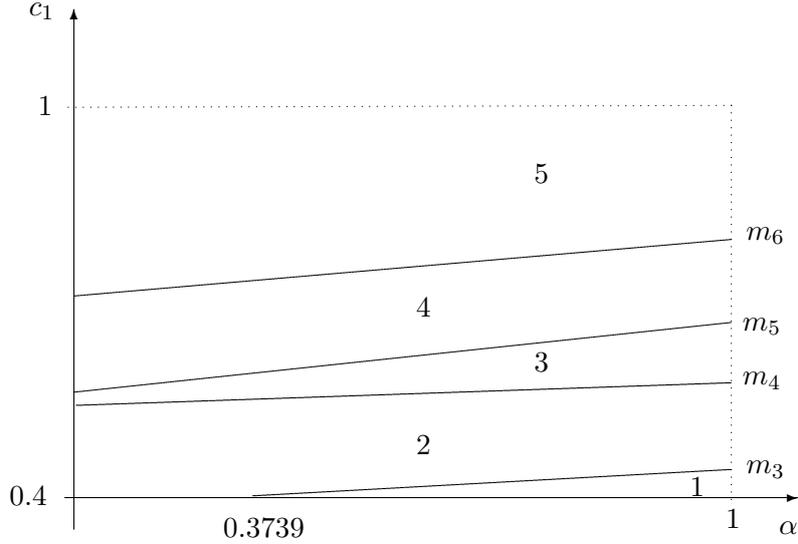


Figure 9: Government has 5 different cases to consider.

Again, in this range the peak of the welfare parabola and otherwise a corner solution is chosen.

Finally, $0 < k_1 < c_1 - \bar{c}_1$ equilibrium market structure is a blockaded monopoly with welfare $W_{1s}^{M2} = 0.0555556\alpha$ independent of k_1 .

The government chooses $k_1 \in [0, c_1]$ depending on α and c_1 . We plot the lines m_3, m_4, m_5, m_6 and $c_1 = 0.4$ in the (α, c_1) space and we notice that there are 5 cases to consider.

- The first case is for $0.4 < c_1 < m_3$, it is possible only for $\alpha > 0.3739$. Here, the government has a choice between W_1^{E1*} , $W_1^D(k_1 = c_1 - 0.299953)$, $W_1^{E2}(k_1 = c_1 - \tilde{c}_1)$ and monopoly welfare W_1^{M2} . Simple calculation shows that W_1^{E1*} is always the highest, so this is the optimal choice of the government.
- The second case is for $m_3 < c_1 < m_4$. Here, optimal duopoly choice is possible. The government compares W_1^{E1*} with $W_1^D(k_1^D)$, $W_1^{E2}(k_1 = c_1 - \tilde{c}_1)$ and W_1^{M2} and finds that W_1^{E1*} is the highest for $c_1 < \min[f_1(\alpha), f_6(\alpha)]$ where

$$f_1 = 0.33359 + 0.0366975\alpha - 0.024098\sqrt{(\alpha - 28.168411)(0.50916 + \alpha)}$$

$$f_6 = 0.47224 + 0.0467012\alpha$$

Duopoly generates highest welfare for $f_1(\alpha) < c_1 < \min[m_4(\alpha), f_6(\alpha)]$ and entry-

deterrence by firm 2 is the best for $\max[f_5, f_6] < c_1 < m_4$, where

$$f_5(\alpha) = 0.56271 + 0.02856\alpha - 1.38407\sqrt{\frac{(\alpha - 28.1684)^2(\alpha - 0.27334)}{153377 - 5445\alpha}} \quad \text{for } \alpha \in (0.27334, 1)$$

- The third case is for $m_4 < c_1 < m_5$. Here, the government chooses the highest of W_1^{E1*} , $W_1^D(k_1 = c_1 - \underline{c}_1)$, $W_1^{E2}(k_1 = c_1 - \tilde{c}_1)$ and W_1^{M2} . We check that $W_1^D(k_1 = c_1 - \underline{c}_1) < W_1^{E2}(k_1 = c_1 - \tilde{c}_1)$ when

$$c_1 > 0.5829 - 0.109912\alpha = f_2.$$

The line f_2 divides the area into two parts. To the right of it, the government will choose such k_1 which will lead to entry-deterrence by firm 2. To the left there is duopoly.

- The fourth case is for $m_5 < c_1 < m_6$. Here k_1^{E2} is available. The government compares W_1^{E1*} , $W_1^D(k_1 = c_1 - \underline{c}_1)$, $W_1^{E2}(k_1^{E2})$ and W_1^{M2} and chooses duopoly if

$$c_1 < 0.485749 + 0.10285\alpha + 0.062697\sqrt{(\alpha - 5)(\alpha - 0.35357)} = f_3 \quad \text{and} \quad \alpha < 0.35357$$

or monopoly of firm 2 if

$$c_1 > 1 - 0.14907\sqrt{5 - \alpha} = f_4$$

- The final case is for $m_6 < c_1 < 1$. Here the government chooses the highest of W_1^{E1*} , $W_1^D(k_1 = c_1 - \underline{c}_1)$, $W_1^{E2}(k_1 = c_1 - \bar{c}_1)$ and W_1^{M2} . For such a high c_1 , it is always most profitable not to subsidize and achieve W_1^{M2} .

Putting the choice of the government together, we get Proposition 4.

References

1. Aghion P., M. Schankerman (2004), On Welfare Effects and Political Economy of Competition Enhancing Policies, *Economic Journal* 114(127), 800-824.
2. Aiginger K., Pfaffermayr (1997), Looking at the Cost Side of "Monopoly", *Journal of International Economics* 45(3), 245-267.
3. D'Aspremont C., A. Jacquemin (1988), Cooperative and Noncooperative R&D in Duopoly with Spillovers, *American Economic Review* 78, 1133-1137.
4. Besley T., P. Seabright (1999), The effects and policy implications of state aids to industry: an economic analysis, *Economic Policy* 14(28), 13-42.
5. Brander J., B. Spencer (1983), Strategic Commitment with R&D: the symmetric case, *Bell Journal of Economics* 14, 225-235.
6. Brander J., B. Spencer (1985), Export Subsidies and International Market Share Rivalry, *Journal of International Economics* 18, 83-100.
7. Collie D. (2000), State Aid in the European Union: The Prohibition of Subsidies in an Integrated Market, *International Journal of Industrial Organization* 18(6), 867-884.
8. Collie D. (2002), Prohibiting State Aid in a Integrated Market: Cournot and Bertrand Oligopolies with Differentiated Products, *Journal of Industry, Competition and Trade* 2, 215-231.
9. Cournot A., *Researches into the Mathematical Principles of the Theory of Wealth*, Original: L. Hachette, Paris, 1838. English translation by Nathaniel T. Bacon, New York, The Macmillan Company, 1897; reprinted by Augustus M. Kelly, New York, 1971.
10. Dixit A., The Role of Investment in Entry-Deterrence, *The Economic Journal* 90, 95-106.
11. ESCRIHUELA-VILLAR (2004), Innovation and Market Concentration with Asymmetric Firms, *CFS Working Paper* No. 2004/03.

12. European Commission, Community Guidelines on State Aid for Rescuing and Restructuring Firms in Difficulty, *Official Journal of the European Union*, 2004/C 244/02.
13. European Commission, EU competition policy and the consumer, *Luxembourg: Office for Official Publications of the European Communities*, 2004.
14. Lahiri S., Y. Ono (1988), Helping minor firms reduces welfare, *The Economic Journal* 98, 1199-1202.
15. Lahiri S., Y. Ono (2004), Trade and Industrial Policy under International Oligopoly, *Cambridge University Press*.
16. Leahy D., C. Montagna (2001), Strategic Trade Policy with Heterogenous Costs, *Bulletin of Economic Research* 53:3, 3307-3378.
17. Leahy D., J. P. Neary (1997), Public Policy Towards R&D in Oligopolistic Industries, *American Economic Review* 87, 642-662.
18. Mollgaard P. (2004), Competitive Effects of State Aid in Oligopoly, *mimeo*.
19. Neary J. P. (1994), Cost asymmetries in international subsidy games: should governments help winners or losers?, *Journal of International Economics* 37, 197-218.
20. Röller L-H., B. Sinclair-Desgagne (1996), On the heterogeneity of firms, *European Economic Review* 40, 531-539.
21. Schumpeter J. (1942), *Capitalism, Socialism, and Democracy*, New York: Harper and Row.
22. Spence M. (1984), Cost Reduction, Competition and Industry Performance, *Econometrica* 52, 101-122.
23. Spencer B., J. Brander (1983), International R&D Rivalry and Industrial Strategy, *Review of Economic Studies* 50, 707-722.
24. Vickers J. (1995), Concepts of Competition, *Oxford Economic Papers* 47, 1-23.

25. Zhao J. (2001), A Characterization for the Negative Welfare Effects of Cost Reduction in Cournot Oligopoly, *International Journal of Industrial Organization* 19, 455-469.