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Auctioning Process Innovations when Losers' Bids Determine Royalty Rates*

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We consider a licensing mechanism for process innovations that combines a license auction with royalty contracts to those who lose the auction. Firms' bids are dual signals of their cost reductions: the winning bid signals the own cost reduction to rival oligopolists, whereas the losing bid influences the beliefs of the innovator who uses that information to set the royalty rate. We derive conditions for existence of a separating equilibrium, explain why a sufficiently high reserve price is essential for such an equilibrium, and show that the innovator generally benefits from the proposed mechanism.

KEYWORDS: Patents, licensing, auctions, royalty, innovation, R&D, mechanism design.
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1 Introduction

This paper revisits the analysis of license auctions of a non-drastic process innovation by an outside innovator to a Cournot oligopoly. The cost reductions induced by that innovation are of the private values type and are firms' private information. After the auction the innovator and firms update their prior beliefs based on observed bids. The main novel feature is that we replace the standard license auction by a superior mechanism that combines a standard license auction with a mandatory royalty contract for those who lose the auction. There, the innovator has two sources of revenue: the equilibrium price paid by the winner of the license auction, and the royalty income paid by those who lose the auction.

This licensing mechanism gives rise to a dual signaling problem: If a bidder wins the auction, his bid signals the own cost reduction to rival firms, whereas if he loses, his bid signals the own cost reduction to the innovator who sets the royalty rate equal to the expected cost reduction. Bidders take into account that they can influence others' beliefs with their bid. Specifically, a bidder gains a strategic advantage in the oligopoly game with an "inflated bid" that signals a higher than true cost reduction, provided this bid happens to win the auction. In turn, a bidder can fool the innovator to set the royalty rate below the true cost reduction with a "deflated bid", provided this bid happens to lose the auction.

Of course, no such "misleading" signaling occurs on the equilibrium path of a separating equilibrium. If a separating equilibrium exists, the marginal benefit of signaling a cost reduction that deviates from the true cost reduction must be matched by a corresponding marginal cost in such a way that both kinds of signaling are deterred in all states of the world.

If bids can only influence the beliefs of rival firms, misleading signals are easily deterred by choosing an appropriate steepness of the bid function. As a result, the possibility to influence the beliefs of rival firms with a winning bid, simply exerts an upward pressure on equilibrium bids.

Similarly, the possibility to influence the beliefs of the innovator with the losing bid exerts a downward pressure on equilibrium bids. However, one cannot deter bidders from signaling to the innovator a lower than true cost reduction by adjusting the steepness of the bid function alone. In addition, the innovator must set a sufficiently high reserve price. Without it, bidders with a low cost reduction would always signal a zero cost reduction, then they lose the auction, yet obtain the innovation for free.

In the face of the dual signaling problem, where bids can influence the beliefs of rival firms and of the innovator, a separating equilibrium bid function exists only in combination with a sufficiently high reserve price. Of course, a seller can typically increase his revenue by adding a reserve price requirement. However, in the present framework, the role of the reserve price is more fundamental, because without it, no separating equilibrium exists.

We analyze two specifications of the model that differ exclusively in the information available to firms in the downstream oligopoly game. In the first highly stylized specification, referred to as *model I*, firms' cost reductions become common knowledge among firms after the auction and before the oligopoly game is played. However, the innovator remains uninformed about cost reductions, and he can only update his prior beliefs after observing bids. The innovator uses this information to set royalty rates for losers, and firms use their knowledge about the information available to the innovator to predict the royalty rate to be paid by those who lose the auction.

In the second specification, referred to as *model II*, firms remain uninformed about their rivals' cost reductions after the auction. Like the innovator they can, however, update their prior beliefs from the observed bids. Firms use this updated information to assess the expected costs of their rivals, to predict the royalty rates the losers have to pay and to predict their rivals' beliefs about the royalty rate they themselves have to pay if they lose the auction.

Altogether, *model II* is more plausible. Nevertheless, *model I* is useful as a benchmark for comparison with the literature, and it prepares nicely the more complex analysis of *model II*.

There is large literature on patent licensing in oligopoly by an outside innovator, among which the following contributions are closely related to the present paper.

In their seminal contributions, Kamien (1992), Kamien and Tauman (1984, 1986), Katz and Shapiro (1985, 1986) show that auctioning a restricted number of licenses is strictly more profitable for the innovator than other mechanisms, such as pure royalty contracts, fixed-fee licensing, and two-part tariffs.[1] The limitation of the classical literature is that it assumes complete information both in the auction and in the subsequent oligopoly game.

Later Jehiel and Moldovanu (2000) introduce incomplete information at the bidding stage, combined with complete information in the oligopoly game, (which is the information structure to which we already referred as *model I*). They show that patent licensing under incomplete information is a prime example of an auction with negative externalities, where bidders' payoffs depend not only on their own types, but also on the type of the player who wins the auction.

In an auction with negative externalities, the theory of optimal mechanism design does not apply, which suggests a piecemeal approach to searching for "good" mechanisms. And, unlike in standard private value auctions, the reserve price plays a much less prominent role, since the seller in an auction that is subject to negative externalities has a stronger incentive to induce participation. As Jehiel and Moldovanu (2000, p. 777) put it succinctly: "when the seller sells more often, the buyers are more afraid that the good will fall into the hands of the competitor, and they bid more aggressively".

More recently, Das Varma (2003) and Goeree (2003) reconsider that model under the more plausible assumption that firms do not know each other's cost reductions after the auction and before the oligopoly game is played, which corresponds to the information structure in our *model II*. This introduces the possibility to signal the own cost reduction to rival firms.

Das Varma (2003) considers both Cournot and Bertrand competition (with product differentiation). He shows that a separating equilibrium exists under Cournot competition with linear demand, but generally fails to exist under Bertrand competition when goods are substitutes.

[1] Recently, Sen (2005) shows that if one takes into account that the number of licenses must be an integer, pure royalty contracts can be superior to license auctions. However, this result is again reversed if one generalizes the format of license auctions (see Giebe and Wolfstetter, 2008). See also Sen and Tauman (2007) who analyze an auction of royalty contracts.

Goeree (2003) assumes reduced form payoff functions of the oligopoly "subgames" that are generally satisfied for Cournot but not for Bertrand market games with substitutes. He compares all three standard auction formats – first-, second–price, and English auctions. Interestingly, he finds that the second–price auction is the most profitable auction form for the innovator, since the upward pressure on equilibrium bids due to the possibility of signaling to rival firms is the most dramatic there.

We follow this insight, and assume in our analysis that the innovator adopts a second-price auction. With this format the potential of signaling with the winning bid gives rise to the most dramatic upward pressure on equilibrium bids, and thus maximizes the auction revenue, while the potential to signal with the losing bid is not affected by the choice of auction format.

In a preceding paper, Giebe and Wolfstetter (2008) introduce an optional royalty scheme to the license auction under complete information, and show that such a mechanism is strictly more profitable than the standard license auction.[2] However, under incomplete information, adding royalty contracts for the losers of the auction works in a different way. First of all, one cannot grant royalty contracts as an option, but one must make them mandatory, second, adding royalty contracts for the losers leads to a reduction in auction revenue, and third, adding royalty contracts leads to a complex dual signaling problem where both the winning and the losing bids signal information to rival firms resp. to the innovator.

Our main findings can be summarized as follows: 1) The equilibrium bid function is strictly monotone increasing under fairly standard conditions concerning the probability distribution of firms' cost reductions, provided the innovator sets a sufficiently high reserve price. 2) The reserve price plays a crucial role in assuring existence of a separating equilibrium; without it, no separating equilibrium exists, unlike in the standard license auction where the reserve price plays a minor role . 3) Adding the royalty contract for losers adversely affects bidding, which lowers the innovator's revenue from the auction. 4) However, the additional royalty income weighs more than that loss in auction revenue, unless the probability distribution of cost reductions exhibits an extreme concentration on low values. Therefore, the proposed mechanism is generally more profitable. 5) Adding the royalty scheme has a stronger effect on equilibrium bids for low than for high cost reductions. Therefore, adding the royalty scheme is particularly profitable when the probability distribution exhibits a concentration on high cost reductions, since it entails a relatively small loss in auction revenue together with a high royalty income.

The paper is organized as follows: In Section 2 we present the model and introduce basic assumptions. In Section 3 we analyze model I where we also show in detail why the reserve price plays a crucial role in assuring that the first–order conditions of the equilibrium bidding problem yield global maxima. In Section 4 we analyze the more plausible model II which nicely complements and extends the analysis of model I. We find stronger results for model II, and show that the royalty scheme is even more profitable in model II. In Section 5 we discuss our results and sketch some extensions. Some of the proofs are contained in the Appendix.

[2] In a companion paper Fan, Jun, and Wolfstetter (2009) consider the licensing problem assuming firms draw imperfect signals of an unknown *common value* cost reduction.

2 The Model

An outsider innovator auctions the right to use a process innovation protected by a patent to a Cournot duopoly. Prior to the innovation, firms have the same constant unit cost c. The innovation reduces firms' unit cost by an amount that depends on who uses it. At the time when auction game is played, firms' cost reductions are their private information (private values case) and unknown both to their competitor and to the innovator.

The innovator employs the following licensing mechanism: one license is auctioned to the highest bidder in a second-price auction with the provision that the loser must accept a royalty contract. The winner of the auction can use the innovation at no further cost. The loser of the auction can also use the innovation, and has to pay a fixed royalty rate per output unit. The royalty rate is set to be equal to the loser's expected cost reduction, conditional on the information revealed to the innovator by his bid.

After the auction, the innovator publishes all bids. Having observed the bids the duopolists compete in a Cournot market game.

We consider two models that differ in the information available in the duopoly game: In *model I*, firms know each other's cost reductions after the auction and before the duopoly game is played, and in *model II* cost reductions remain private information, and firms can only update their beliefs about each other's cost reductions after observing their rival's bid. In both models, the innovator generally does not know firms' cost reductions, and can only infer cost reductions from observed bids.

Firms and the innovator are risk neutral and inverse market demand $P(Q)$ is a decreasing and concave function of aggregate output $Q := q_1 + q_2$.[3]

The firm specific cost reductions induced by the innovation, denoted by x, y are *iid* random variables, drawn from the c.d.f. $F : [0, c] \to [0, 1]$, with positive density f everywhere. Both F and the reliability function $1 - F$ are assumed to be log-concave which rules out that F has parts that are highly convex and highly concave. The log-concavity of $1 - F$ is equivalent to the well-known hazard rate monotonicity.[4]

We consider only non-drastic innovations whose exclusive use does not propel a monopoly. If $P(Q) = 1 - Q$, innovations are non-drastic if and only if $c \in [0, 1/2)$.

We denote the two highest order statistics of the sample of cost reductions by $X_{(1)}, X_{(2)}$, and the associated p.d.f. of $X_{(2)}$ by $g_2(x_2) = 2(1 - F(x_2)) f(x_2)$ and the joint p.d.f. of $X_{(1)}, X_{(2)}$ by $g_{12}(x_1, x_2) = 2 f(x_1) f(x_2)$. And we denote the equilibrium duopoly profits of the winner and the loser of the auction by π_W and π_L, respectively; and the equilibrium profit when both firms abstain from bidding by π_{nn}.

[3] This assures that the duopoly game has a unique equilibrium (see Szidarovszky and Yakowitz, 1977).

[4] A sufficient condition for the log-concavity of both F and $1 - F$ is the log-concavity of f. See Lemma A2 in Goeree and Offerman (2003) together with Theorem 1 in Bagnoli and Bergstrom (2005).

3 Model I (complete information in the duopoly game)

Following Jehiel and Moldovanu (2000) we first consider a highly stylized model in which firms learn about each other's cost reductions before they play the duopoly game. Jehiel and Moldovanu referred to the auctioning of a patent license as a prime example of auctions with negative externalities. Their main finding was that in the presence of a negative externality, using a reserve price becomes less attractive for the auctioneer.

Among other results we will show that the reserve price plays a crucial role if one replaces the simple license auction considered by Jehiel and Moldovanu (2000) by the generally superior mechanism proposed here.

3.1 Benchmark: The game without royalty contract for the loser

The auctioning of one license without royalty contract for the loser has already been analyzed in Jehiel and Moldovanu (2000, Sect. 4). We briefly review their results for the case of a duopoly, which serve as a benchmark.

Since only the winner of the auction has access to the innovation, the equilibrium profits in the duopoly subgames depend only on the cost reduction of the winner of the license auction, denoted by x. And the equilibrium profit of the winner, $\pi_W(x)$, is increasing and that of the loser, $\pi_L(x)$, is decreasing in x.

In a second-price auction with reserve price, R, the winner pays the reserve price if he is the only bidder and otherwise pays the second highest bid. The game has an equilibrium in weakly dominant strategies. There, bidders play cutoff strategies, and bid truthfully if and only if their cost reduction is at least as high as the cutoff value r, and do not bid otherwise. The equilibrium cutoff value is such that the marginal bidder, with a cost reduction equal to r, is indifferent between bidding and not bidding, i.e., $\pi_W(r) - R = \pi_{nn}$. This yields a unique relationship between r and R, that allows us to eliminate R, and compute the innovator's expected profit as a function of r.

The equilibrium bid function, β_n, and the innovator's expected revenue, $G_n(r)$ are (see Sect. 4, Jehiel and Moldovanu, 2000), where $R = \pi_W(r) - \pi_{nn}$,

$$\beta_n(x) = \pi_W(x) - \pi_L(x), \quad \text{for } x \geq r \tag{1}$$

$$G_n(r) = 2F(r)(1 - F(r))R + \int_r^c (\pi_W(x_2) - \pi_L(x_2))\, g_2(x_2) dx_2. \tag{2}$$

And the optimal cutoff value r_n solves the following condition:

$$\frac{1 - F(r)}{f(r)} = \frac{\pi_W(r) - \pi_{nn}}{\pi_W'(r)} + \frac{1 - F(r)}{F(r)} \frac{(\pi_{nn} - \pi_L(r))}{\pi_W'(r)}. \tag{3}$$

It is useful to compare this with a hypothetical world in which there is no negative externality in the sense that the loser's equilibrium duopoly profit is not affected by the winner's cost reduction.

In that hypothetical world, the equilibrium bidding strategy and the innovator's expected revenue would be equal to

$$\beta_0(x) = \pi_W(x) - \pi_{nn} \tag{4}$$

$$G_0(r) = 2\,(\pi_W(r) - \pi_{nn})\,F(r)\,(1 - F(r)) + \int_r^c (\pi_W(x_2) - \pi_{nn})\,g_2(x_2)dx_2, \tag{5}$$

yielding the optimal cutoff value r_0 that solves the equation,

$$\frac{1 - F(r)}{f(r)} = \frac{\pi_W(r) - \pi_{nn}}{\pi'_W(r)}. \tag{6}$$

Comparing (3) and (6) and using the assumed hazard rate monotonicity it follows immediately:

Proposition 1 (Jehiel and Moldovanu (2000))**.** *The optimal reserve price with negative externality is lower than that in a standard auction without negative externality: $r_n < r_0$.*

Essentially, in the presence of the negative externality, the innovator has an incentive to lower the reserve price, because a lower reserve price makes it more likely that both firms bid, and the winner pays $\pi_W(r) - \pi_L(r)$, which is more than the reserve price, $R = \pi_W(r) - \pi_{nn}$ that the winner pays if only one firm bids.

3.2 The game with royalty contract for the loser

For the moment, suppose cost reductions become common knowledge after the auction to firms *as well as* to the innovator. Then neither the duopoly nor the bidding game is affected by adding the royalty contract. And it follows immediately:

Proposition 2. *When costs reductions are common knowledge after the auction, adding the royalty scheme to the auction is profitable for the innovator and increases welfare.*

Adding the royalty scheme leaves the loser's cost unchanged, since the royalty rate is equal to his cost reduction. Therefore, neither the duopoly nor the bidding game is affected. The innovator then earns the same income in the auction, yet the royalty contract adds a positive royalty income. The equilibrium payoffs of the duopolists remain unchanged, but the innovator's equilibrium expected revenue increases; hence, welfare increases.

However, if cost reductions become common knowledge only among firms, while the innovator remains uninformed, the innovator can only update his prior beliefs about cost reductions from observed bids, and then sets the royalty rate equal to the inferred cost reduction of the loser. This, in turn, induces bidders to use their bids to influence the innovator's beliefs about their cost reduction and thus the royalty rate they have to pay in the event when they lose the auction.

We employ the following procedure to solve the game. As a working hypothesis we assume that the bidding game has a symmetric and monotone increasing equilibrium, $\beta : [r, c] \to \Re$ (which we will confirm), which allows the innovator to draw a perfect inference from observed bids to the underlying cost reductions and allows the winner of the auction to infer the royalty rate paid by the loser. We consider one bidder, say bidder 1 with cost reduction x, who assumes that his rival plays the strictly increasing equilibrium strategy β.

Without loss of generality we restrict deviating bids to $b \in [\beta(r), \beta(c)]$, because bidding outside that range is obviously dominated. Therefore, bidding according to the strategy β *as if* the cost reduction were equal to $z \in (r, c)$ captures all relevant deviating bids. We first solve all duopoly subgames that may occur if bidder 1 unilaterally deviates from the equilibrium bid while everyone (rival and innovator) believes that firms play the equilibrium bidding strategy β.

3.2.1 Downstream duopoly "subgames"

Suppose firm 1 has drawn the cost reduction x but bids as if it had drawn cost reduction $z \geq x$, while firm 2 has played the strictly increasing equilibrium bidding strategy.[5] In the continuation duopoly game, the following "subgames" occur, depending upon the true and pretended cost reductions of firm 1, x, z, and the cost reduction of firm 2, y.

When both firms bid and firm 1 has won the auction Let $z > y$ and $x, y \geq r$. The innovator infers that firm 2 has cost reduction y and charges a royalty rate equal to y. It is then common knowledge among firms that the profile of unit costs is $(c_1, c_2) = (c - x, c)$. Denote the Cournot equilibrium strategies for this cost profile by $(q_{W_1}(x), q_{L_2}(x))$. Therefore, the reduced form profit function of firm 1, conditional on winning the auction, is $\pi_W(x) := \big(P(q_{W_1}(x) + q_{L_2}(x)) - c + x\big)\, q_{W_1}(x)$.

When both firms bid and firm 1 has lost the auction Let $y > z$ and $x, y \geq r$. The innovator infers that the cost reduction of firm 1 is equal to z and then charges a royalty rate equal to z. It is then common knowledge among firms that the profile of unit costs is $(c_1, c_2) = (c - x + z, c - y)$. Denote the equilibrium strategies of that duopoly subgame by $(q_{L_1}(x, z, y)), q_{W_2}(x, z, y)$. Therefore, the reduced form profit function of firm 1, conditional on losing the auction, is $\pi_L(x, z, y) := \big(P(q_{W_2}(x, z, y) + q_{L_1}(x, z, y)) - c + x - z\big)\, q_{L_1}(x, z, y)$. On the equilibrium path, i.e., for $z = x$, the equilibrium strategy of firm 1 when it lost the auction and the associated reduced form payoff are only a function of firm 2's cost reduction, y; therefore, we write: $q_L^*(y) = q_{L_1}(x, z, y)\big|_{z=x}$ and $\pi_L^*(y) = \pi_L(x, z, y)|_{z=x}$. Similarly, we write $q_W^*(y) = q_{W_2}(x, z, y)\big|_{z=x}$.

When at least one firm did not bid If no one has made a bid, the game is just the default game without innovation; in this case the equilibrium profit of firm 1 is equal to π_{nn}. If firm 1 was the only bidder, its equilibrium profit is the same as in the event when both firms bid and firm 1 has won the auction, and if firm 2 was the only bidder, the equilibrium profit of firm 1 is the same as in the game without royalty scheme, and is exclusively a function of the winner's cost reduction, $\pi_L(y)$, as explained in section 3.1.

Lemma 1. *In the relevant duopoly subgames one has* $\partial_z \pi_L(x, z, y)|_{z=x} = -q_L^*(y)\gamma(y)$, *where*

$$\gamma(y) := 1 - \big(P'(q_{W_2}(\cdot) + q_{L_1}(\cdot))\partial_z q_{W_2}(x, z, y)\big)\big|_{z=x} > 1. \tag{7}$$

[5] The case of $z \leq x$ is slightly different, yet yields the same payoff function, $\Pi(x, z)$ and differential equation, and hence is omitted.

and $\pi'_W(x) > 0$, $\pi^{*'}_L(y) < 0$. If demand is linear, $\gamma(y) = 4/3$ (for a summary account of the linear model see Appendix A.5, A.6).

The proof is in Appendix A.1.

3.2.2 Auction "subgame"

Using the equilibria of the continuation duopoly subgames, firm 1's payoff function is

$$\begin{aligned}\Pi(x,z) = F(r)\,(\pi_W(x) - R) + \int_r^z (\pi_W(x) - \beta(y))\,dF(y) \\ + \int_z^c \pi_L(x,z,y)dF(y), \quad \text{where} \quad R = \pi_W(r) - \pi_{nn}.\end{aligned} \tag{8}$$

In equilibrium, the bid function β is such that it is the best reply of firm 1 to bid $\beta(x)$, rather than $\beta(z), z \neq x$. Therefore, β is an equilibrium if and only if $x = \arg\max_z \Pi(x,z)$, for all $x \in [r,c]$.

Proposition 3. *In equilibrium firms with $x \geq r$ bid according to the strictly increasing strategy, $\beta(x)$, provided the reserve price is sufficiently high, and abstain from bidding if $x < r$,*

$$\beta(x) = \pi_W(x) - \pi^*_L(x) + \frac{1}{f(x)} \int_x^c \partial_z \pi_L(x,x,y)dF(y) < \beta_n(x) \tag{9}$$

Proof. 1) Suppose $x \geq r$. Differentiating the expected payoff function of firm 1, $\Pi(x,z)$, (8), with respect to z, and then setting $z = x$, one obtains,

$$(\pi_W(x) - \beta(x))\,f(x) + \int_x^c \partial_z\, \pi_L(x,z,y)|_{z=x}\, dF(y) - \pi^*_L(x)f(x) = 0. \tag{10}$$

This implies the asserted bid function (9) and, by Lemma 1, $\beta(x) < \beta_n(x)$ for all $x \geq r$.

2) The proof of the asserted monotonicity of β is in Appendix A.2,

3) The above assumes that bidders play a cutoff strategy, and bid if and only if $x \geq r$. In Appendix A.3 we prove formally that the equilibrium strategy is indeed such a cutoff strategy.

4) Having established sufficient conditions for the monotonicity of the bid function one must also confirm that the underlying first-order conditions (10) yield a global maximum for each x. This is assured if the function $\Pi(x,z)$ is pseudoconcave in z, which is assured if and only if $\partial_{zx}\Pi(x,z) \geq 0$.[6] As we explain below, this "second-order condition" is satisfied only if the innovator adopts a sufficiently high reserve price. □

[6] A function of one variable is pseudoconcave if it is increasing to the left of the stationary point and decreasing to the right. Bidders' payoff function $\Pi(x,z)$ is pseudoconcave in z if $\partial_{zx}\Pi \geq 0$, for all x,z, since the sign of that cross derivative implies $z < x \Rightarrow \partial_z\Pi(x,z) > \partial_z\Pi(z,z) = 0$, $z > x \Rightarrow \partial_z\Pi(x,z) < \partial_z\Pi(x,x) = 0$. Pseudoconcavity obviously implies that every stationary point is a global maximum.

From the point of view of the innovator, adding the royalty scheme has an adverse effect on equilibrium bids. This signalling effect has the following interpretation. If the introduction of the royalty scheme did not affect the equilibrium strategy, each bidder would benefit from bidding below equilibrium in order to signal a cost reduction that is lower than true cost reduction. This incentive to signal can only be eliminated by pointwise lowering the equilibrium bid function and by introducing a sufficiently high reserve price. This indicates that the innovator faces a trade-off between income earned in the auction and royalty income earned from the duopoly game.

To see why a sufficiently high reserve price is needed, suppose no reserve price is used and hence $r = 0$, and assume *per absurdum* that β is the equilibrium bid function. Then, a bidder with a cost reduction $x > 0$ who bids $\beta(x)$ earns the payoff $\Pi(x, x)$, whereas if he deviates and bids $\beta(0)$ he earns $\Pi(x, 0)$, with

$$\begin{aligned}\Pi(x,0) - \Pi(x,x) = \int_0^c (\pi_L(x,0,y) - \pi_L(x,x,y))\,dF(y) + \int_0^x \pi_L(x,x,y)dF(y) \\ - \int_0^x (\pi_W(x) - \beta(y))dF(y).\end{aligned}$$

Evidently, $\pi_L(x, 0, y) > \pi_L(x, x, y)$ and for all $y < x$, $\pi_W(x) > \beta(x) > \beta(y)$. Therefore, if x is sufficiently small, it follows that $\Pi(x, 0) > \Pi(x, x)$, which contradicts the assumption that β is an equilibrium strategy.

Specifically,

Proposition 4. *If demand is linear, the smallest reserve price and the associated cutoff value $r_{\min}$ that assure that the second-order conditions are satisfied is implicitly defined as the unique solution of*

$$r - \frac{1 - F(r)}{f(r)} \frac{2}{3} = 0. \tag{11}$$

Proof. Substituting $\pi_W(x)$, R, $\beta(y)$ and $\pi_L(x, z, y)$ from Appendix A.5 into the payoff function of firm 1 (8), one can easily confirm that

$$\forall x, z \geq r : \partial_{zx}\Pi(x,z) = \frac{4}{3}zf(z) - \frac{8}{9}(1 - F(z)) \geq 0 \quad \iff \quad r \geq r_{\min}.$$

$\Pi(x, z)$ is pseudoconcave and hence has a global maximum at $z = x$ for all x if and only if $\partial_{zx}\Pi(x, z) \geq 0$ (see footnote 6), which is equivalent to the condition that $r \geq r_{\min}$. Finally, the existence and uniqueness of $r_{\min}$ follows from the fact that the LHS of (11) is negative at $r = 0$, positive at $r = c$, and strictly increasing in r by the hazard rate monotonicity. □

To illustrate the role of the reserve price, consider the following example which indicates why (9) is an equilibrium only if $r \geq r_{\min}$.

Example 1. *Let* $F(x) := {}^{x}/_{c}$ *(uniform distribution), in which case* $r_{\min} = {}^{2c}/_{5}$*, and suppose* $c = 0.49$*,* $x = 0.296$*. If* $r = 0$*, the stationary point* $\partial_z \Pi(x,z)|_{z=x} = 0$ *is a local but not a global maximum (see the left side of Figure 1). Therefore, the best reply is to bid* $\beta(0)$*, thus lose the auction yet obtain the innovation for free. Whereas if* $r = r_{\min}$*, that stationary point is a global maximum (see the right side of Figure 1).*

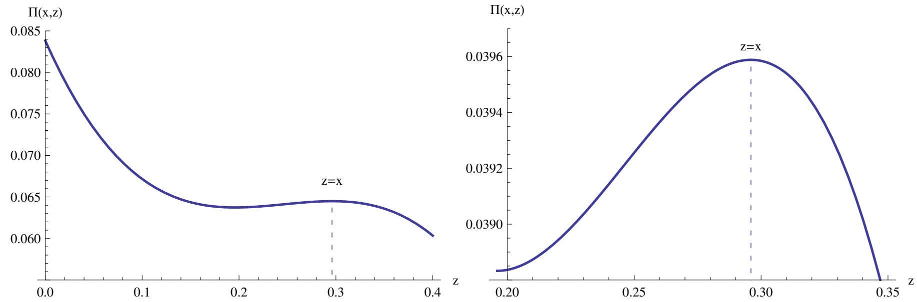

Figure 1: Left: Stationary point at $z = x$ is not a global maximum for $r = 0$; Right: Stationary point at $z = x$ is a global maximum for $r \geq r_{\min}$

The purpose of a sufficiently high reserve price is to deter firms with low cost reductions from bidding as if they had drawn no cost reduction, by making a bid equal to $\beta(0)$. Of course, the innovator could also live without a reserve price and an equilibrium that involves pooling at low levels of x. However, in that case he would earn neither royalty income nor income in the auction whenever at least one bidder has a low level of x. Whereas, if he adds the reserve price, he earns at least the reserve price from the firm who has a cost reduction $x \geq r$.

3.2.3 Is it profitable to add royalty contracts for the loser?

For the innovator, adding the royalty scheme has a benefit and a cost. The benefit is that he earns royalty income from the loser whenever both firms bid in the auction, without directly affecting the payoff of the winner in the downstream duopoly game, since, in equilibrium, the loser pays a royalty rate equal to his cost reduction. The cost is that the innovator needs to set a relatively high reserve price, which involves the risk that only one or even no firm bids, and that bidding is pointwise lower than without royalty scheme. Nevertheless, adding the royalty scheme is profitable for all standard probability distributions, even though one can construct examples in which it is not profitable. The latter occurs if the probability distribution exhibits a high concentration on low cost reductions.

The innovator's expected revenue in the game including royalties, $G(r)$, has three components: the collected reserve price, the expected payment from the winning bidder when both firms participate, and the royalty income from the loser:

$$G(r) = 2F(r)(1 - F(r))R + \int_r^c \beta(x_2) g_2(x_2) dx_2 + \int_r^c \int_r^{x_1} x_2 q_L^*(x_1) g_{12}(x_1, x_2) dx_2 dx_1.$$

Define $\Delta(r) := G(r) - G_n(r)$. Substituting $G_n(r)$ (see (2)), β, and $R = \pi_W(r) - \pi_{nn}$, one finds after changing the order of integration and a bit of rearranging:

$$\begin{aligned}\Delta(r) &= 2\int_r^c \int_{x_2}^c \left(x_2 q_L^*(x_1) f(x_2) + (1 - F(x_2))\, \partial_z \pi_L(x_2, z, x_1)|_{z=x_2}\right) dF(x_1) dx_2 \\ &= 2\int_r^c \int_{x_2}^c q_L^*(x_1) \left(x_2 - \frac{1 - F(x_2)}{f(x_2)} \gamma(x_1)\right) dF(x_1) dF(x_2), \quad \text{(by Lemma 1)}. \qquad (12)\end{aligned}$$

To obtain further results, we now assume that the demand function is linear. We denote the maximizers of G_n, G, and Δ by r_n, r_r, r_Δ, respectively, and the root of $\Delta(r)$ by $\hat{r}$.

Lemma 2. *Suppose demand is linear. Then, $r_\Delta > r_{\min}$ and $\Delta(r) > 0$ for all $r \in (\hat{r}, c)$.*

The proof is in Appendix A.4.

Proposition 5. *Suppose demand is linear. Adding the royalty scheme increases the innovator's expected revenue: $G(r_r) > G_n(r_n)$ if $r_n > \hat{r}$.*

Proof. The assumption that $r_n > \hat{r}$ implies $\Delta(r_n) > 0$ by Lemma 2. Therefore, $G(r_r) \geq G(r_n) \equiv G_n(r_n) + \Delta(r_n) > G_n(r_n)$. □

Whether r_n is either smaller or greater than $\hat{r}$ depends on the probability distribution F. We now state two examples in which $r_n > \hat{r}$, and then show how one must change the probability distribution to make the royalty scheme unprofitable for the innovator.

In Figure 2 we plot the innovator's expected revenue with and without royalty scheme, $G(r), G_n(r)$, for the case of the uniform distribution, $F : [0, c] \to [0, 1]$, $F(x) = x/c$, assuming $c = 0.49$. In this case, if the royalty scheme is adopted, the innovator must set a high reserve price resp. cut-off value $r \geq r_{\min} = 0.196$. Although this exposes the innovator to a high risk of not selling his innovation, altogether, adding the royalty scheme is profitable, since $G(r_r) > G(r_n)$.

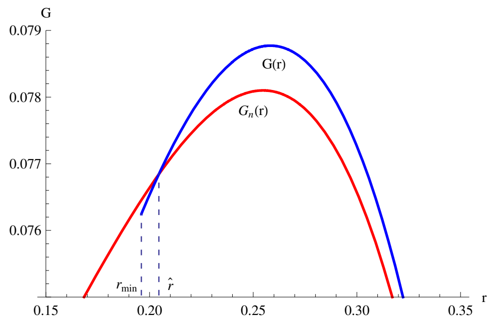

Figure 2: Uniform distribution

Adding the royalty scheme is profitable for many standard probability distributions. Starting from the uniform distribution, one finds that if probability mass is shifted to high cost reductions, the royalty scheme becomes even more profitable. However, if the probability distribution exhibits a sufficiently high concentration on low cost reductions, the royalty schemes becomes less profitable, and the revenue ranking can be reversed.

We illustrate this with truncated exponential distributions in Figures 3 and 4.[7] The probability distribution in Figure 3 exhibits a concentration on high cost reductions; whereas the probability distribution in Figure 4 exhibits a high concentration on low cost reductions.

Figures 3 and 4 plot the innovator's expected revenue for the corresponding probability distributions. As one skews the distribution towards high cost reductions, the royalty scheme becomes more profitable; however, as one concentrates probability mass on low cost reductions, as in Figure 4, the revenue ranking is reversed and it no longer pays to employ the royalty scheme.

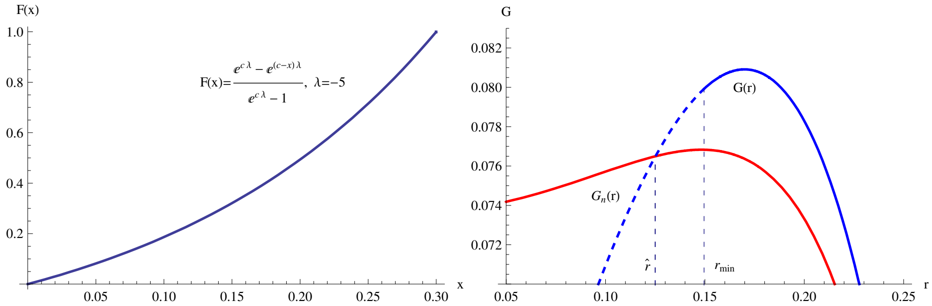

Figure 3: Truncated exponential distribution with a concentration on high cost reductions: cdf (left) and associated expected revenue of the innovator (right)

The intuition for these findings is as follows. We know from the analysis of the bid functions β and β_n that adding the royalty scheme has a stronger effect on equilibrium bids for low than for high cost reductions (see part 2 of the proof of Proposition 3). Also, equilibrium royalty rates are smaller for low than for high cost reductions. Therefore, if the probability distribution exhibits a high concentration on low cost reductions, adding the royalty scheme is not very appealing since it gives rise to a high loss in auction revenue together with a low expected royalty income. Whereas if the probability distribution has a high concentration on high cost reductions, adding the royalty scheme is appealing since it entails a relatively small loss in auction revenue combined with a high gain in royalty income.

4 Model II (incomplete information in the duopoly game)

We now turn to the more plausible model in which firms do not learn each other's cost reduction after the auction and before they play the duopoly game. In this case, bids not only signal firms' cost reductions to the innovator but also to their rival. Like in model I, each firm would like to signal weakness to the innovator, and make the innovator believe that their cost reduction is "low", since a low cost reduction translates into a low royalty rate in the event when they lose the auction. However, unlike in model I, each firm would also like to signal strength to its rival and make him believe that one's cost reduction is "high", since this will make the rival play less

[7] Both examples assume $c = 0.3$, and both distributions are consistent with our assumption of log-concave reliability functions.

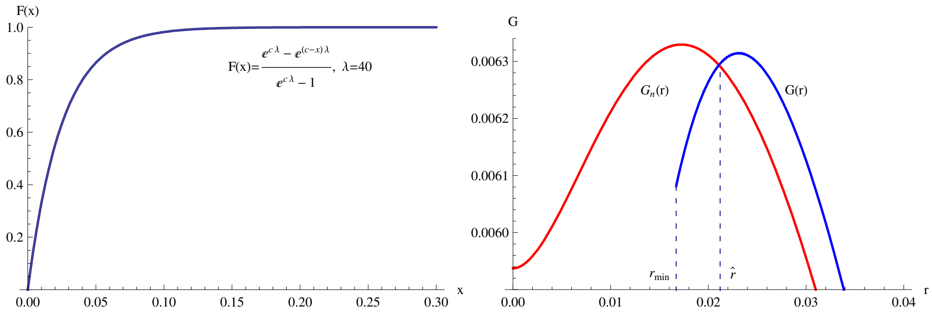

Figure 4: Truncated exponential distribution with a concentration on low cost reductions: cdf (left) and associated expected revenue of the innovator (right)

aggressively in the duopoly game. Both signaling considerations will affect equilibrium bids in the auction.

As we will show, model II yields more general results and altogether adding the royalty scheme is more profitable than in model I.

We solve the perfect equilibrium of the auction game followed by the incomplete information duopoly game. Thereby we employ the same methodology as in model I. Note, however, that unlike in model I, firms do not learn each other's cost reductions after the auction. Therefore, both the innovator and firms use the information revealed by observed bids to update their prior beliefs. The innovator uses this information to set the royalty rate for the loser, and firms use it to predict their rival's cost reduction and the royalty rate paid by the loser.

Since the game without royalty scheme corresponds to a game analyzed by Goeree (2003) and Das Varma (2003), we focus on the game with royalty scheme and only mention casually what changes if no royalty scheme is adopted.

4.1 Downstream duopoly "subgames"

Suppose firm 1 has drawn cost reduction x but bids as if it had drawn cost reduction $z \geq x$, while firm 2 has played the strictly monotone increasing equilibrium bidding strategy β.[8] In the continuation duopoly game, the following "subgames" occur, depending upon x,z and y.

4.1.1 When both firms bid and firm 1 has won the auction

Let $z > y$ and $x, y \geq r$. Firm 1 privately knows its cost reduction is x; whereas firm 2 (the loser) believes that the winner's cost reduction is equal to z. Therefore, firm 2 believes to play a duopoly subgame with the profile of unit costs $(c_1, c_2) = (c - z, c)$. Denote the associated equilibrium strategies of that game the loser believes to play by $(q_W(z), q_{L_2}(z))$.

[8]Like in model I, the case of $z \leq x$ is slightly different, yet yields the same payoff function, $\Pi(x, z)$, and differential equation, and hence is omitted.

Firm 1 anticipates that the loser plays $q_{L_2}(z)$. But since firm 1 privately knows that its cost reduction is equal to x rather than z it plays the best reply:

$$q_{W_1}(x,z) = \arg\max_q \big(P(q + q_{L_2}(z)) - c + x\big)\, q. \tag{13}$$

The reduced form profit function of firm 1, conditional on winning the auction, is $\pi_W(x,z) := \big(P(q_{W_1}(x,z) + q_{L_2}(z)) - c + x\big)\, q_{W_1}(x,z)$.

4.1.2 When both firms bid and firm 1 has lost the auction

Let $y > z$ and $x, y \geq r$. Firm 2 believes to play a Cournot duopoly subgame with unit costs $(c_1, c_2) = (c, c - y)$. Denote the associated equilibrium strategies of the game that firm 2 (the winner) believes to play by $(q_L(y), q_{W_2}(y))$.

If the royalty scheme is adopted, firm 1 privately knows that its cost reduction is x yet pays a royalty rate z that exceeds its cost reduction x. Therefore, firm 1 plays the following best reply to $q_{W_2}(y)$:

$$q_{L_1}(x,z,y) = \arg\max_q \big(P(q + q_{W_2}(y)) - c + x - z\big)\, q. \tag{14}$$

The associated reduced form profit function of firm 1 conditional on losing the auction is then $\pi_L(x,z,y) := \big(P(q_{W_2}(y) + q_{L_1}(x,z,y)) - c + x - z\big)\, q_{L_1}(x,z,y)$. Also note that on the equilibrium path, for $z = x$, that payoff is only a function of the cost reduction of firm 2, y; therefore, we write $q_L^*(y) = q_L(x,z,y)|_{z=x}$ and $\pi_L^*(y) = \pi_L(x,z,y)|_{z=x}$.

Whereas if no royalty scheme is used, the equilibrium play of firm 1 depends only on its rival's cost reduction and therefore is independent of x and z. Hence, in this case, the reduced form profit function of firm 1 conditional on losing the auction is $\pi_L(y) := \big(P(q_{W_2}(y) + q_L(y)) - c\big)\, q_L(y)$.

We stress that in model II the equilibrium strategy that firm 2 plays in the event when firm 1 lost the auction, q_{W_2}, is only a function of its own cost reduction, y, whereas in model I it is a function of x, z, and y. This is due to the fact that in model II both firm 2 and the innovator believe that the cost reduction of firm 1 is equal to z, and therefore firm 2 believes that the effective unit costs are equal to $c_1 = c - z + z = c$, and $c_2 = c - y$. Whereas in model I, the innovator believes that the cost reduction of firm 1 is equal to z but firm 2 knows that it is equal to x; therefore, firm 2 believes that the effective unit costs are equal to $c_1 = c - x + z$, and $c_2 = c - y$.

4.1.3 When at least one firm did not bid

If no one has made a bid, the game is just the default game without innovation; in this case the equilibrium profit of firm 1 is equal to π_{nn}. If firm 1 was the only bidder, its equilibrium profit is the same as in subgame 4.1.1, and if firm 2 was the only bidder, the equilibrium profit of firm 1 is the same as in the game without royalty scheme, and is exclusively a function of the winner's cost reduction, $\pi_L(y)$, as explained in section 4.1.2.

Lemma 3. *In the relevant duopoly subgames one has: 1)* $\partial_z \pi_L(x,z,y)|_{z=x} = -q_L^*(y)$*, and 2)* $\frac{d}{dx}\pi_W(x,x) > 0$*,* $\pi_L^{*'}(y) < 0$*.*

Proof. The proof of part 1) is the same as the proof of Lemma 1 (spelled out in Appendix A.1) if one takes into account that in model II one has $\partial_z q_{W_2}(\cdot) = 0$ and therefore $\gamma(y) = 1$, see (7). Part 2) follows from the envelope theorem:

$$\begin{aligned}
\frac{d}{dx}\pi_W(x,x) &= \Big(P'(\cdot)\partial_x q_{L_2}(z) q_{W_1}(x,z) + q_{W_1}(x,z) + P'(\cdot)\partial_z q_{L_2}(z) q_{W_1}(x,z)\Big)\Big|_{z=x} \\
&= \Big(q_{W_1}(x,z) + P'(\cdot)\partial_z q_{L_2}(z) q_{W_1}(x,z)\Big)\Big|_{z=x} \qquad (\text{since } \partial_x q_{L_2}(z) = 0) \\
&> 0 \quad (\text{since } P'(\cdot) < 0 \text{ and } \partial_z q_{L_2}(z) < 0) \\
\pi_L^{*'}(y) &= P'(\cdot)\partial_y q_{W_2}(y) q_{L_1}(x,z,y) < 0 \quad (\text{since } \partial_y q_{W_2}(y) > 0)\,.
\end{aligned}$$

□

4.2 Auction "subgame"

Suppose firm 1 has drawn the cost reduction x but bids as if its cost reduction were equal to $z \geq x$. Assume that firm 2 plays the strictly monotone increasing equilibrium strategy $\beta(y)$. Then, the payoff function of firm 1 in the mechanism with royalty scheme is

$$\Pi(x,z) = F(r)(\pi_W(x,z) - R) + \int_r^z (\pi_W(x,z) - \beta(y))\, dF(y) + \int_z^c \pi_L(x,z,y) dF(y),$$

where similar to model I, the following relationship applies to the reserve price R and the cutoff value r:

$$R = \pi_W(r,r) - \pi_{nn}.$$

In the mechanism without royalty scheme, the payoff function is the same, except that in the last term $\pi_L(x,z,y)$ is replaced by $\pi_L(y)$, as explained in the above analysis of the relevant duopoly subgames.

Proposition 6. *The equilibrium bidding strategies with resp. without royalty scheme, $\beta(x)$, $\beta_n(x)$, are, for all $x \geq r$:*

$$\beta(x) = \beta_n(x) + \frac{1}{f(x)} \int_x^c \partial_z \pi_L(x,z,y)|_{z=x}\, dF(y) \tag{15}$$

$$\beta_n(x) = \pi_W(x,x) - \pi_L^*(x) + \frac{F(x)}{f(x)}\, \partial_z \pi_W(x,z)|_{z=x} \tag{16}$$

Whereas β is strictly monotone increasing only if r is sufficiently large, β_n is strictly monotone increasing for all r.

Proof. For the derivation of β_n see Goeree (2003, Proposition 2). To proof the monotonicity of β_n, compute the derivative of β_n. This derivative has two parts. The first part is positive, $\big(\pi_W(x,x) - \pi_L^*(x)\big)' > 0$, by Lemma 3. The second part, which is equal to $d\big(\frac{F(x)}{f(x)}\,\partial_z \pi_W(x,z)|_{z=x}\big)/dx$, is also positive by the assumption that F is log-concave.

The derivation of β is similar to that in model I. To prove monotonicity of β, it is sufficient to show that $\beta(x) - \beta_n(x)$ is strictly monotone increasing, i.e. $(\beta(x) - \beta_n(x))'$, is also positive, as we confirm below:

$$
\begin{aligned}
&\frac{d}{dx}\left(\frac{1}{f(x)}\int_x^c \partial_z \pi_L(x,x,y)dF(y)\right) \\
&\quad = \frac{1}{f(x)}\int_x^c \left(\partial_{zx}\pi_L(x,x,y) + \partial_{zz}\pi_L(x,x,y)\right)dF(y) \\
&\qquad - \partial_z\pi_L(x,x,x) - \frac{f'(x)}{f(x)^2}\int_x^c \partial_z\pi_L(x,x,y)dF(y) \\
&\quad = -\partial_z\pi_L(x,x,x) - \frac{f'(x)}{f(x)^2}\int_x^c \partial_z\pi_L(x,x,y)dF(y) \quad \text{(step a)} \\
&\quad > -\partial_z\pi_L(x,x,x) + \frac{1}{1-F(x)}\int_x^c \partial_z\pi_L(x,x,y)dF(y) \quad \text{(step b)} \\
&\quad > -\partial_z\pi_L(x,x,x) + \frac{1}{1-F(x)}\int_x^c \partial_z\pi_L(x,x,x)dF(y) \quad \text{(step c)} \\
&\quad = -\partial_z\pi_L(x,x,x) + \partial_z\pi_L(x,x,x) = 0.
\end{aligned}
$$

The different steps in this assessment are explained as follows: step a) follows from the facts that $\partial_{zx}\pi_L(x,x,y) = -\partial_x q_L^*(y) = 0$ and $\partial_{zz}\pi_L(x,x,y) = -\partial_z q_L^*(y) = 0$ (see Lemma 3); step b) follows from the assumed log-concavity of the reliability function, which implies that $f'(x) > -f(x)^2/(1-F(x))$, together with the fact that $\partial_z\pi_L(x,x,y) < 0$; step c) follows from the fact that $\partial_z\pi_L(x,x,y) = -q_L^*(y)$ (by Lemma 3), which is monotone increasing in y. Therefore, the proof of monotonicity of β applies to all concave inverse demand functions. Also note that $\beta(x) < \beta_n(x)$ by the fact that $\partial_z\pi_L(x,x,y) = -q_L^*(y) < 0$.

The role of the reserve price, resp. r, to assure that the second-order conditions are satisfied, is similar to model I. □

Proposition 7. *The introduction of the royalty scheme reduces equilibrium bids pointwise by a smaller amount than in model I.*

Proof. Distinguish the equilibrium bid functions in models I resp. II by writing β_n^I, β^I, resp. β_n^{II}, β^{II} and define $\Delta\beta^I := \beta_n^I(x) - \beta^I(x)$, resp. $\Delta\beta^{II} := \beta_n^{II}(x) - \beta^{II}(x)$. Recall that due to $\gamma(y) > 1$:

$$\partial_z \pi_L^I(x,z,y)\big|_{z=x} = -q_L^*(y)\gamma(y) < -q_L^*(y) \quad \text{(model I)} \tag{17}$$

$$\partial_z \pi_L^{II}(x,z,y)\big|_{z=x} = -q_L^*(y) \quad \text{(model II)} \tag{18}$$

Therefore, for all x from the intersection of the domains of these bid functions,

$$
\begin{aligned}
\Delta\beta^I &= -\frac{1}{f(x)}\int_x^c \partial_z\pi_L^I(x,z,y)\big|_{z=x} dF(y) \\
&= \frac{1}{f(x)}\int_x^c \gamma(y) q_L^*(y) dF(y) \quad \text{(by Lemma 1)}
\end{aligned}
$$

$$
\begin{aligned}
&> \frac{1}{f(x)}\int_x^c q_L^*(y)dF(y) \quad \text{(since } \gamma(y) > 1) \\
&= \Delta\beta^{II} > 0.
\end{aligned}
$$

□

The intuition for this result is as follows. Consider the equilibrium bid function in model II without royalty scheme, which by definition of an equilibrium exhibits equality of the marginal benefit and the marginal cost of an incremental change in the bid from $\beta_n^{II}(x)$ to $\beta_n^{II}(x+\varepsilon)$.[9] Now introduce the royalty scheme, while maintaining the candidate equilibrium bid function. Then, the marginal benefit of bidding higher does not change while it gives rise to a positive marginal cost, due to the fact that when the bidder loses the auction, he pays a royalty rate that exceed his cost reduction. This is also true in model I. However, whereas in model II, the rival firm believes that the effective cost is equal to c, in model I the rival knows that the royalty rate exceeds the cost reduction so that the effective cost is higher than c. To reestablish an equilibrium, the bid function has to be lowered pointwise, to bring the unchanged marginal benefit in balance with the higher marginal cost. But since that marginal cost is higher in model I than in model II, the bid function has to be lowered more in model I than in model II.

4.3 The innovator's expected revenue

The above result suggests that adding the royalty scheme is more profitable in model II than in model I, since it has a smaller adverse effect on the equilibrium bid and thus on the innovator's auction revenue.

The expected revenue of the innovator in the mechanism with resp. without royalty scheme, $G(r)$, $G_n(r)$, is

$$
\begin{aligned}
G_n(r) &= 2\,(1-F(r))\,F(r)R + \int_r^c \beta_n(x_2)g_2(x_2)dx_2 \\
G(r) &= 2(1-F(r))F(r)R + \int_r^c \beta(x_2)g_2(x_2)dx_2 + \int_r^c \left(\int_r^{x_1} x_2 q_L^*(x_1)g_{12}(x_1,x_2)dx_2\right)dx_1 \\
&= G_n(r) + \int_r^c (\beta(x_2)-\beta_n(x_2))g_2(x_2)dx_2 + \int_r^c \left(\int_r^{x_1} x_2 q_L^*(x_1)g_{12}(x_1,x_2)dx_2\right)dx_1.
\end{aligned}
$$

Define $\Delta(r) := G(r) - G_n(r)$. After substituting β and R and using the fact that $\partial_z \pi_L(x_2,x_2,x_1) = -q_L^*(x_1)$, by Lemma 3, one obtains,

$$
\Delta(r) = 2\int_r^c \int_{x_2}^c q_L^*(x_1)\left(x_2 - \frac{1-F(x_2)}{f(x_2)}\right)dF(x_1)dF(x_2),
$$

Proposition 8. *The introduction of the royalty scheme increases the innovator' expected revenue more than in model I, for all r from the intersection of the domains of these functions.*

[9] The benefit is the expected revenue from winning the auction, net after deducting the expected price, and the cost is the loss in the event of losing the auction and paying a royalty rate that exceed the own cost reduction.

Proof. Distinguish Δ in model I resp. II by writing $\Delta^I(r), \Delta^{II}(r)$. Using (17), (18), and the fact that $\gamma(y) > 1$ in model I, one finds for all x from the intersection of the domains of these functions,

$$\begin{aligned}\Delta^I(r) &= 2\int_r^c \int_{x_2}^c q_L^*(x_1)\left(x_2 - \frac{1-F(x_2)}{f(x_2)}\gamma(x_1)\right) dF(x_1)dF(x_2) \\ &< 2\int_r^c \int_{x_2}^c q_L^*(x_1)\left(x_2 - \frac{1-F(x_2)}{f(x_2)}\right) dF(x_1)dF(x_2) = \Delta^{II}(r).\end{aligned}$$

□

As in model I, define $\hat{r}$ as the root of $\Delta(r)$, and r_r, r_n, r_Δ as the maximizers of G, G_n, Δ, respectively. The following results hold for all concave inverse demand functions (unlike in model I where the corresponding results hold only for a class of concave demand functions).

Lemma 4. $\Delta(r) > 0$ *for all* $r \in (\hat{r}, c)$.

Proof. Let $\varphi(r) := r - {}^{(1-F(r))}/_{f(r)}$. It is straightforward to see that r_Δ is the unique solution of $\varphi(r) = 0$. The remainder of the proof is the same as the proof of Lemma 2. (Note, however, that the functions φ differ in models I and II.) □

Proposition 9. *Adding the royalty scheme increases the innovator's expected revenue,* $G(r_r) > G_n(r_n)$ *if* $r_n > \hat{r}$.

Proof. The assumption that $r_n > \hat{r}$ implies $\Delta(r_n) > 0$ by Lemma 4. Therefore, $G(r_r) \geq G(r_n) \equiv G_n(r_n) + \Delta(r_n) > G_n(r_n)$. □

For more specific results turn to the case of linear demand, which was assumed in model I. In that case we find that adding the royalty scheme is profitable for each of the probability distributions considered in model I, even for the distribution that exhibits a high concentration on low cost reductions. This fact is illustrated in Figures 5. There, the figure on the left corresponds to the probability distribution plotted in Figure 3, and the figure on the right corresponds to the distribution plotted in Figure 4. Evidently, and unlike in model I, adding the royalty scheme is profitable even for the probability distribution that exhibits a high concentration on low cost reductions.

These examples illustrate that adding the royalty scheme is more profitable in model II than in model I, and is particularly appealing if the probability distribution exhibits a high concentration on high cost reductions, as already explained intuitively at the end of section 3.

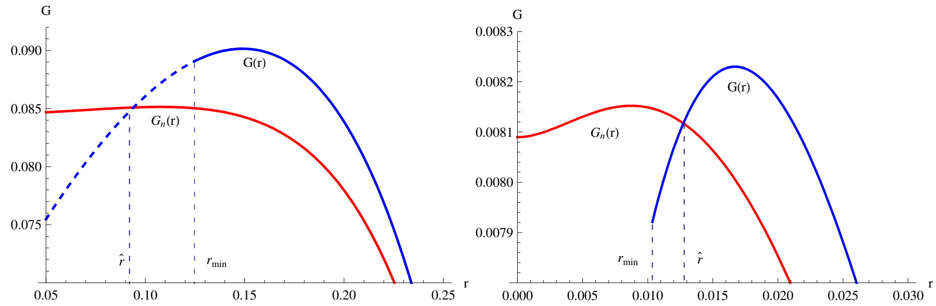

Figure 5: Expected revenue for truncated exponential distributions with a concentration on high cost reductions (left) and on low cost reductions (right)

5 Discussion

In the present paper we reconsider the licensing of a process innovation to a Cournot duopoly under incomplete information assuming the private values paradigm. Unlike the previous literature, we assume that the innovator combines a restrictive license auction with a mandatory royalty contracts for the firm who loses the auction. We consider two specifications of the model: one in which the cost reductions become common knowledge among firms after the auction and bids only serve as signals of the underlying cost reduction to the innovator and one in which cost reductions remain private information and both the innovator and firms can only update their beliefs using the information revealed by bids. In both models, the innovator uses the information revealed by bids to set the royalty rate to be paid by the loser of the auction. Our main finding is that adding the royalty scheme to the license auction adversely affects equilibrium bidding in the auction, yet is generally profitable unless cost reductions are highly concentrated on low values, and is more profitable in model II than in model I.

The limitation of the present paper is that we only consider the case of two firms. If the oligopoly consists of more than two firms it remains attractive to add the royalty scheme to the license auction. However, it becomes also an issue how many licenses should be auctioned. This issue has been at center stage in the classical literature on patent licensing under complete information (see Kamien, 1992, Giebe and Wolfstetter, 2008), but has not been addressed as yet in the framework of incomplete information.

Another concern is whether the private value paradigm is appropriate to analyze the cost reductions for firms that serve the same market and employ similar technologies. If instead one assumes a common value framework, each firm's expected cost reduction is a function of the signals observed by all firms, and so is the royalty rate set by the innovator, unless the largest signal is a sufficient statistic of the unknown cost reduction, in which case only the largest signal matters (see Fan, Jun, and Wolfstetter, 2009).

A Appendix

A.1 Proof of Lemma 1

Proof. We show that $\partial_z \pi_L(x, z, y)|_{z=x} = -q_L^*(y)\gamma(y)$, where $\gamma(y) > 1$ for all y.

By the envelope theorem we have

$$\begin{aligned}
\partial_z \pi_L(x,z,y)|_{z=x} &= -q_{L_1}(x,z,y)\Big(1 - P'(q_{W_2}(\cdot)+q_{L_1}(\cdot))\partial_z q_{W_2}(x,z,y)\Big)\Big|_{z=x} \\
&= -q_L^*(y)\gamma(y) \\
\text{where} \quad \gamma(y) :&= 1 - \Big(P'(q_{W_2}(\cdot)+q_{L_1}(\cdot))\partial_z q_{W_2}(x,z,y)\Big)\Big|_{z=x} \\
&> 1 \quad (\text{since } P' < 0 \text{ and } \partial_z q_{W_2}(x,z,y)\big|_{z=x} > 0).
\end{aligned}$$

Note, both $P'(\cdot)$ and $\partial_z q_{W_2}(x,z,y)\big|_{z=x}$ are only functions of y.

If demand is linear, $P'(\cdot) = -1$, $\partial_z q_{W_2}(x,z,y)\big|_{z=x} = 1/3$; hence, $\gamma(y) = 4/3$ (see Appendix A.5).

Next we show that $\pi_W'(x) > 0$ and $\pi_L^{*'}(y) < 0$. Again, using the envelope theorem and the fact that $q_{L_2}'(x) < 0$ and $q_{W_2}^{*'}(y) > 0$ one has:

$$\begin{aligned}
\pi_W'(x) &= P'(\cdot)q_{L_2}'(x)q_{W_1}(x) + q_{W_1}(x) > 0 \\
\pi_L^{*'}(y) &= P'(\cdot)q_{W_2}^{*'}(y)q_L^*(y) < 0.
\end{aligned}$$

□

A.2 Part 2 of the proof of Proposition 3

Compute $\beta'(x)$ from (9). By Lemma 1 and the assumed log-concavity of the reliability function, which implies that $f'(x) > -f(x)^2/(1-F(x))$, this derivative can be written as

$$\begin{aligned}
\beta'(x) &= (\pi_W'(x) - \pi_L^{*\prime}(x)) + \frac{d}{dx}\left(\frac{1}{f(x)}\int_x^c \partial_z \pi_L(x,x,y)dF(y)\right) \\
&> (\pi_W'(x) - \pi_L^{*\prime}(x)) + q_L^*(x)\gamma(x) - \frac{1}{1-F(x)}\int_x^c q_L^*(y)\gamma(y)dF(y).
\end{aligned}$$

There, $\pi_W(x)$ and $\pi_L^*(x)$ are the equilibrium profits in the Cournot subgames when the winning firm's signal is x:

$$\begin{aligned}
\pi_W(x) &= \max_q \big(P(q + q_L^*(x)) - c + x\big)q, \\
\pi_L^*(x) &= \max_q \big(P(q_W^*(x) + q) - c\big)q,
\end{aligned}$$

and $q_W^*(x)$ and $q_L^*(x)$ are the corresponding equilibrium outputs.[10]

The first–order conditions of the above maximization problem are:

$$\begin{aligned}
P'(q_W^*(x)+q_L^*(x))q_W^*(x) + P(q_W^*(x)+q_L^*(x)) - c + x &= 0 \\
P'(q_W^*(x)+q_L^*(x))q_L^*(x) + P(q_W^*(x)+q_L^*(x)) - c &= 0.
\end{aligned} \tag{19}$$

[10] Note that $q_W^*(x) = q_{W_1}(x) = q_{W_2}(x,z,y)|_{z=x=y}$ and $q_L^*(x) = q_{L_2}(x) = q_{L_1}(x,z,y)|_{z=x=y}$.

Differentiating (19) w.r.t. x, one obtains

$$\left(P''(\cdot)q_W^*(x)+P'(\cdot)\right)(q_W^{*'}(x)+q_L^{*'}(x))+P'(\cdot)q_W^{*'}(x)=-1$$
$$\left(P''(\cdot)q_L^*(x)+P'(\cdot)\right)(q_W^{*'}(x)+q_L^{*'}(x))+P'(\cdot)q_L^{*'}(x)=0,$$

from which one can derive

$$\begin{aligned} q_W^{*'}(x) &= -\frac{2P'(\cdot)+P''(\cdot)q_L^*(x)}{P'(\cdot)[3P'(\cdot)+P''(\cdot)\left(q_W^*(x)+q_L^*(x)\right)]} \\ q_L^{*'}(x) &= \frac{P'(\cdot)+P''(\cdot)q_L^*(x)}{P'(\cdot)[3P'(\cdot)+P''(\cdot)\left(q_W^*(x)+q_L^*(x)\right)]}. \end{aligned} \tag{20}$$

By the envelope theorem, one obtains

$$\begin{aligned} \pi_W'(x) &= \left(P'(\cdot)q_L^{*'}(x)+1\right)q_W^*(x) \\ \pi_L^{*\prime}(x) &= P'(\cdot)q_W^{*'}(x)q_L^*(x). \end{aligned} \tag{21}$$

To compute $\gamma(x)$, which is defined as $\gamma(y):=1-\left(P'(q_{W_2}(\cdot)+q_{L_1}(\cdot))\partial_z q_{W_2}(x,z,y)\right)\big|_{z=x}$ (see equation (7)), consider the Cournot subgame in the event when both firms bid and firm 1 has lost the auction. The first–order conditions of that subgame are:

$$\begin{aligned} P'(\cdot)q_{W_2}(\cdot)+P(\cdot)-c+y &= 0 \\ P'(\cdot)q_{L_1}(\cdot)+P(\cdot)-c+x-z &= 0. \end{aligned} \tag{22}$$

Differentiating (22) w.r.t. z, one obtains

$$\partial_z q_{W_2}(x,z,y) = -\frac{P'(\cdot)+P''(\cdot)q_{W_2}(x,z,y)}{P'(\cdot)[3P'(\cdot)+P''(\cdot)\left(q_{W_2}(x,z,y)+q_{L_1}(x,z,y)\right)]}. \tag{23}$$

Setting $z=y=x$, we get

$$\gamma(x) = 1+\frac{P'(\cdot)+P''(\cdot)q_W^*(x)}{3P'(\cdot)+P''(\cdot)\left(q_W^*(x)+q_L^*(x)\right)} \tag{24}$$

Combining (20), (21) and (24), we have

$$\begin{aligned} (\pi_W'(x)-\pi_L^{*\prime}(x))+q_L^*(x)\gamma(x) &= 2q_L^*(x)+q_W^*(x)+\frac{\left(P'(\cdot)+P''(\cdot)q_L^*(x)\right)q_W^*(x)}{3P'(\cdot)+P''(\cdot)\left(q_W^*(x)+q_L^*(x)\right)} \\ &> 2q_L^*(x)+q_W^*(x) \ \ \text{(since } P'\le 0,\ P''\le 0\text{)}. \end{aligned} \tag{25}$$

From (23) evaluating at $z=x$ and (7), one obtains $\gamma(y)$, which has the same form as (24) with x replaced by y. Hence, we have

$$-q_L^*(y)\gamma(y) = -2q_L^*(y)+\frac{\left(2P'(\cdot)+P''(\cdot)q_L^*(y)\right)q_L^*(y)}{3P'(\cdot)+P''(\cdot)\left(q_W^*(y)+q_L^*(y)\right)} > -2q_L^*(y). \tag{26}$$

From (25) and (26), it follows that

$$
\begin{aligned}
\beta'(x) &> 2q_L^*(x) + q_W^*(x) - \frac{1}{1-F(x)} \int_x^c 2q_L^*(y)dF(y) \\
&> 2q_L^*(x) + q_W^*(x) - \frac{1}{1-F(x)} \int_x^c 2q_L^*(x)dF(y) \\
&= q_W^*(x) > 0,
\end{aligned}
$$

the second inequality holds because $q_L^*(\cdot)$ is a decreasing function.

A.3 Part 3 of the proof of Proposition 3

Proof. Denote the payoff from bidding by $\Pi_b(x)$ and that from non-bidding by $\Pi_n(x)$, both for $x \geq r$. One obtains,

$$
\begin{aligned}
\Pi_b(x) &= F(r)\,(\pi_W(x) - R) + \int_r^x (\pi_W(x) - \beta(y))\,dF(y) + \int_x^c \pi_L^*(y)dF(y) \\
&= -F(r)R + F(x)\pi_W(x) - \int_r^x \beta(y)dF(y) + \int_x^c \pi_L^*(y)dF(y) \\
\Pi_n(x) &= F(r)\pi_{nn} + \int_r^c \pi_L^*(y)dF(y).
\end{aligned}
$$

Let

$$
\begin{aligned}
\psi(x) &:= \Pi_b(x) - \Pi_n(x) \\
&= -F(r)(\pi_{nn} + R) + F(x)\pi_W(x) - \int_r^x \big(\pi_L^*(y) + \beta(y)\big)\,dF(y).
\end{aligned}
$$

Differentiate ψ with respect to x, and one obtains, using (9):

$$
\begin{aligned}
\psi'(x) &= f(x)\pi_W(x) + F(x)\pi_W'(x) - \big(\pi_L^*(x) + \beta(x)\big)\,f(x) \\
&= F(x)\pi_W'(x) - \int_x^c \partial_z \pi_L(x,x,y)dF(y) > 0.
\end{aligned}
$$

By definition of r, one has $\psi(r) = 0$, and the assertion follows immediately. $\square$

A.4 Proof of Lemma 2

Proof. By Lemma 1, $\gamma(x) = 4/3$. Define $\varphi(r) := r - 4(1-F(r))/3f(r)$, We first show that Δ has a unique global maximum at r_Δ, which is implicitly defined as the solution of $\varphi(r) = 0$.

Notice that φ is strict monotone increasing (by the assumed hazard rate monotonicity) and

$$
\varphi(r) \gtreqqless 0 \iff r \gtreqqless r_\Delta.
$$

Differentiating (12) it follows immediately that

$$
\Delta'(r) = -2f(r) \int_r^c q_L^*(x_1)\varphi(r)dF(x_1) \gtreqqless 0 \iff r \lesseqqgtr r_\Delta.
$$

Therefore, $\Delta(r)$ has a unique global maximum at r_Δ.

Suppose $r \in [r_\Delta, c)$. Then, $\varphi(r) \geq 0$ and since $\varphi(r)$ is increasing,

$$\begin{aligned}\Delta(r) &= 2\int_r^c \int_{x_2}^c q_L^*(x_1)\varphi(x_2)dF(x_1)dF(x_2) \\ &> 2\int_r^c \int_{x_2}^c q_L^*(x_1)\varphi(r)dF(x_1)dF(x_2) \geq 0.\end{aligned}$$

Suppose $r \in (\hat{r}, r_\Delta)$. Then $\Delta'(r) > 0$ and since $\Delta(\hat{r}) = 0$, it follows that $\Delta(r) > 0$.

Therefore, $\Delta(r) > 0$ for all $r \in (\hat{r}, c)$.

Finally, $r_\Delta > r_{\min}$ follows from the monotonicity of $\varphi(r)$ combined with

$$\varphi(r_\Delta) - \varphi(r_{\min}) = -\varphi(r_{\min}) = -r_{\min} + \frac{1-F(r_{\min})}{f(r_{\min})}\frac{4}{3} = \frac{2}{3}\frac{1-F(r_{\min})}{f(r_{\min})} > 0.$$

□

A.5 Model I with linear demand

In the duopoly games with royalty scheme, the equilibrium strategies of firm 1 are: $q_{W_1}(x) = {}^{(1-c+2x)}/_3$, $q_{L_1}(x,z,y) = {}^{(1-c+2x-2z-y)}/_3$. The associated equilibrium profits are $\pi_W(x) = q_{W_1}(x)^2$ and $\pi_L(x,z,y) = q_{L_1}(x,z,y)^2$. The equilibrium profit when both firms do not bid is $\pi_{nn} = {}^{(1-c)^2}/_9$.

In the game without royalty scheme, $q_{L_1}(x,z,y)$ should be replaced by $q_L(y) = {}^{(1-c-y)}/_3$ and $\pi_L(y) = q_L(y)^2$.

The equilibrium bid functions are,

$$\beta_n(x) = \frac{x(2-2c+x)}{3}, \quad \beta(x) = \beta_n(x) - \frac{4}{9f(x)}\int_x^c (1-c-y)dF(y).$$

The relationship between the reserve price R and the critical valuation r induced by R is: $R = \pi_W(r) - \pi_{nn} = {}^{4(1-c+r)r}/_9$.

A.6 Model II with linear demand

In the game with royalty scheme, the Cournot equilibrium strategies of firm 1, are $q_{W_1}(x,z) = {}^{(2-2c+3x+z)}/_6$, $q_{L_1}(x,z,y) = {}^{(2-2c+3x-3z-2y)}/_6$. The associated equilibrium profits are $\pi_W(x,z) = q_{W_1}(x,z)^2$ and $\pi_L(x,z,y) = q_{L_1}(x,z,y)^2$. The equilibrium profit when both firms do not bid, π_{nn} is the same as in model I.

In the game without royalties, $q_{L_1}(x,z,y)$ should be replaced by $q_L(y) = {}^{(1-c-y)}/_3$ and $\pi_L(x,z,y)$ by $\pi_L(y) = (q_L(y))^2$.

The equilibrium bid functions are,

$$\beta_n(x) = \frac{x(2-2c+x)}{3} + \frac{(1-c+2x)F(x)}{9f(x)}, \quad \beta(x) = \beta_n(x) - \frac{1}{3f(x)}\int_x^c (1-c-y)dF(y).$$

The relationship between R and r is $R = \pi_W(r,r) - \pi_{nn} = {}^{4(1-c+r)r}/_9$.

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