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On Competition and the Strategic Management of Intellectual Property in Oligopoly

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Abstract

An innovative firm with private information about its indivisible process innovation chooses strategically whether to apply for a patent with probabilistic validity or rely on secrecy. By doing so, the firm manages its rivals' beliefs about the size of the innovation, and affects the incentives in the product market. A Cournot competitor tends to patent big innovations, and keep small innovations secret, while a Bertrand competitor adopts the reverse strategy. Increasing the number of firms gives a greater (smaller) patenting incentive for Cournot (Bertrand) competitors. Increasing the degree of product substitutability increases the incentives to patent the innovation.

Keywords: Bertrand and Cournot competition, oligopoly, product differentiation, asymmetric information, strategic disclosure, stochastic patent, trade secret, process innovation, imitation
JEL Codes: D82, L13, O31, O32

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1 Introduction

This paper studies the incentives of an innovative firm to patent its process innovation in an oligopoly. A patent discloses the technology to the firm's competitors, and gives some protection against expropriation of the disclosed technology. However, patents are imperfect. They only give protection with a certain probability (Lemley and Shapiro, 2005). Moreover, surveys in the US (Levin *et al.*, 1987, and Cohen *et al.*, 2000) and Europe (Arundel, 2001) find that high-level executives do not consider patenting the most effective appropriability mechanism for process innovations. Instead, secrecy was often considered as a more effective way to protect those innovations. In spite of the perceived weak protection, firms do apply for patents (e.g., Kim and Marschke, 2004, and Hall, 2005). One reason for this is that a patent enables a firm to signal information about its innovation in a credible, verifiable way (Long, 2002).[1] In this paper, I analyze this motive and explore its economic consequences.

I analyze the patenting incentives in a model of asymmetric information about the size of an innovation. In such a setting an innovative firm faces the following trade-off. On the one hand, patenting a technology is a way to persuade the competitor of the technology's efficiency. This creates a signaling effect. On the other hand, the potential expropriation of a patented technology yields a more efficient, and more "aggressive" competitor in the product market. This expropriation effect gives the innovative firm a disincentive to apply for a patent. The innovative firm manages the expectations of its competitor in the product market, and thereby affects his conduct, by patenting certain technologies while keeping other technologies secret.

A firm can patent selectively by making different patenting choices for different innovations. Selective patenting of an indivisible innovation gives either a patented innovation or a trade secret.[2] This makes patents and trade secrets substitutes. By contrast, if an innovation is divisible, the firm can also choose to patent only certain parts of any given innovation, while keeping the remaining parts secret. In this case, patenting and secrecy are complementary strategies. Whether the intellectual property strategies are substitutes or complements in practice remains an open issue. At first sight, data from the Yale Survey and Carnegie Mellon Survey (CMS) in the US, and the European Community Innovation Survey (CIS) seem to support at most weak

[1] Of course, there are alternative explanations for this so-called patent paradox, such as the build-up of patent portfolios to improve a firm's bargaining position (e.g., Hall and Ziedonis, 2001).

[2] If a process innovation cannot be broken in small parts, then the full disclosure requirement of a patent only leaves the choice between truthful disclosure or complete concealment of the technology.

complementarity between the appropriation strategies (Levin *et al.*, 1987, Cohen *et al.*, 2000, Arundel and Kabla, 1998, Hussinger, 2006, and Pajak, 2010).[3] However, there are at least two reasons why these studies tend to over-estimate the complementarity between patenting and secrecy. First, the studies are based on firm-level data. A firm may treat patents and secrets as substitutes at the innovation level by using patenting exclusively for some of its innovations and secrecy exclusively for its other innovations. However, it would contribute to a positive correlation between patenting and secrecy at the firm level.[4] Second, when different intellectual property instruments are used exclusively in different stages of an innovation's development, this will also give an over-estimation of complementarity (Arundel and Kabla, 1998).

Since there does not appear to be strong evidence for either relationship between patenting and secrecy, and there tend to be significant differences between industries (e.g., Arundel and Kabla, 1998, and Moser, 2010), substitutability could be expected to fit well with some industries, while complementarity would fit better with other industries. In this paper, I adopt the assumption of substitutability, as in Horstmann *et al.* (1985), Gill (2008), and Jansen (2006, 2010).[5] By contrast, Anton and Yao (2003, 2004) analyze the trade-off between expropriation and signaling for an innovation that can be subdivided into arbitrary small parts. Thereby, my analysis is complementary to the analyses of Anton and Yao. Interestingly, my analysis gives different predictions on a firm's intellectual property strategy.

The paper shows that the strategy of an innovative firm depends on the mode of competition in the industry. When firms compete in output levels, a firm has an incentive to appear as an efficient, "tough" competitor in the product market to discourage its competitors (strategic substitutes). Consequently, in the absence of expropriation,

[3] The principal components analysis of the Yale Survey data in Levin *et al.* (1987), and the factor analysis on the 1994 CMS in Cohen *et al.* (2000) suggest that there is no strong correlation between the importance of patenting and secrecy as methods of appropriation in US data. Arundel and Kabla (1998) find in the 1993 CIS that there is substitutability between patenting and secrecy for product innovations, and only weak complementarity for process innovations. In data from the 2004 CIS, Pajak (2010) finds only a small positive pairwise correlation between patenting and secrecy for product innovations of small, innovative firms in France. By contrast, Hussinger (2006) finds a strong correlation between the patent propensity and the use of secrecy in data from the 2000 CIS among German firms. At the same time, however, she finds that 35% of the firms use one of the instruments exclusively (i.e., 15% use only patents, and 20% use only secrecy), while 41% of the firms used both patenting and secrecy (the remaining 24% use neither patents nor secrets).

[4] Pajak (2010) mitigates this problem by focusing on small firms with one innovation on average.

[5] The former two papers study models of entry deterrence, whereas I consider accommodating strategies. Jansen (2006) studies a simple model with two types, and therefore cannot analyze selective patenting strategies. Jansen (2010) studies a complementary problem of technology sharing by competing innovative Cournot duopolists in the absence of intellectual property protection.

a Cournot oligopolist has an incentive to patent big innovations, and keep small innovations secret. Indeed, an innovative firm may adopt this strategy, if a patent gives a moderate risk of expropriation. Further, if patenting gives a high risk of expropriation, the expropriation effect tends to dominate, and a Cournot competitor keeps its innovation secret. Conversely, for patents with a low risk of expropriation, the signaling effect tends to dominate, and all innovations are patented (Okuno-Fujiwara *et al.*, 1990). Empirical findings by Mäkinen (2007), Moser (2010), and Pajak (2010) are consistent with the strategy of patenting big innovations to a greater extent than small innovations.[6,7] Interestingly, Anton and Yao (2003) obtain the opposite prediction in a related model with divisible innovations, i.e., small innovations are patented to a greater extent than big innovations.[8] A firm with a divisible innovation can signal the innovation's size by patenting only a small part of its innovation. However, in my paper such a strategy is not feasible, since I consider an indivisible innovation.

A change of the mode of competition from competition in quantities to competition in prices changes the direction of the signaling effect. A Bertrand oligopolist only discloses inefficient technologies to persuade the competitor that he will face relaxed competition in the product market (strategic complements). That is, competition in prices gives an incentive to patent different technologies than competition in quantities. Pajak (2010) finds that small firms in the French intermediate goods industry are more likely to patent small innovations than big innovations, which is consistent with these incentives. As far as I know, my paper is the first to analyze the trade-off between signaling and expropriation in a model of Bertrand competition, and to compare the two modes of competition.[9]

A switch from a market where firms strategically set output levels (Cournot com-

[6] In innovation-level data on Finnish product innovations from 1985-1998, Mäkinen (2007) finds that more novel and significant product innovations were patented more often than smaller innovations. Moser (2010) finds in innovation-level data on UK and US innovations at World Fairs from 1851-1915 that award-winning innovations were more likely to be patented. When considering all small innovative firms in France together, Pajak (2010) finds in firm-level data of the 2004 CIS that large product innovations are more likely to be patented.

[7] Alternatively, such a patenting strategy could also be consistent with a model in which a non-strategic firm chooses between patenting at a fixed cost and secrecy. However, such a model would yield no patenting for industries where secrecy gives better protection against expropriation than patenting. That is, it would not resolve the patent paradox. Moreover, Moser (2010) finds only a weak elasticity with respect to patenting fees, when comparing patenting by UK and US innovators.

[8] Anton and Yao (2003) differs in a second respect from my paper. Whereas Anton and Yao study a drastic innovation, I consider a non-drastic innovation. However, Appendix C suggests that Anton and Yao's qualitative result also holds in a model with a non-drastic (divisible) innovation.

[9] Anton and Yao (2003, 2004), Gill (2008), and Jansen (2006, 2010) analyze the trade-off between expropriation and signaling in models with strategic substitutability in the production stage.

petition), to a market where they set prices (Bertrand competition) increases the competitive pressure for the firms (e.g., see Singh and Vives, 1984). By comparing the patenting strategies of a Cournot competitor with the strategy of a Bertrand competitor, I make a first step in characterizing the effect of competitive pressure on patenting strategies. In addition, by applying insights from the theory of monotone comparative statics (e.g., see Vives, 2005, for an excellent survey), I characterize the effects of changes in two alternative measures of competitive pressure. First, increasing the number of non-innovative firms in the industry intensifies product market competition. Second, an increase in the degree of product substitutability is an alternative way of increasing the competitive pressure. These analyses try to contribute to the current debate on the effects of competitive pressure on innovative activity. The existing literature typically focuses on the relationship between competitive pressure and incentives to create new knowledge (see, e.g., Belleflamme and Vergari, 2006, Gilbert, 2006, Vives, 2008, and Schmutzler, 2010, for overviews). By contrast, I study the effects of competitive pressure on the incentives to diffuse new knowledge. In other words, my analysis is complementary to the existing literature.

The different measures of competitive pressure affect the patenting incentives in different ways, since they have different effects on the responsiveness of the firms' product market strategies. Whereas non-innovative firms become less responsive to changes in the prices or output levels of an innovative firm when their number grows, they become more responsive when products become closer substitutes. More responsive competitors tend to adjust their strategies more drastically when they become informed about the size of the innovation. Hence, more responsive product market strategies of competitors tend to give a relatively stronger signaling effect, and thereby a greater incentive to patent an innovation. The paper confirms that a greater substitutability between goods gives more patenting. Moreover, an increase in the number of non-innovative firms gives less patenting when firms compete in prices. By contrast, when firms compete in output levels, an increase in the number of non-innovative competitors gives more patenting. This is due to a greater responsiveness of the innovative firm's output strategy to output changes of its competitors.[10]

[10] Empirical analyses that consider the effect of competitive pressure on the propensity to patent are scarce and inconclusive. Mäkinen (2007) finds only a weakly significant negative effect (i.e., as for entry in a Bertrand oligopoly), by taking competition to be intense if price competition is at least an important factor for initiating the development of an innovation. Duguet and Kabla (1998) find no significant effect from the logarithms of the average market share and the average Herfindahl concentration index on patent propensity. A careful, empirical analysis of the relationship between the intensity of product market competition and the propensity to patent awaits future research.

My paper also relates to recent literature on endogenous knowledge spillovers. For example, De Fraja (1993), Katsoulacos and Ulph (1998), Kamien and Zang (2000), Gersbach and Schmutzler (2003), Fosfuri and Rønde (2004), Encaoua and Lefouili (2006), and Milliou (2009) analyze the choice of technology diffusion in oligopoly models of complete information.[11] Whereas expropriation of technological knowledge affects the spillover choice in these papers, there is no role for signaling. By contrast, signaling plays a central role in my model.

The paper is organized as follows. The next section describes the model. Section 3 characterizes the equilibrium product market strategies under patenting and trade secrecy, and the equilibrium patenting strategies. Section 4 discusses the effects of competitive pressure on the incentive to patent an innovation. Finally, section 5 concludes the paper. Appendix A contains the proofs of the paper's propositions, Appendix B gives more detailed derivations, and Appendix C covers three extensions.

2 The Model

Consider $N+1$ risk-neutral firms, firm I and firms $1, .., N$, producing differentiated goods, with $N \geq 1$. Firm I, the innovative firm, obtains a patentable non-drastic process innovation, which yields a production cost $\theta_I \in [\underline{\theta}, \overline{\theta}]$, drawn from p.d.f. $f : [\underline{\theta}, \overline{\theta}] \rightarrow \mathbb{R}_+$ (and corresponding c.d.f. $F : [\underline{\theta}, \overline{\theta}] \rightarrow [0,1]$), with $0 \leq \underline{\theta} < \overline{\theta}$.[12] The production cost θ_I is private information to firm I. Firms $1, .., N$, the non-innovative firms, have an inefficient, non-patentable technology, with the production cost $\overline{\theta}$, i.e., $\theta_1 = ... = \theta_N = \overline{\theta}$.[13]

After firm I learns its cost, it makes its patent choice. Firm I chooses whether to file for a patent and consequently reveal its cost truthfully, $s(\theta_I) = \theta_I$, or to keep its cost secret and send the uninformative message $s(\theta_I) = \varnothing$. The firm's patenting strategy can be written as follows:

$$s(\theta_I) = \begin{cases} \varnothing, & \text{if } \theta_I \in \mathcal{S} \\ \theta_I, & \text{if } \theta_I \notin \mathcal{S} \end{cases} \tag{2.1}$$

where $\mathcal{S} \subseteq [\underline{\theta}, \overline{\theta}]$ denotes the set of technologies that are kept secret.[14]

[11] Papers in this line of research often build on the seminal work by d'Aspremont and Jacquemin (1988) about research joint ventures.

[12] This specification allows for uncertainty about the existence of an innovation by assigning a positive probability mass to the atom $\theta_I = \overline{\theta}$.

[13] The assumption that there is only one innovative firm is made for simplicity. Section 5 discusses the patenting incentives when there are more innovative firms in the industry.

[14] Alternatively, instead of the uninformative message, $\varnothing$, a secretive firm could choose to release

Patents are always granted, but their validity is challenged in court.[15] The firm's patent for the new technology is successfully defended in court with probability γ^P, where $0 \leq \gamma^P < 1$. However, with probability $1 - \gamma^P$ the patent is invalid, and the firms $1, .., N$ can imitate the patent holder's technology without incurring any cost.[16] A trade secret remains secret with probability γ^S, but with probability $1 - \gamma^S$ the secret leaks out to the competitors, enabling them to imitate the leaked technology at no additional cost. To make the problem interesting, I assume that imitation is more likely under patenting than under secrecy $\gamma^P < \gamma^S \leq 1$.[17] For the analysis of patent incentives there is no loss of generality to set $\gamma^S = 1$ and $\gamma^P = \gamma$ with $\gamma < 1$.[18] The parameter γ measures the relative protection of patents *vis-à-vis* secrets.

Finally, after messages are received and the validity of the patent is determined, firms set the output levels of their differentiated goods simultaneously (Cournot competition).[19] Firm ℓ with cost θ_ℓ chooses its output, $q_\ell \geq 0$, and earns the profit:

$$\pi_\ell(\mathbf{q}; \theta_\ell) = (P_\ell(\mathbf{q}) - \theta_\ell)q_\ell \tag{2.2}$$

with $\ell \in \{I, 1, .., N\}$. At the outputs $\mathbf{q} \equiv (q_I, q_1, .., q_N)$, the inverse demand for the good of firm ℓ is linear in quantities:

$$P_\ell(\mathbf{q}) = \alpha - q_\ell - \beta \sum_{k \neq \ell} q_k, \tag{2.3}$$

with $\ell, k \in \{I, 1, .., N\}$.[20] Firm I's innovation is non-drastic, i.e., I assume that

the commonly available technology $\overline{\theta}$. This would give the same effects on competition, i.e., no expropriation and no precise signal about the firm's actual technology. When secretive types pool with the worst type of the innovative firm, and the worst type has a positive probability mass (i.e., there is uncertainty about the existence of an innovation), this has an effect on the beliefs of the non-innovative firms. However, this does not change the qualitative results of the paper.

[15]The assumption that a firm with the worst technology draw (i.e., $\theta_I = \overline{\theta}$) can get a patent is made for simplicity. Given free access to the existing, old technology, a patent of $\theta_I = \overline{\theta}$ only serves as a certification device, since imitation is irrelevant for $\theta_I = \overline{\theta}$. Hence, this assumption gives the same results as the alternative assumption that a patent of technology $\overline{\theta}$ is always invalid.

[16]Clearly, the probability of holding an invalid patent can also be interpreted as the probability with which the patent validity is challenged in court, and the defense of the patent fails.

[17]A model with stronger patent protection (i.e., $\gamma^P \geq \gamma^S$) would yield the patenting of all technologies in equilibrium, since the signaling benefits of patenting (e.g., Okuno-Fujiwara *et al.,* 1990) would be reinforced by the benefit of less or equally frequent expropriation. Assuming $\gamma^P < \gamma^S$ is consistent with theoretical work (Anton and Yao, 2003) and empirical findings (Cohen et al., 2000).

[18]If Π^P is the profit from a valid patent, Π^S is the profit from a secret, and Π^I is the profit after imitation, then the expected profit gain from patenting instead of secrecy is: $[\gamma^P\Pi^P + (1-\gamma^P)\Pi^I] - [\gamma^S\Pi^S + (1-\gamma^S)\Pi^I]$. This profit difference equals: $[\gamma^P\Pi^P + (\gamma^S - \gamma^P)\Pi^I] - \gamma^S\Pi^S$. Clearly, the sign of this net profit is the same as the sign of: $[\gamma\Pi^P + (1-\gamma)\Pi^I] - \Pi^S$, with $\gamma \equiv \gamma^P/\gamma^S$.

[19]Later, in section 3.2, I also consider the model in which firms set prices (Bertrand competition).

[20]In this model of Cournot competition, one can also interpret the innovation as a product innovation, where the representative consumer's intrinsic willingness-to-pay for the innovation is $\alpha - \theta_I$.

$\alpha \geq 2\overline{\theta} - \underline{\theta}$. Parameter β represents the degree of product differentiation, with $0 < \beta \leq 1$. The greater β, the more substitutable the firms' goods.[21]

I solve the game backwards, and restrict the analysis to pure-strategy equilibria.

3 Equilibrium Strategies

3.1 Cournot Competition

First, I derive the equilibrium output levels for any given patent choice and belief. Subsequently, I characterize the equilibrium patenting strategies.

3.1.1 Output Strategies

Suppose that firm ℓ anticipates that its competitor k has a marginal cost θ_k in the technology subset $\mathcal{T}_k \subseteq [\underline{\theta}, \overline{\theta}]$, and uses the output strategy $q_k(\theta_k)$ for $\theta_k \in \mathcal{T}_k$. Maximization of expected profits by firm ℓ with marginal cost θ_ℓ then yields the following best response function (for $\ell, k \in \{I, 1, .., N\}$):

$$r_\ell^c(\mathbf{q}_{-\ell}; \theta_\ell) = \frac{1}{2}\left(\alpha - \theta_\ell - \beta \sum_{k \neq \ell} E\{q_k(\theta_k) | \theta_k \in \mathcal{T}_k\}\right). \tag{3.1}$$

If firm ℓ has marginal cost θ_ℓ, it expects marginal costs $\sum_{k \neq \ell} E\{\theta_k | \theta_k \in \mathcal{T}_k\}$ from its competitors, and the competitors believe that firm ℓ's marginal cost is in the subset $\mathcal{T}_\ell$, then the firm sets the following output in equilibrium:

$$q_\ell^c\left(\theta_\ell, \sum\nolimits_{k \neq \ell} E\{\theta_k | \theta_k \in \mathcal{T}_k\}; \mathcal{T}_\ell\right) \equiv \frac{1}{(2 + N\beta)(2 - \beta)}\Bigg((2 - \beta)(\alpha - \theta_\ell) + \beta \sum_{k \neq \ell}(E\{\theta_k | \theta_k \in \mathcal{T}_k\} - \theta_\ell) + \frac{\beta}{2} \cdot \beta N [\theta_\ell - E\{\theta_\ell | \theta_\ell \in \mathcal{T}_\ell\}]\Bigg) \tag{3.2}$$

In particular, three situations can emerge. In the first two situations, the firms choose outputs under complete information. These situations emerge after firm I patents its technology θ_I, i.e., $\mathcal{T}_I = \{\theta_I\}$. The cost of non-innovative firms $1, .., N$ depends on the validity of firm I's patent. First, if the patent is valid, then the non-innovative firms cannot adopt the new technology, i.e., $\theta_n = \overline{\theta}$ and $\mathcal{T}_n = \{\overline{\theta}\}$ for $n = 1, ..., N$. In equilibrium, the outputs are $q_I^c(\theta_I, N\overline{\theta}; \{\theta_I\})$ and $q_n^c(\overline{\theta}, \theta_I + (N-1)\overline{\theta}; \{\theta_I\})$ for $n = 1, .., N$. Second, if the patent is invalid, then imitation gives all firms the

[21] For example, the markets are independent for $\beta = 0$, and the goods are homogeneous for $\beta = 1$.

marginal cost θ_I. In this case, each firm sets the symmetric equilibrium output level $q_\ell^c(\theta_I, N\theta_I; \{\theta_I\})$ for $\ell \in \{I, 1, .., N\}$.

Finally, after firm I adopts secrecy there is asymmetric information about firm I's marginal cost θ_I (i.e., $\mathcal{T}_I = \mathcal{S}$ for some $\mathcal{S} \subseteq [\underline{\theta}, \overline{\theta}]$), and no imitation is possible (i.e., $\theta_n = \overline{\theta}$ and $\mathcal{T}_n = \{\overline{\theta}\}$ for $n = 1, ..., N$). Consequently, the equilibrium output levels are $q_I^c(\theta_I, N\overline{\theta}; \mathcal{S})$ and $q_n^c(\overline{\theta}, E\{\theta_I | \theta_I \in \mathcal{S}\} + (N-1)\overline{\theta}; \{\overline{\theta}\})$ for $n = 1, .., N$.

In any case, firm ℓ's profit equals: $\pi_\ell^c(\bullet) = q_\ell^c(\bullet)^2$, for $\ell \in \{I, 1, .., N\}$.

3.1.2 Patenting Strategies

Firm I with innovation θ_I bases its patenting decision on the comparison of the profit from secrecy, $\pi_I^c(\theta_I, N\overline{\theta}; \mathcal{S})$, and the expected profit from patenting. Patenting generates the profit from a valid patent, $\pi_I^c(\theta_I, N\overline{\theta}; \{\theta_I\})$, with probability γ, and the profit from an invalid patent, $\pi_I^c(\theta_I, N\theta_I; \{\theta_I\})$, with probability $1-\gamma$. The firm prefers secrecy whenever $\pi_I^c(\theta_I, N\overline{\theta}; \mathcal{S}) \geq \gamma \pi_I^c(\theta_I, N\overline{\theta}; \{\theta_I\}) + (1-\gamma)\pi_I^c(\theta_I, N\theta_I; \{\theta_I\})$, which can be written as:

$$\pi_I^c(\theta_I, N\overline{\theta}; \{\theta_I\}) - \pi_I^c(\theta_I, N\overline{\theta}; \mathcal{S}) \leq (1-\gamma)\left[\pi_I^c(\theta_I, N\overline{\theta}; \{\theta_I\}) - \pi_I^c(\theta_I, N\theta_I; \{\theta_I\})\right] \quad (3.3)$$

for $\theta_I \in [\underline{\theta}, \overline{\theta}]$ and $\mathcal{S} \subseteq [\underline{\theta}, \overline{\theta}]$. A firm that switches from secrecy to patenting changes the beliefs of the competitors. The competitors learn from a patent that the technology is actually θ_I instead of the expected technology $E\{\theta_I | \theta_I \in \mathcal{S}\}$. This changes their conduct in the product market (i.e., a non-innovative firm "moves along" its best response curve). Consequently, the innovative firm replaces the profit $\pi_I^c(\theta_I, N\overline{\theta}; \mathcal{S})$ with the profit $\pi_I^c(\theta_I, N\overline{\theta}; \{\theta_I\})$ if the patent is valid. The left hand side of inequality (3.3) captures this signaling effect of patenting. If a patent would always be valid, then this would be the only effect of patenting as compared to secrecy.

However, a patent is not always valid. The patent turns out to be invalid with probability $1-\gamma$. In this case, the switch from secrecy to patenting gives a profit loss of $\pi_I^c(\theta_I, N\overline{\theta}; \{\theta_I\}) - \pi_I^c(\theta_I, N\theta_I; \{\theta_I\})$. Imitation makes firms $1, .., N$ more "aggressive" competitors (i.e., the firms' best response functions shift outwards, to the right), which reduces the innovative firm's output and profit. The right hand side of inequality (3.3) captures this expected loss from expropriation.

In short, firm I chooses secrecy if (3.3) holds, i.e., the signaling effect is weaker than the expropriation effect. Before stating the proposition, which results from the trade-off between signaling and expropriation, I define the following critical value:

$$\gamma^o \equiv 1 - \frac{\beta}{2}\left(\frac{E\{\theta_I\} - \underline{\theta}}{\overline{\theta} - \underline{\theta}}\right) \frac{q_I^c(\underline{\theta}, N\overline{\theta}; \{\underline{\theta}\}) + q_I^c(\underline{\theta}, N\overline{\theta}; [\underline{\theta}, \overline{\theta}])}{q_I^c(\underline{\theta}, N\overline{\theta}; \{\underline{\theta}\}) + q_I^c(\underline{\theta}, N\underline{\theta}; \{\underline{\theta}\})} \quad (3.4)$$

where $0 < \gamma^o < 1$.

Proposition 1 *For any equilibrium, there exists a critical value θ^c, with $\underline{\theta} \leq \theta^c \leq \overline{\theta}$, such that firm I chooses the patenting strategy s^c in (2.1) with $\mathcal{S} = [\theta^c, \overline{\theta}]$. In particular,* ***(a)*** *there exists an equilibrium in which firm I keeps any technology secret (i.e., $\theta^c = \underline{\theta}$) if and only if $\gamma \leq \gamma^o$, with γ^o as in (3.4);* ***(b)*** *there exists an equilibrium in which firm I patents all technologies (i.e., $\theta^c = \overline{\theta}$) if and only if $\gamma \geq 1 - \frac{1}{2}\beta$;* ***(c)*** *if $\gamma^o < 1 - \frac{1}{4}\beta$, then for any $\gamma^o < \gamma < 1 - \frac{1}{4}\beta$ there exists an equilibrium with $\underline{\theta} < \theta^c < \overline{\theta}$.*

The intuition for this result lies in the analysis of the signaling effect. Since firms compete in output levels in the product market, their product market strategies are strategic substitutes. Consequently, if firm I discloses a technology which is less efficient than expected, then the non-innovative firms adjust their outputs upwards (i.e., they "move up" along their best response curves), and become more aggressive competitors. That is, in this case the expropriation effect and the signaling effect reinforce each other, and give a disincentive to apply for a patent. Conversely, disclosure of a technology which is more efficient than expected makes the non-innovative firms less aggressive competitors in the product market (strategic substitutes). That is, in this case the expropriation and signaling effect conflict, and the patenting incentives are determined by their trade-off.

Extreme strengths of intellectual property right give the following incentives. On the one hand, perfect protection (i.e., $\gamma \to 1$) eliminates the expropriation effect of patenting a technology. The remaining signaling effect gives firm I an incentive to patent any technologies with above-average efficiency levels. This drives the expected cost level of secret technologies up to the highest cost level (i.e., $\mathcal{S} = \{\overline{\theta}\}$). In other words, for γ approaching 1 the unraveling result applies (Okuno-Fujiwara *et al.*, 1990), yielding full patenting in equilibrium (i.e., $\theta^c = \overline{\theta}$).

On the other hand, in the absence of patent protection ($\gamma = 0$), the expropriation effect outweighs the signaling effect. Patenting of technology θ_I would enable the non-innovative firms to imitate, and set output levels $q_n^c(\theta_I, N\theta_I; \{\theta_I\})$ for $n = 1, .., N$. By contrast, trade secrecy enables non-innovative firms to set an equilibrium output level of at most $q_n^c(\overline{\theta}, N\overline{\theta}; \{\overline{\theta}\})$. That is, trade secrecy yields less aggressive competitors when imitation is certain, since $q_n^c(\overline{\theta}, E\{\theta_I|\theta_I \in \mathcal{S}\} + (N-1)\overline{\theta}; \{\overline{\theta}\}) \leq q_n^c(\overline{\theta}, N\overline{\theta}; \{\overline{\theta}\}) \leq q_n^c(\theta_I, N\theta_I; \{\theta_I\})$ for any θ_I and $\mathcal{S}$. Consequently, firm I adopts secrecy for any technology in equilibrium. For intermediate strengths of intellectual property protection, a more subtle trade-off emerges between signaling and expropriation.

The comparison between the critical value γ^o, on the one hand, and the values $1-\frac{1}{2}\beta$ and $1-\frac{1}{4}\beta$, on the other, depends on the size of the average technology, $E\{\theta_I\}$. In particular, γ^o is decreasing in the average technology $E\{\theta_I\}$, and several equilibrium outcomes can emerge, as Figure 1 illustrates for $\beta = 1$. The abbreviations FP, FS,

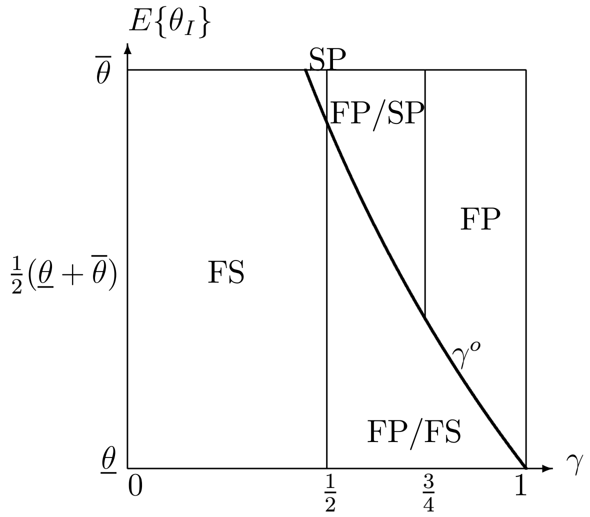

Figure 1: Equilibrium patenting (Cournot competition)

and SP stand for full patenting, full secrecy, and selective patenting, respectively.

These equilibrium strategies differ from the strategies in Anton and Yao (2003). The innovative firm in Anton and Yao patents small innovations to a greater extent than big innovations, whereas here the reverse tends to happen. The model of Anton and Yao (2003) has a divisible and drastic innovation, whereas my model has an indivisible, non-drastic innovation. The analysis in Appendix C suggests that also a model with a divisible, non-drastic innovation yields more patenting of small innovations than big innovations, as in Anton and Yao (2003). In particular, in the absence of protection (i.e., $\gamma = 0$), the innovative firm chooses the equilibrium strategy $\widehat{\varphi}(\theta_I) = \frac{\beta}{2}\theta_I + (1-\frac{\beta}{2})\overline{\theta}$, which means that the firm patents only technologies of relatively low efficiency (i.e., $\theta_I \geq \frac{\beta}{2}\underline{\theta} + (1-\frac{\beta}{2})\overline{\theta}$), while it does not patent technologies that are more efficient. Similar equilibrium strategies emerge for weak patent protection (for more details, see Appendix C). Clearly, such a strategy is not feasible for a firm with an indivisible innovation. That is, the assumption of indivisibility of the innovation has a non-trivial effect on the strategies that the innovative firm chooses in equilibrium. By contrast, for sufficiently strong protection (i.e., $\gamma \geq 1-\frac{\beta}{2}$), the signaling effect dominates, which gives the innovative firm an incentive to patent its innovation completely. This is analogous to my result in Proposition 1(b).

3.2 Bertrand Competition

Now I turn to the model in which firms choose the prices of their goods simultaneously in the last stage (Bertrand competition). In particular, firm ℓ with cost θ_ℓ chooses its price, $p_\ell \geq 0$, and earns the profit:

$$\pi_\ell(\mathbf{p};\theta_\ell) = D_\ell(\mathbf{p})(p_\ell - \theta_\ell) \tag{3.5}$$

for $\ell \in \{I, 1, .., N\}$. Here $D_\ell(\mathbf{p})$ is the direct demand at prices $\mathbf{p} \equiv (p_I, p_1, .., p_N)$:

$$D_\ell(\mathbf{p}) = \frac{1}{(1-\beta)(1+N\beta)}\left((1-\beta)\alpha - [1+(N-1)\beta]p_\ell + \beta\sum_{k\neq\ell} p_k\right), \tag{3.6}$$

where $\ell, k \in \{I, 1, .., N\}$. I assume that the goods are sufficiently differentiated (i.e., β is sufficiently low), such that all firms produce in equilibrium.

3.2.1 Pricing Strategies

Firm ℓ with marginal cost θ_ℓ, who anticipates that its competitor's marginal cost θ_k is in the subset $\mathcal{T}_k \subseteq [\underline{\theta}, \overline{\theta}]$, has the following best response function:

$$r_\ell^b(\mathbf{p}_{-\ell};\theta_\ell) = \frac{1}{2}\theta_\ell + \frac{(1-\beta)\alpha + \beta\sum_{k\neq\ell} E\{p_k(\theta_k)|\theta_k \in \mathcal{T}_k\}}{2[1+(N-1)\beta]}. \tag{3.7}$$

If firm ℓ has marginal cost θ_ℓ, it expects marginal costs $\sum_{k\neq\ell} E\{\theta_k|\theta_k \in \mathcal{T}_k\}$ from its competitors, and the competitors believe that firm ℓ's marginal cost is in the subset $\mathcal{T}_\ell$, then the firm sets the following price in equilibrium

$$p_\ell^b\left(\theta_\ell, \sum\nolimits_{k\neq\ell} E\{\theta_k|\theta_k \in \mathcal{T}_k\}; \mathcal{T}_\ell\right) = \theta_\ell + \frac{m_\ell^b\left(\theta_\ell, \sum_{k\neq\ell} E\{\theta_k|\theta_k \in \mathcal{T}_k\}; \mathcal{T}_\ell\right)}{[2+(N-2)\beta][2+(2N-1)\beta]} \tag{3.8}$$

for $\ell, k \in \{I, 1, .., N\}$, with the equilibrium margin:[22]

$$\begin{aligned} m_\ell^b\left(\theta_\ell, \sum\nolimits_{k\neq\ell} E\{\theta_k|\theta_k \in \mathcal{T}_k\}; \mathcal{T}_\ell\right) &\equiv (1-\beta)[2+(2N-1)\beta](\alpha - \theta_\ell) \\ + [1+(N-1)\beta]\beta\sum_{k\neq\ell}(E\{\theta_k|\theta_k \in \mathcal{T}_k\} - \theta_\ell) &+ \frac{\beta}{2}\cdot\beta N(E\{\theta_\ell|\theta_\ell \in \mathcal{T}_\ell\} - \theta_\ell). \end{aligned} \tag{3.9}$$

After firm I patents its technology θ_I, the firms set prices under complete information. If the patent is valid, then firm I chooses the margin $m_I^b(\theta_I, N\overline{\theta}; \{\theta_I\})$ in equilibrium, while the non-innovative firm n set $m_n^b(\overline{\theta}, \theta_I + (N-1)\overline{\theta}; \{\overline{\theta}\})$ for $n = 1, ..., N$. If

[22]As usual, the equilibrium price is increasing in the expected costs. The equilibrium margin is decreasing in the own cost, since only part of a firm's cost increase is passed through to consumers.

the patent is invalid, each firm has the marginal cost θ_I, and chooses $m^b_\ell(\theta_I, N\theta_I; \{\theta_I\})$ for $\ell \in \{I, 1, .., N\}$.

Finally, if firm I adopts secrecy, the non-innovative firms remain uninformed about the technology θ_I, and anticipate the patenting strategy (2.1) for some $\mathcal{S} \subseteq [\underline{\theta}, \overline{\theta}]$. In equilibrium the firms I and $n = 1, .., N$ choose the margins $m^b_I(\theta_I, N\overline{\theta}; \mathcal{S})$ and $m^b_n(\overline{\theta}, E\{\theta_I|\theta_I \in \mathcal{S}\} + (N-1)\overline{\theta}; \{\overline{\theta}\})$, respectively.

In any case, in equilibrium firm ℓ supplies the following output level and earns the following expected profit, respectively (for $\ell \in \{I, 1, .., N\}$):

$$q^b_\ell(\bullet) \equiv \frac{1+(N-1)\beta}{(1-\beta)(1+N\beta)} \cdot \frac{m^b_\ell(\bullet)}{[2+(N-2)\beta][2+(2N-1)\beta]} \tag{3.10}$$

$$\pi^b_\ell(\bullet) \equiv \frac{1+(N-1)\beta}{(1-\beta)(1+N\beta)} \left(\frac{m^b_\ell(\bullet)}{[2+(N-2)\beta][2+(2N-1)\beta]}\right)^2. \tag{3.11}$$

3.2.2 Patenting Strategies

The patenting choice of a firm that competes in prices (strategic complements) also trades off the expropriation effect and a signaling effect. For technologies with above-average efficiency levels both effects of patenting are negative. In particular, potential expropriation of the technology makes the rivals (firms $1, .., N$) compete more aggressively. Moreover, the rivals update their beliefs in an unfavorable direction, since they learn that firm I is more efficient (and aggressive) than expected. This makes the rivals compete even more aggressively, since the actions are strategic complements. In short, the firm has no incentive to patent any efficient technologies. This brief description of the patenting incentives already suggests that the firm's patenting strategies under Bertrand competition differ from the patenting strategies under Cournot competition. Whereas the firm may choose to patent only efficient technologies under Cournot competition, it has a clear disincentive to do so under Bertrand competition.

For technologies with a below-average efficiency level the two effects of patenting are in conflict. On the one hand, the expropriation effect still gives firm I an incentive to keep the technology secret. However, on the other hand, now the signaling effect gives an incentive to apply for a patent. For sufficiently high cost parameters the signaling effect outweighs the expropriation effect, and disclosure softens the conduct of the non-innovative firms in the product market. That is, although imitation of a minor innovation makes the firms $1, .., N$ slightly more productive competitors, the firms charge a higher price, since they drastically downgrade their beliefs about the aggressiveness of firm I's pricing strategy.[23] As a result, firm I has an incentive to patent such a technology. In short, firm I has an incentive to patent inefficient technologies, and keep efficient technologies secret.

Proposition 2 *In any equilibrium, there exists a critical value θ^b, with $\underline{\theta} < \theta^b < \overline{\theta}$, such that firm I chooses the patenting strategy s^b in (2.1) with $\mathcal{S} = [\underline{\theta}, \theta^b]$.*

In other words, a Bertrand competitor always patents some technologies in equilibrium. In the limit, for $\gamma \to 1$, the expropriation effect vanishes, and firm I patents all technologies (i.e., $\lim_{\gamma\to 1} \theta^b = \underline{\theta}$). As before, the unraveling result holds in this case. Interestingly, even in the absence of intellectual property rights (i.e., $\gamma = 0$) firm I shares some technologies in equilibrium. In spite of the full expropriation of any disclosed technology, the innovative firm still has an incentive to share some technologies with its competitors (i.e., any $\theta_I > \theta^b$), as is shown in Proposition 2. This results from the firm's incentive to strategically manage its competitors' expectations in the product market.

Figure 2 illustrates the equilibrium patenting incentives for a Bertrand duopolist ($N = 1$) in the absence of patent protection ($\gamma = 0$). The bold lines represent the best

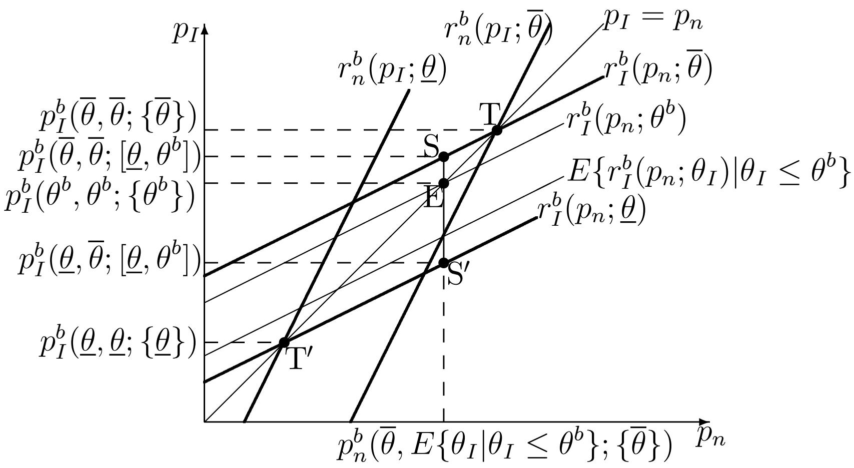

Figure 2: Equilibrium patenting (Bertrand competition, $\gamma = 0$)

response functions of the firms for extreme technologies, i.e., $r_I^b(p_n;\underline{\theta})$ and $r_I^b(p_n;\overline{\theta})$ for firm I, and $r_n^b(p_I;\underline{\theta})$ and $r_n^b(p_I;\overline{\theta})$ for firm n. If firm I shares its technology, the

[23] For example, for a firm with the least efficient technology ($\theta_I = \overline{\theta}$) the expropriation effect is absent, while the signaling effect remains, if firms $1, ..., N$ do not hold degenerate beliefs about firm i's cost (i.e., $E\{\theta_I|\theta_I \in \mathcal{S}\} \neq \overline{\theta}$), and is at its strongest.

equilibrium prices correspond to a point on the line T-T′. For example, if the firm has technology $\underline{\theta}$ and shares it, the equilibrium prices correspond to point T′; if it shares θ^b, then the firms reach equilibrium point E; sharing technology $\overline{\theta}$ yields point T. The adoption of secrecy gives the following. Firm n has technology $\overline{\theta}$ and it believes that firm I has a pricing strategy that corresponds to the expected best response $E\{r_I^b(p_n;\theta_I)|\theta_I \leq \theta^b\}$, which lies between the curves $r_I^b(p_n;\theta^b)$ and $r_I^b(p_n;\underline{\theta})$. The point where firm I's expected best response crosses firm n's best response $r_n^b(p_I;\overline{\theta})$ determines firm n's equilibrium price level, $p_n^b(\overline{\theta}, E\{\theta_I|\theta_I \leq \theta^b\};\{\overline{\theta}\})$. Firm I plays a best response against the price $p_n^b(\overline{\theta}, E\{\theta_I|\theta_I \leq \theta^b\};\{\overline{\theta}\})$, which yields a point along the line S-S′. For example, if the firm keeps technology $\underline{\theta}$ secret, the equilibrium prices correspond to point S′; if it hides θ^b, then the firms reach equilibrium point E; hiding technology $\overline{\theta}$ yields point S. Comparing the equilibrium prices that firm I sets after technology sharing with the firm's prices under secrecy gives the following. If firm I has a lower cost than θ^b, then it can reach a higher equilibrium price by adopting secrecy. For example, the firm that hides technology $\underline{\theta}$ sets price $p_I^b(\underline{\theta},\overline{\theta};[\underline{\theta},\theta^b])$ which is greater than the price it would set if it were to share the technology, $p_I^b(\underline{\theta},\underline{\theta};\{\underline{\theta}\})$, since point S′ lies above point T′. By contrast, if firm I's technology is less productive than θ^b, then technology sharing gives higher equilibrium prices. For example, the least efficient type sets $p_I^b(\overline{\theta},\overline{\theta};\{\overline{\theta}\})$ after it discloses, which is greater than its price under secrecy, $p_I^b(\overline{\theta},\overline{\theta};[\underline{\theta},\theta^b])$, since point T lies above point S. The threshold value for patenting, θ^b, is exactly the cost at which firm I is indifferent between patenting and trade secrecy (point E), given beliefs of firm n consistent with patenting strategy s^b in (2.1) for $\mathcal{S} = [\underline{\theta},\theta^b]$.

3.3 Comparative Statics

In the next sections I consider comparative statics results for the extremal equilibrium thresholds (Milgrom and Roberts, 1994), since there may be multiple equilibria.[24] That is, I consider the effects of changing a parameter value on the lowest and highest equilibrium thresholds of the patenting strategies s^c in Proposition 1 and s^b in Proposition 2.

An increase of the patent validity parameter γ (i.e., stronger patent protection) yields more patenting in equilibrium, as the following proposition shows.

Proposition 3 *Any extremal equilibrium threshold θ^c in patenting strategy s^c is non-decreasing in γ, and any extremal equilibrium threshold θ^b in s^b is decreasing in γ. In*

[24]If the equilibrium is unique, then this reduces to the standard monotonicity results.

the limit, for $\gamma \to 1$, firm I chooses the patenting strategy $s^c(\theta_I) = s^b(\theta_I) = \theta_I$ for any $\theta_I \in [\underline{\theta}, \overline{\theta}]$ in the unique equilibrium.

In other words, the stronger the patent protection, the weaker the expropriation effect, and the stronger firm I's incentive to patent the technology. This is intuitive.

The uniform technology distribution (i.e., $F(\theta_I) = (\theta_I - \underline{\theta})/(\overline{\theta} - \underline{\theta})$ for $\theta_I \in [\underline{\theta}, \overline{\theta}]$) yields an easy solution. Figure 3 illustrates the proposition for a uniformly distributed technology θ_I. The bold lines in Figure 3(a) illustrate the threshold values θ^c of the

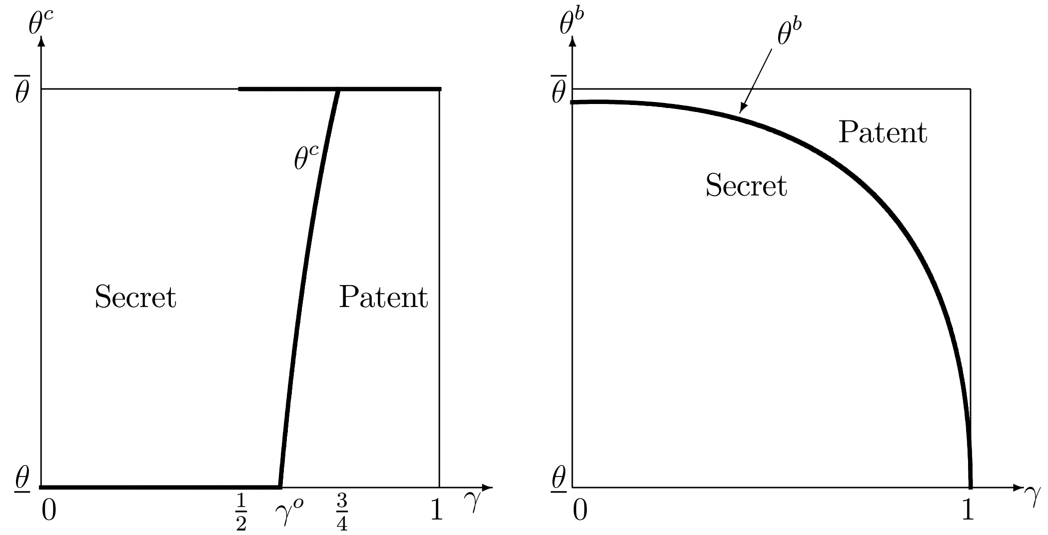

Fig. 3(a): Cournot competition   Fig. 3(b): Bertrand competition

Figure 3: Effect of protection (uniform distribution)

Cournot competitor's patenting strategy s^c in Proposition 1 for $\beta = 1$. Technologies above the curve remain secret, while technologies below the curve are patented. In Figure 3(b) the bold, downward-sloping curve sketches the threshold level θ^b of the Bertrand competitor's strategy in Proposition 2. Here technologies above the curve are patented, while technologies below the curve are kept secret.

A change of the technology distribution function has the following effects:

Proposition 4 *Let λ be a parameter of distribution F such that $E\{\theta_I|\theta_I > x\}$ and $E\{\theta_I|\theta_I \leq x\}$ are increasing in λ for all $x \in [\underline{\theta}, \overline{\theta}]$. Then any extremal equilibrium threshold θ^c of strategy s^c is non-decreasing in λ, while any extremal threshold θ^b of strategy s^b is increasing in λ.*

Skewing the distribution towards inefficient technologies (by increasing λ) gives a stronger signaling effect to a Cournot competitor. The disclosure of an efficient technology by a patent creates a more drastic update of the non-innovative firms' beliefs, and thereby a greater output effect. The stronger signaling effect gives a greater incentive to patent technologies. The reverse holds for a Bertrand competitor that considers patenting an inefficient technology. An increase of λ yields a weaker signaling effect, which gives the innovative firm a smaller patenting incentive.

For example, truncated exponential distributions satisfy the condition in Proposition 4. Assume that the technology θ_I lies in interval $\left[0, \overline{\theta}\right]$, and has the distribution $F(\theta_I; \lambda) = \left(1 - e^{-\theta_I/\lambda}\right) / \left(1 - e^{-\overline{\theta}/\lambda}\right)$. An increase of the hazard rate parameter λ increases the conditional expected costs $E\{\theta_I | \theta_I > x\}$ and $E\{\theta_I | \theta_I \leq x\}$ for all $x \in [0, \overline{\theta}]$.[25] Then Proposition 4 implies that the equilibrium patenting threshold θ^c is non-decreasing in λ, while threshold θ^b is increasing in λ.

4 Competitive Pressure

In this section I analyze the effects of competitive pressure on the incentives to patent the technology θ_I. First, I increase the competitive pressure by switching from competition in output levels (Cournot) to competition in prices (Bertrand). Second, I increase the number of non-innovative firms in the industry. Finally, I increase the degree of substitutability between products.

4.1 Mode of Competition

The competitive pressure on the innovative firm increases when the firms switch from competition in quantities to competition in prices (Singh and Vives, 1984). The comparison of equilibrium patenting strategies of Propositions 1 and 2 depends on the strength of intellectual property right protection (γ).

In particular, for sufficiently weak patent protection (e.g., $\gamma \leq \min\{1 - \frac{1}{2}\beta, \gamma^o\}$) an innovative firm patents more technologies under Bertrand than under Cournot competition. For these parameter values a firm adopts full secrecy under Cournot competition, while it adopts a selective patenting strategy, where the worst technologies are patented, under Bertrand competition. In other words, there is a greater diffusion of technology under Bertrand competition with weak protection.

[25] In particular, the conditional expected costs are $E\{\theta_I | \theta_I > x\} = \lambda + \frac{x \exp\{-x/\lambda\} - \overline{\theta} \exp\{-\overline{\theta}/\lambda\}}{\exp\{-x/\lambda\} - \exp\{-\overline{\theta}/\lambda\}}$ and $E\{\theta_I | \theta_I \leq x\} = \lambda - \frac{x \exp\{-x/\lambda\}}{1 - \exp\{-x/\lambda\}}$. It is straightforward to show that these conditional expected costs are increasing in λ for all $0 < x \leq \overline{\theta}$.

If, however, protection is sufficiently strong (e.g., $\gamma \geq \max\{1 - \frac{1}{4}\beta, \gamma^o\}$), then an innovative firm patents fewer technologies under Bertrand competition than under Cournot competition. Whereas firm I patents any technology (full patenting) for marginally weaker than perfect patent protection under Cournot competition, it keeps the most efficient technologies secret under Bertrand competition. That is, technology diffusion is smaller under Bertrand competition when patent protection is strong.

4.2 Number of Competitors

The competitive pressure on the innovative firm increases when the number of non-innovative firms, N, increases (Boone, 2000). Increasing N gives the following.

Proposition 5 *Any extremal equilibrium threshold θ^c in patenting strategy s^c is non-decreasing in N, and any extremal equilibrium threshold θ^b in s^b is increasing in N.*

In other words, in equilibrium the entry of non-innovative firms gives a Cournot competitor a greater or equal incentive to patent its innovation, while it gives a Bertrand competitor a smaller incentive to apply for a patent.

An analysis of the best response functions can provide some intuition for these results. The best response function $r_I^c(\mathbf{q}_{-I}; \theta_I)$ in (3.1), which captures the output strategy of firm I, is only a function of the cumulative output of the non-innovative firms, $Q_N \equiv \sum_{k=1}^{N} q_k$. Therefore, it can be redefined as the best response to the cumulative output per non-innovative firm:

$$R_I^c(\widehat{q}_N; \theta_I) = \frac{1}{2}\bigg(\alpha - \theta_I - \beta N \widehat{q}_N\bigg) \tag{4.1}$$

with $\widehat{q}_N \equiv Q_N/N$. Adding the best response functions of non-innovative firms, $r_n^c(\mathbf{q}_{-n}; \theta_n)$ in (3.1) for $n = 1, .., N$ with $\theta_1 = ... = \theta_N$, and solving for the sum of their outputs, Q_N, at any output of firm I, and dividing by N, gives the cumulative best response per non-innovative firm:

$$R_N^c(q_I; \theta_n) = \frac{\alpha - \theta_n - \beta q_I}{2 + (N-1)\beta} \tag{4.2}$$

The solution of (4.1) and (4.2) gives the equilibrium output levels of the innovative firm and a non-innovative firm. Analogously for Bertrand competition, the system of best response functions $r_\ell^b(\mathbf{p}_{-\ell}; \theta_\ell)$ in (3.7) for $\ell \in \{I, 1, .., N\}$ can be transformed into firm I's best response to the cumulative price per non-innovative firm, and the cumulative best response per non-innovative firm, respectively:

$$R_I^b(\widehat{p}_N;\theta_I) = \frac{1}{2}\left(\theta_I + \frac{(1-\beta)\alpha + \beta N \widehat{p}_N}{1+(N-1)\beta}\right) \tag{4.3}$$

$$R_N^b(p_I;\theta_n) = \frac{[1+(N-1)\beta]\,\theta_n + (1-\beta)\alpha + \beta p_I}{2+(N-1)\beta} \tag{4.4}$$

with $\widehat{p}_N \equiv \sum_{k=1}^N p_k/N$. Figure 4 illustrates these best responses for a given belief about firm I's technology (i.e., for some given subset $\mathcal{S}$), and $N' > N$. In particular,

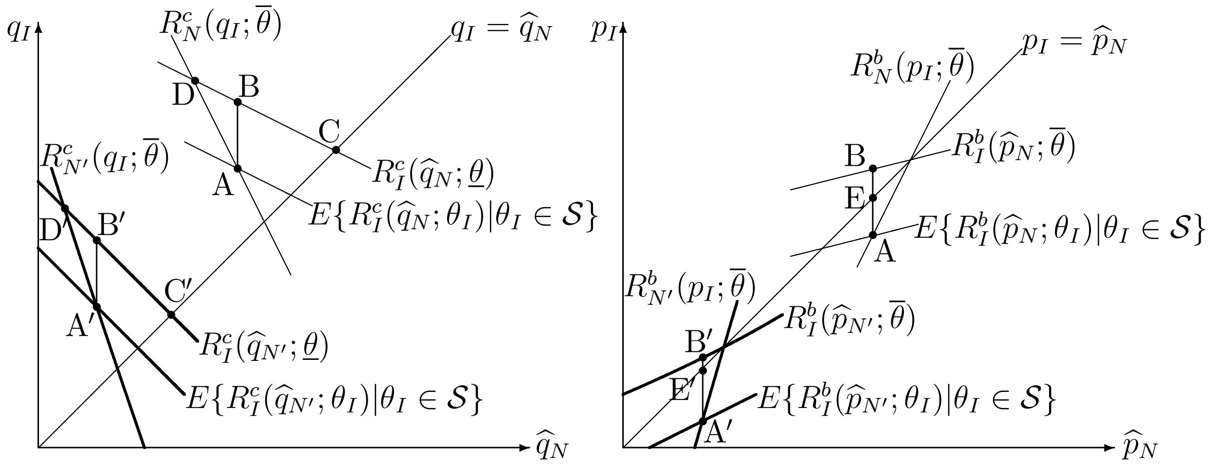

Fig. 4(a): Cournot competition　　Fig. 4(b): Bertrand competition

Figure 4: Effects of entry ($N' > N$)

Figure 4(a) sketches the best response curves of Cournot competitors, and Figure 4(b) gives these curves for Bertrand competitors. The thin (bold) curves represent the best response curves when there are N (respectively, N') non-innovative firms.

Figure 4(a) illustrates the equilibrium output levels of the innovative firm with the most efficient technology, $\theta_I = \underline{\theta}$. If this firm adopts secrecy and it has N competitors, it supplies the output corresponding to point B in Fig. 4(a). If firm I patents the technology $\underline{\theta}$ and has N competitors, it reaches point D when the patent is valid, and it reaches point C when the patent is invalid. An increase in the number of non-innovative firms has the following effects on firm I's patenting incentive. On the one hand, it makes the best response R_I^c steeper. All else equal, this makes the output difference $q_I^c(\theta_I, N\overline{\theta}; \{\theta_I\}) - q_I^c(\theta_I, N\overline{\theta}; \mathcal{S})$ relatively bigger in comparison to the difference $q_I^c(\theta_I, N\overline{\theta}; \{\theta_I\}) - q_I^c(\theta_I, N\theta_I; \{\theta_I\})$. In Fig. 4(a) this would correspond to a disproportional increase of vertical distance B-D compared to the vertical distance C-D. Moreover, a steeper R_I^c also increases the difference between $q_I^c(\theta_I, N\overline{\theta}; \{\theta_I\}) + q_I^c(\theta_I, N\overline{\theta}; \mathcal{S})$ and $q_I^c(\theta_I, N\overline{\theta}; \{\theta_I\}) + q_I^c(\theta_I, N\theta_I; \{\theta_I\})$. In Fig. 4(a) this would correspond to an increase in the vertical distance B-C. Both effects of a steeper own best response R_I^c augment the relative size of the right hand side of in (3.3) in comparison to the left hand side. In other words, it gives a stronger signaling effect. On the other hand, an increase in N makes the best response R_N^c less steep. All else equal, this gives the opposite effects (i.e., vertical distance B-D decreases relative to distance C-D, and vertical distance B-C decreases), which is favorable for the expropriation effect. Proposition 5 shows that the former effect dominates the latter. That is, the overall effect of increasing N is to strengthen the signaling effect relative to the expropriation effect, and thereby give a greater incentive to apply for a patent. As Fig. 4(a) illustrates for an increase from N to N', the best response R_I^c becomes steeper, and R_N^c becomes flatter.[26] The vertical distance B′-C′ is greater than the vertical distance B-C, whereas the proportions between the vertical distances B′-D′ and C′-D′ are equal to the proportions between the distances B-D and C-D.

Figure 4(b) considers Bertrand competition in the absence of patent protection ($\gamma = 0$). First, I consider the case in which firm I competes with N non-innovative firms. Analogous to the discussion of Figure 2, if the firm hides a technology of below-average efficiency, then it can reach some price along the line A-B in Fig. 4(b). If firm I has a technology such that its best response curve runs through point E, then the firm is indifferent between secrecy and technology sharing. The firm prefers to keep more efficient technologies secret, while it shares less efficient technologies. Second, similar incentives emerge in case there are N' non-innovative firms. An increase in the number of non-innovative firms (e.g. from N to N') makes the innovative firm's best response function R_I^b steeper, whereas it makes a non-innovative firm's cumulative best response R_N^b less steep, as is illustrated in the figure. Both effects give a higher cost θ_I at which firm I is indifferent between secrecy and technology sharing, for a given belief. In Fig. 4(b) this is captured by the fact that the distance A′-E′ exceeds the distance A-E, whereas the distance A′-B′ equals the distance A-B for a given belief about firm I's technology. Therefore, all else equal, firm I has an incentive to keep more technologies secret after the number of non-innovative firms grows.[27] The

[26] An increase of N also shifts both best response functions inwards (towards the origin), but this does not affect firm I's incentives to patent its technology.

[27] For example, the uniform technology distribution gives a unique patenting equilibrium. In the absence of protection, the patenting strategy for a uniformly distributed technology has the threshold value: $\theta_U^b = \overline{\theta} - \beta(\overline{\theta} - \underline{\theta}) / [4 + (4N-3)\beta]$. Clearly, θ_U^b increases in N.

proposition shows that this holds also in the presence of patent protection.

4.3 Product Differentiation

An alternative way of increasing the competitive pressure on the innovative firm is to increase the degree of substitutability between products, β (Boone, 2000).

The following proposition shows that an increase of the product substitutability tends to increase patenting in equilibrium.

Proposition 6 *Any extremal equilibrium threshold θ^c in patenting strategy s^c is non-decreasing in β. There exists a critical degree of substitutability, $\beta^b > 0$, such that for all $\beta < \beta^b$, any extremal equilibrium threshold θ^b in strategy s^b is decreasing in β. Moreover, if $\gamma = 0$ (no protection), and the firms choose accommodating pricing strategies, then any extremal equilibrium threshold θ^b in s^b is decreasing in β.*

A greater product substitutability gives an innovative firm a (weakly) greater incentive to patent its innovation when the firms compete in quantities. An increase of β makes the best response functions R_I^c in (4.1) and R_N^c in (4.2) steeper. As argued in the previous subsection, both effects yield a relatively stronger signaling effect. Therefore, the innovative firm has a greater incentive to patent an innovation.

When firms compete in prices, the patenting incentive follows from a trade-off between two opposing effects. An increase of β makes the best responses (4.3) and (4.4) steeper. On the one hand, a steeper best response of firm I, R_I^b in (4.3), reduces the incentive to share technologies. The previous subsection illustrates this point. On the other hand, a steeper cumulative best response per non-innovative firm, R_N^b in (4.4), gives a greater incentive to patent technologies. Proposition 6 shows that the latter effect tends to dominate.[28]

At the extreme where goods approach independence (i.e., $\beta \to 0$), the signaling effect diminishes. The remaining expropriation effect gives firm I a disincentive to patent its technology. In the limit firm I does not patent any technology (i.e., $\lim_{\beta\to 0}\theta^b = \overline{\theta}$ and $\lim_{\beta\to 0}\theta^c = \underline{\theta}$).[29] For positive degrees of substitutability the firm may have an incentive to patent some technologies in equilibrium (Propositions 1 and

[28]For example, in the absence of patent protection, the uniform technology distribution gives the unique threshold $\theta_U^b = \overline{\theta} - \beta(\overline{\theta} - \underline{\theta})/\left[4 + (4N-3)\beta\right]$. Clearly, θ_U^b is decreasing in β.

[29]Clearly, if $\beta = 0$, the markets are independent, and firm I is indifferent between patenting and secrecy. As a consequence, any patenting strategy can be sustained as an equilibrium strategy. If $\beta < 0$, then the goods are complements. As before, imitation gives the non-innovative firms an incentive to set lower prices (higher outputs). In the case of complementary goods, the competitors' price reduction (resp., output expansion) increases the demand and profit of the innovative firm. In

2). This suggests that, at least locally (for β close to zero), patenting incentives are growing in the degree of substitutability. Proposition 6 confirms this.

5 Conclusion

In this paper I analyzed the effects of probabilistic patent validity on strategic patent choices in an oligopoly with asymmetric information, and differentiated goods. A Cournot competitor tends to patent big innovations, and keep small innovations secret, while a Bertrand competitor adopts the reverse strategy.

Changing the mode of product market competition has interesting effects on the diffusion of knowledge. If the patent protection is weak, then an innovative firm patents more technologies under Bertrand competition than under Cournot competition. For sufficiently weak protection of intellectual property a firm adopts full secrecy under Cournot competition, while it adopts a selective patenting strategy under Bertrand competition. In this case, the bigger diffusion of technology increases the expected consumer surplus under Bertrand competition, which widens the surplus gap between Bertrand and Cournot competition.

If, however, protection is sufficiently strong, but imperfect, then an innovative firm patents more technologies under Cournot competition than under Bertrand competition. Whereas a Cournot competitor patents any innovation (due to an unraveling result), a Bertrand competitor resorts to a selective patenting strategy. In this case the greater technology diffusion under Cournot competition increases the expected consumer surplus under Cournot competition, and reduces the surplus gap between Bertrand and Cournot competition.

Different measures of competitive pressure have different effects on the incentives to patent a process innovation. An increase in the degree of substitutability tends to increase the patenting incentives of accommodating firms. The effect of an increase in the number of firms depends on the mode of competition in the product market. An increase in the number of non-innovative firms gives an innovative firm a greater incentive to patent when firms compete in output levels, but a smaller incentive when firms compete in prices. In the latter case, an increase in the number of non-innovative firms has two conflicting effect on the expected consumer surplus. On the one hand, it increases the expected consumer surplus for a given level of technology diffusion.

other words, expropriation gives the innovative firm an extra incentive to apply for a patent. Hence, the basic trade-off between expropriation and signaling disappears, and the standard unraveling result applies (i.e., the innovative firm patents all technologies), whenever the goods are complementary.

This is a direct effect. On the other hand, it reduces the expected surplus through a reduction in the diffusion of technology. This is an indirect effect. That is, the strategic management of intellectual property reduces the surplus gain from entry of non-innovative firms. This may have implications for the optimal economic policy towards entry in innovative industries with weak intellectual property right protection.

The model assumes that the firms choose their product market variables after the patent validity is determined. Alternatively, one could consider the model where the patent validity is determined after the firms set their product market variables. In the subgame that starts after the innovative firm patents its technology, a non-innovative firm chooses its product market variable that maximizes its expected profit at the expected cost $\gamma\overline{\theta} + (1-\gamma)\theta_I$. That is, in this model with reversed timing, the profit from patenting is the profit at the competitors' expected cost, instead of the expected profit at the competitors' realized costs. Although this changes the size of the profit difference, it does not change the direction in which this difference changes with parameter values (see Appendix C for further details). Therefore, reversing the timing has no effect on the qualitative results.

The model with Bertrand competition can be extended easily by allowing all firms to be innovative. In a simple model where patents are invalid (i.e., $\gamma = 0$), and the technologies of firms are independent draws from their technology distributions, a firm with access to a competitor's technologies adopts the most productive technology. This could be its own or its competitor's technology. Therefore, it is uncertain whether a shared technology will be adopted or not, since this depends on the relative efficiency of both firms' technology draws. Whereas in the model with one-sided asymmetric information the probability of imitation was exogenously fixed, here it depends on the size of the innovation, and the technology distribution of the competitor. In spite of this difference, the firms' incentives to share technologies are similar to the incentives with one-sided asymmetric information (see also Appendix C). As before, a firm's technology-sharing strategy trades off an expropriation effect against a signaling effect. Moreover, a higher cost draw gives a weaker expropriation effect and a stronger signaling effect. Consequently, each firm shares inefficient technologies while it keeps efficient technologies secret as in Proposition 2. Jansen (2010) shows that the introduction of several innovative firms in the model with Cournot competition has more subtle effects on the firms' patenting incentives.

Appendix

A Proofs of Propositions

Proof of Proposition 1

Suppose that the non-innovative firms have beliefs that are consistent with patenting strategy (2.1). Then firm I keeps technology θ_I secret if and only if inequality (3.3) holds. This inequality is equivalent to $\Phi^c(\theta_I; \mathcal{S}) \geq 0$ where:

$$\begin{aligned}\Phi^c(\theta; \mathcal{S}) &\equiv 1 - \gamma - \frac{q_I^c(\theta, N\overline{\theta}; \{\theta\})^2 - q_I^c(\theta, N\overline{\theta}; \mathcal{S})^2}{q_I^c(\theta, N\overline{\theta}; \{\theta\})^2 - q_I^c(\theta, N\theta; \{\theta\})^2} \qquad (A.1)\\ &= 1 - \gamma - \frac{\frac{\beta}{2}\left(E\{\theta_I | \theta_I \in \mathcal{S}\} - \theta\right)}{\overline{\theta} - \theta} \cdot \frac{q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\overline{\theta}; \mathcal{S})}{q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\theta; \{\theta\})}\end{aligned}$$

For any given set $\mathcal{S}$, the function $\Phi^c(\theta; \mathcal{S})$ is increasing in θ (see Appendix B). Therefore, there can exist only equilibria with $\mathcal{S} = [\theta^c, \overline{\theta}]$ for some $\underline{\theta} \leq \theta^c \leq \overline{\theta}$. In particular, three situations may emerge in equilibrium.

(a) Firm I keeps all technologies secret in equilibrium (i.e., $\theta^c = \underline{\theta}$), if and only if $\Phi^c(\theta; [\underline{\theta}, \overline{\theta}]) \geq 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. This inequality holds for all θ if and only if $\Phi^c(\underline{\theta}; [\underline{\theta}, \overline{\theta}]) \geq 0$, since $\Phi^c(\theta; [\underline{\theta}, \overline{\theta}])$ is increasing in θ. The latter inequality holds if and only if $\gamma \leq \gamma^o$, where γ^o is defined as (3.4).

(b) Firm I patents all technologies in equilibrium (i.e., $\theta^c = \overline{\theta}$), if and only if $\Phi^c(\theta; \{\overline{\theta}\}) \leq 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. This inequality holds if and only if $\Phi^c(\overline{\theta}; \{\overline{\theta}\}) \leq 0$, since $\Phi^c(\theta; \{\overline{\theta}\})$ is increasing in θ, which reduces to $\gamma \geq 1 - \frac{\beta}{2}$.

(c) Suppose that $\gamma^o < 1 - \frac{\beta}{4}$, and take $\gamma^o < \gamma < 1 - \frac{\beta}{4}$. If there exists an equilibrium with $\mathcal{S} = [\theta^c, \overline{\theta}]$ for some $\underline{\theta} < \theta^c < \overline{\theta}$, then the threshold θ^c is a root of function:

$$\widetilde{\Phi}^c(\theta) \equiv \Phi^c(\theta; [\theta, \overline{\theta}]) \qquad (A.2)$$

for $\theta \in [\underline{\theta}, \overline{\theta}]$. Evaluation of $\widetilde{\Phi}^c$ for extreme variable values gives the following. First, $\widetilde{\Phi}^c(\underline{\theta}) = \Phi^c(\underline{\theta}; [\underline{\theta}, \overline{\theta}]) < 0$ for $\gamma > \gamma^o$, as follows from part (b). Second, from (A.1) and application of the De L'Hospital rule it follows that:

$$\begin{aligned}\lim_{\theta \uparrow \overline{\theta}} \widetilde{\Phi}^c(\theta) &= 1 - \gamma - \lim_{\theta \uparrow \overline{\theta}} \frac{\frac{\beta}{2}\left(E\{\theta_I | \theta_I > \theta\} - \theta\right)}{\overline{\theta} - \theta} \cdot \frac{q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\overline{\theta}; [\theta, \overline{\theta}])}{q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\theta; \{\theta\})}\\ &= 1 - \gamma - \lim_{\theta \uparrow \overline{\theta}} \frac{\frac{\beta}{2}\left(E\{\theta_I | \theta_I > \theta\} - \theta\right)}{\overline{\theta} - \theta} = \left(1 - \frac{\beta}{4}\right) - \gamma,\end{aligned}$$

since

$$\begin{aligned}
\lim_{\theta\uparrow\overline{\theta}}\frac{\partial E\{\theta_I|\theta_I>\theta\}}{\partial\theta} &= \lim_{\theta\uparrow\overline{\theta}}\frac{\partial}{\partial\theta}\left(\int_{\theta}^{\overline{\theta}}\frac{f(z)}{1-F(\theta)}zdz\right)\\
&= \lim_{\theta\uparrow\overline{\theta}}\frac{f(\theta)}{1-F(\theta)}\Big(E\{\theta_I|\theta_I>\theta\}-\theta\Big)\\
&= f(\overline{\theta})\lim_{\theta\uparrow\overline{\theta}}\frac{E\{\theta_I|\theta_I>\theta\})-\theta}{1-F(\theta)}=\frac{1}{2}
\end{aligned}$$

where the last equality follows from the application of the De L'Hospital rule, i.e.,

$$\begin{aligned}
\lim_{\theta\uparrow\overline{\theta}}\frac{E\{\theta_I|\theta_I>\theta\})-\theta}{1-F(\theta)} &= \lim_{\theta\uparrow\overline{\theta}}\frac{\frac{f(\theta)}{1-F(\theta)}\Big(E\{\theta_I|\theta_I>\theta\}-\theta\Big)-1}{-f(\theta)}\\
&= \frac{1}{f(\overline{\theta})}-\lim_{\theta\uparrow\overline{\theta}}\frac{E\{\theta_I|\theta_I>\theta\}-\theta}{1-F(\theta)}
\end{aligned}$$

yielding

$$\lim_{\theta\uparrow\overline{\theta}}\frac{E\{\theta_I|\theta_I>\theta\})-\theta}{1-F(\theta)}=\frac{1}{2f(\overline{\theta})}$$

Therefore, $\lim_{\theta\uparrow\overline{\theta}}\widetilde{\Phi}^c(\theta)>0$ for $\gamma<1-\frac{\beta}{4}$. Hence, for any $\gamma^o<\gamma<1-\frac{\beta}{4}$, the intermediate value theorem implies that there exists an interior θ^c such that $\widetilde{\Phi}^c(\theta^c)=0$, since $\widetilde{\Phi}^c(\underline{\theta})<0<\lim_{\theta\uparrow\overline{\theta}}\widetilde{\Phi}^c(\theta)$, and $\widetilde{\Phi}^c$ is continuous in θ. □

Proof of Proposition 2

Suppose that the non-innovative firms have beliefs that are consistent with patenting strategy (2.1). Then firm I keeps technology θ_I secret if and only if inequality (3.3) holds, with π^c replaced by π^b. This inequality is equivalent to $\Phi^b(\theta_I;\mathcal{S})\geq 0$ where:

$$\Phi^b(\theta;\mathcal{S})\equiv 1-\gamma-\frac{m_I^b(\theta,N\overline{\theta};\{\theta\})^2-m_I^b(\theta,N\overline{\theta};\mathcal{S})^2}{m_I^b(\theta,N\overline{\theta};\{\theta\})^2-m_I^b(\theta,N\theta;\{\theta\})^2} \tag{A.3}$$

Using (3.9), this function can be written as:

$$\Phi^b(\theta;\mathcal{S})=1-\gamma-\frac{\frac{\beta}{2}\left(\theta-E\{\theta_I|\theta_I\in\mathcal{S}\}\right)}{[1+(N-1)\beta](\overline{\theta}-\theta)}\cdot\frac{m_I^b(\theta,N\overline{\theta};\{\theta\})+m_I^b(\theta,N\overline{\theta};\mathcal{S})}{m_I^b(\theta,N\overline{\theta};\{\theta\})+m_I^b(\theta,N\theta;\{\theta\})}$$

It is straightforward to show that Φ^b is decreasing in θ (see Appendix B). As a consequence, there can only exist equilibria in which the patenting strategy (2.1) has $\mathcal{S}=[\underline{\theta},\theta^b]$ for some $\theta^b\in[\underline{\theta},\overline{\theta}]$.

The evaluation of Φ^b for extreme values of θ yields the following. $\Phi^b(\underline{\theta},[\underline{\theta},\theta^b]) > 0 > \Phi^b(\overline{\theta},[\underline{\theta},\theta^b])$ for any $\theta^b \in [\underline{\theta},\overline{\theta}]$. Consequently (due to continuity), only critical values $\underline{\theta} < \theta^b < \overline{\theta}$ can be consistent with the equilibrium patenting strategy. The equilibrium threshold value θ^b is the root of:

$$\widetilde{\Phi}^b(\theta) \equiv \Phi^b(\theta,[\underline{\theta},\theta]) \tag{A.4}$$

Clearly, $\widetilde{\Phi}^b(\underline{\theta}) = \Phi^b(\underline{\theta},\{\underline{\theta}\}) > 0$ for any $\gamma < 1$, and $\widetilde{\Phi}^b(\overline{\theta}) = \Phi^b(\overline{\theta},[\underline{\theta},\overline{\theta}]) < 0$. The intermediate value theorem implies that a θ^b exists, with $\underline{\theta} < \theta^b < \overline{\theta}$, such that $\widetilde{\Phi}^b(\theta^b) = 0$. □

Proof of Proposition 3

In case of Cournot competition, notice that $\widetilde{\Phi}^c$ in (A.2) is continuous in θ and decreasing in parameter γ.

First, consider the minimal threshold, θ^c_L. For $\gamma \le \gamma^o$, Proposition 1 (a) shows that $\theta^c_L = \underline{\theta}$. For $\gamma^o < \gamma < 1 - \frac{\beta}{4}$, Proposition 1 (c) shows that $\widetilde{\Phi}^c(\underline{\theta}) < 0 < \lim_{\theta\uparrow\overline{\theta}} \widetilde{\Phi}^c(\theta)$, and there exists an equilibrium with an interior threshold $\theta^c_L > \underline{\theta}$. In this case, it follows from Milgrom and Roberts (1994, Theorem 1) that the minimal interior threshold is increasing in γ. For $\gamma \ge \max\{\gamma^o, 1 - \frac{\beta}{4}\}$, both $\widetilde{\Phi}^c(\underline{\theta})$ and $\lim_{\theta\uparrow\overline{\theta}} \widetilde{\Phi}^c(\theta)$ are non-positive. In that case, if $\widetilde{\Phi}^c(\theta)$ has an interior root, then $\widetilde{\Phi}^c$ crosses zero from below at the minimal root, and consequently the minimal root increases in γ. Otherwise, i.e., if $\widetilde{\Phi}^c(\theta)$ has no interior root, $\theta^c_L = \overline{\theta}$.

Second, consider the maximal threshold, θ^c_H. For $\gamma \le \min\{\gamma^o, 1 - \frac{\beta}{2}\}$, both $\widetilde{\Phi}^c(\underline{\theta})$ and $\lim_{\theta\uparrow\overline{\theta}} \widetilde{\Phi}^c(\theta)$ are non-negative. In that case, if $\widetilde{\Phi}^c(\theta)$ has an interior root, then $\widetilde{\Phi}^c$ crosses zero from below at the maximal root, and consequently the maximal root increases in γ. Otherwise, i.e., if $\widetilde{\Phi}^c(\theta)$ has no interior root, $\theta^c_H = \underline{\theta}$. For $\gamma^o < \gamma < 1 - \frac{\beta}{2}$, Proposition 1 (b)-(c) show that the maximal root is interior (i.e., $\underline{\theta} < \theta^c_H < \overline{\theta}$), and $\widetilde{\Phi}^c(\underline{\theta}) < 0 < \lim_{\theta\uparrow\overline{\theta}} \widetilde{\Phi}^c(\theta)$. In this case, it follows from Milgrom and Roberts (1994, Theorem 1) that the maximal interior threshold is increasing in γ. Finally, for $\gamma \ge 1 - \frac{\beta}{2}$, Proposition 1 (b) shows that $\theta^c_H = \overline{\theta}$.

Finally, in case of Bertrand competition, notice that the function $\widetilde{\Phi}^b$ in (A.4) is continuous in θ, and it is decreasing in γ, i.e., $\partial\widetilde{\Phi}^b/\partial\gamma < 0$ as follows immediately from (A.3). An equilibrium threshold θ^b is the root of $\widetilde{\Phi}^b(\theta) = 0$. In the proof of Proposition 2 I show that $\widetilde{\Phi}^b(\underline{\theta}) > 0 > \widetilde{\Phi}^b(\overline{\theta})$. Then Milgrom and Roberts (1994, Theorem 1) show that any extremal equilibrium threshold must be decreasing in γ.

For $\gamma \to 1$, $\Phi^c(\theta; [\theta, \overline{\theta}]) = 0$ iff $\theta = E\{\theta_I | \theta_I > \theta\}$. Clearly, the only possible equilibrium strategy is (2.1) with $\mathcal{S} = \{\overline{\theta}\}$. Similarly, for $\gamma \to 1$, $\Phi^b(\theta; [\underline{\theta}, \theta]) = 0$ iff $\theta = E\{\theta_I | \theta_I \leq \theta\}$, which gives uniquely $\mathcal{S} = \{\underline{\theta}\}$.[30] □

Proof of Proposition 4

The proof is similar to the proof of Proposition 3. It is easy to show that the function $\widetilde{\Phi}^c(\theta)$ is decreasing in λ, since $\partial\widetilde{\Phi}^c / \partial E\{\theta_I | \theta_I > \theta\} < 0$, as follows from (A.1). Further, $\widetilde{\Phi}^b$ is increasing in λ, since $\partial\widetilde{\Phi}^b / \partial E\{\theta_I | \theta_I \leq \theta\} > 0$ as follows easily from (A.3). Again, using continuity of $\widetilde{\Phi}^c$ and $\widetilde{\Phi}^b$, and applying Milgrom and Roberts (1994, Theorem 1) gives the comparative statics results with respect to λ. □

Proof of Proposition 5

The proof is similar to the proof of Proposition 3. In Appendix B, I show that the function $\widetilde{\Phi}^c(\theta)$ is decreasing in N for any given θ. Further, Appendix B shows that $\widetilde{\Phi}^b$ is increasing in N. Again, using continuity of $\widetilde{\Phi}^c$ and $\widetilde{\Phi}^b$, and applying Milgrom and Roberts (1994, Theorem 1) gives the comparative statics results on N. □

Proof of Proposition 6

The proof is similar to the proof of Proposition 3. First, Appendix B shows that the function $\widetilde{\Phi}^c(\theta)$ is decreasing in β for any given θ. Second, Appendix B shows that $\lim_{\beta \to 0} \partial\widetilde{\Phi}^b / \partial\beta < 0$. Due to continuity of $\partial\widetilde{\Phi}^b / \partial\beta$ in β, there exists a critical degree $\beta^b > 0$, such that $\widetilde{\Phi}^b$ is decreasing in β for all $\beta < \beta^b$. Finally, if $\gamma = 0$, then the root of $\widetilde{\Phi}^b(\theta) = 0$ equals the root of:

$$\begin{aligned} \widetilde{\phi}^b(\theta) &\equiv \frac{1}{\beta^2}\Big(m_I^b(\theta, N\overline{\theta}; [\underline{\theta}, \theta]) - m_I^b(\theta, N\theta; \{\theta\}) \Big) \\ &= \left(\frac{1}{\beta} + N - 1\right) N(\overline{\theta} - \theta) - \frac{N}{2}(\theta - E\{\theta_I | \theta_I \leq \theta\}). \end{aligned}$$

This function is non-decreasing in θ at extremal roots, since it is continuous in θ, and $\widetilde{\phi}^b(\underline{\theta}) > 0 > \widetilde{\phi}^b(\overline{\theta})$. Further, the function $\widetilde{\phi}^b$ is decreasing in parameter β. Using these properties, the continuity of $\widetilde{\Phi}^c$, $\widetilde{\Phi}^b$ and $\widetilde{\phi}^b$, and applying Milgrom and Roberts (1994, Theorem 1) gives the comparative statics results on β. □

[30] Any interior threshold (i.e., $\underline{\theta} < \theta^b < \overline{\theta}$) cannot emerge in equilibrium, since a root of $\widetilde{\Phi}^b$ would require that $E\{\theta_I | \theta_I \leq \theta^b\} = \theta^b$, which is impossible for non-degenerate p.d.f.-s. Also $\theta^b = \overline{\theta}$ cannot emerge in equilibrium, since $\Phi^b(\theta, [\underline{\theta}, \overline{\theta}]) > 0$ for any $\theta > E\{\theta_I\}$.

B Basic Properties of Φ

First, I present the basic properties of Φ^c in (A.1). Second, I present the basic properties of Φ^b in (A.3).

B.1 Cournot Competition

Consider any given set $\mathcal{S} \subseteq [\underline{\theta}, \overline{\theta}]$. First, it is useful to show that $\Phi^c(\theta; \mathcal{S})$ in (A.1) is decreasing in $E\{\theta_I | \theta_I \in \mathcal{S}\}$:

$$
\begin{aligned}
\frac{\partial \Phi^c(\theta; \mathcal{S})}{\partial E\{\theta_I | \theta_I \in \mathcal{S}\}} &= \frac{-\frac{\beta}{2}}{\overline{\theta} - \theta} \cdot \frac{q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\overline{\theta}; \mathcal{S}) + \frac{\frac{\beta}{2}\beta N(\theta - E\{\theta_I|\theta_I \in \mathcal{S}\})}{(2+\beta N)(2-\beta)}}{q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\theta; \{\theta\})} \\
&= \frac{-\frac{\beta}{2}}{\overline{\theta} - \theta} \cdot \frac{2 q_I^c(\theta, N\overline{\theta}; \mathcal{S})}{q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\theta; \{\theta\})} < 0 \qquad \text{(B.1)}
\end{aligned}
$$

Differentiating (B.1) with respect to θ gives:

$$
\begin{aligned}
\frac{\partial^2 \Phi^c(\theta; \mathcal{S})}{\partial E\{\theta_I | \theta_I \in \mathcal{S}\} \partial \theta} &= \frac{-\beta}{(\overline{\theta} - \theta)^2} \cdot \frac{q_I^c(\theta, N\overline{\theta}; \mathcal{S}) - \frac{(2-\beta) + \beta N - \frac{\beta}{2}\beta N}{(2+\beta N)(2-\beta)}(\overline{\theta} - \theta)}{q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\theta; \{\theta\})} \\
&\quad - \frac{\beta}{\overline{\theta} - \theta} \cdot \frac{\frac{2(2-\beta)+\beta N}{(2+\beta N)(2-\beta)} q_I^c(\theta, N\overline{\theta}; \mathcal{S})}{\left[q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\theta; \{\theta\})\right]^2} \\
&< \frac{-\beta}{(\overline{\theta} - \theta)^2} \cdot \frac{q_I^c(\overline{\theta}, N\overline{\theta}; \mathcal{S})}{q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\theta; \{\theta\})} < 0 \qquad \text{(B.2)}
\end{aligned}
$$

Inequality (B.2) is useful to show that $\Phi^c(\theta; \mathcal{S})$ is increasing in θ for any given $\mathcal{S}$.

$$
\begin{aligned}
\frac{\partial \Phi^c(\theta; \mathcal{S})}{\partial \theta} &= \frac{\frac{\beta}{2}\left(\overline{\theta} - E\{\theta_I | \theta_I \in \mathcal{S}\}\right)}{(\overline{\theta} - \theta)^2} \cdot \frac{q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\overline{\theta}; \mathcal{S})}{q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\theta; \{\theta\})} \\
&\quad + \frac{\frac{\beta}{2}(\theta - E\{\theta_I | \theta_I \in \mathcal{S}\})}{\overline{\theta} - \theta} \cdot \frac{\partial}{\partial \theta}\left(\frac{q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\overline{\theta}; \mathcal{S})}{q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\theta; \{\theta\})}\right) \qquad \text{(B.3)}
\end{aligned}
$$

with

$$
\frac{\partial}{\partial \theta}\left(\frac{q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\overline{\theta}; \mathcal{S})}{q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\theta; \{\theta\})}\right) = \frac{\partial}{\partial \theta}\left(1 + \frac{q_I^c(\theta, N\overline{\theta}; \mathcal{S}) - q_I^c(\theta, N\theta; \{\theta\})}{q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\theta; \{\theta\})}\right)
$$

$$
\begin{aligned}
&= \frac{-\frac{\frac{1}{2}\beta N(2-\beta)}{(2+\beta N)(2-\beta)}\left[q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\theta; \{\theta\})\right]}{\left[q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\theta; \{\theta\})\right]^2} \\
&\quad + \frac{\frac{2(2-\beta)+\beta N}{(2+\beta N)(2-\beta)}\left[q_I^c(\theta, N\overline{\theta}; \mathcal{S}) - q_I^c(\theta, N\theta; \{\theta\})\right]}{\left[q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\theta; \{\theta\})\right]^2} \\
&= \frac{-\frac{1}{2}\beta N(2-\beta)\left[2(2-\beta)(\alpha-\theta) + \beta N(\overline{\theta}-\theta)\right]}{(2+\beta N)^2(2-\beta)^2\left[q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\theta; \{\theta\})\right]^2} \\
&\quad + \frac{\beta N[2(2-\beta)+\beta N]\left[\overline{\theta} - \theta - \frac{\beta}{2}(E\{\theta_I|\theta_I \in \mathcal{S}\} - \theta)\right]}{(2+\beta N)^2(2-\beta)^2\left[q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\theta; \{\theta\})\right]^2}
\end{aligned}
$$

Since inequality (B.2) implies $\partial\Phi^c(\theta;\mathcal{S})/\partial\theta > \partial\Phi^c(\theta;\{\overline{\theta}\})/\partial\theta$, the following holds:

$$
\begin{aligned}
\frac{\partial\Phi^c(\theta;\mathcal{S})}{\partial\theta} &> -\frac{\beta}{2}\cdot\frac{\partial}{\partial\theta}\left(\frac{q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\overline{\theta}; \{\overline{\theta}\})}{q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\theta; \{\theta\})}\right) \\
&= -\frac{\beta}{2}\cdot\frac{-\frac{1}{2}\beta N(2-\beta)2(2-\beta)(\alpha-\overline{\theta})}{(2+\beta N)^2(2-\beta)^2\left[q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\theta; \{\theta\})\right]^2} > 0
\end{aligned}
$$

B.1.1 Number of Firms

Consider any given technology $\underline{\theta} < \theta < \overline{\theta}$. The function $\widetilde{\Phi}^c(\theta)$ in (A.2) can be written as:

$$
\begin{aligned}
\widetilde{\Phi}^c(\theta) &= 1 - \gamma - \frac{\frac{\beta}{2}\left(E\{\theta_I|\theta_I > \theta\} - \theta\right)}{\overline{\theta} - \theta}\cdot\left(1 + \frac{q_I^c(\theta, N\overline{\theta}; [\theta, \overline{\theta}]) - q_I^c(\theta, N\theta; \{\theta\})}{q_I^c(\theta, N\overline{\theta}; \{\theta\}) + q_I^c(\theta, N\theta; \{\theta\})}\right) \\
&= 1 - \gamma - \frac{\frac{\beta}{2}\left(E\{\theta_I|\theta_I > \theta\} - \theta\right)}{\overline{\theta} - \theta}\cdot\left(1 + \frac{\beta N\left[\overline{\theta} - (1-\frac{\beta}{2})\theta - \frac{\beta}{2}E\{\theta_I|\theta_I > \theta\}\right]}{2(2-\beta)(\alpha-\theta) + \beta N(\overline{\theta}-\theta)}\right)
\end{aligned}
$$

Hence, differentiating with respect to N gives:

$$
\begin{aligned}
\frac{\partial\widetilde{\Phi}^c(\theta)}{\partial N} &= \frac{-\frac{\beta}{2}\left(E\{\theta_I|\theta_I > \theta\} - \theta\right)}{\overline{\theta} - \theta}\cdot\frac{\partial}{\partial N}\left(1 + \frac{\beta N\left[\overline{\theta} - (1-\frac{\beta}{2})\theta - \frac{\beta}{2}E\{\theta_I|\theta_I > \theta\}\right]}{2(2-\beta)(\alpha-\theta) + \beta N(\overline{\theta}-\theta)}\right) \\
&= \frac{-\frac{\beta}{2}\left(E\{\theta_I|\theta_I > \theta\} - \theta\right)}{\overline{\theta} - \theta}\cdot\frac{2\beta(2-\beta)(\alpha-\theta)\left[\overline{\theta} - (1-\frac{\beta}{2})\theta - \frac{\beta}{2}E\{\theta_I|\theta_I > \theta\}\right]}{\left[2(2-\beta)(\alpha-\theta) + \beta N(\overline{\theta}-\theta)\right]^2} \\
&< 0.
\end{aligned}
$$

B.1.2 Degree of Substitutability

As before, for any given technology $\underline{\theta} < \theta < \overline{\theta}$, the function $\widetilde{\Phi}^c(\theta)$ in (A.2) can be written as:

$$
\widetilde{\Phi}^c(\theta) = 1 - \gamma - \frac{\frac{1}{2}\left(E\{\theta_I|\theta_I > \theta\} - \theta\right)}{\overline{\theta} - \theta}\cdot\beta\left(1 + \frac{\beta N\left[\overline{\theta} - (1-\frac{\beta}{2})\theta - \frac{\beta}{2}E\{\theta_I|\theta_I > \theta\}\right]}{2(2-\beta)(\alpha-\theta) + \beta N(\overline{\theta}-\theta)}\right)
$$

Differentiating this expression with respect to β gives:

$$\begin{aligned}\frac{\partial\widetilde{\Phi}^c(\theta)}{\partial\beta} &= \frac{-\frac{1}{2}\left(E\{\theta_I|\theta_I>\theta\}-\theta\right)}{\overline{\theta}-\theta}\Bigg(1+\frac{\beta N\left[\overline{\theta}-(1-\frac{\beta}{2})\theta-\frac{\beta}{2}E\{\theta_I|\theta_I>\theta\}\right]}{2(2-\beta)(\alpha-\theta)+\beta N(\overline{\theta}-\theta)} \\ &\quad +\beta N\frac{\overline{\theta}-(1-\beta)\theta-\beta E\{\theta_I|\theta_I>\theta\}}{2(2-\beta)(\alpha-\theta)+\beta N(\overline{\theta}-\theta)} \\ &\quad +\beta N\,\frac{\left[\overline{\theta}-(1-\frac{\beta}{2})\theta-\frac{\beta}{2}E\{\theta_I|\theta_I>\theta\}\right]\left[2\beta(\alpha-\theta)-\beta N(\overline{\theta}-\theta)\right]}{\left[2(2-\beta)(\alpha-\theta)+\beta N(\overline{\theta}-\theta)\right]^2}\Bigg) \\ &= \frac{-\frac{1}{2}\left(E\{\theta_I|\theta_I>\theta\}-\theta\right)}{\overline{\theta}-\theta}\Bigg(1+\frac{\beta N\left[\overline{\theta}-(1-\frac{\beta}{2})\theta-\frac{\beta}{2}E\{\theta_I|\theta_I>\theta\}\right]4(\alpha-\theta)}{\left[2(2-\beta)(\alpha-\theta)+\beta N(\overline{\theta}-\theta)\right]^2} \\ &\quad +\beta N\frac{\overline{\theta}-(1-\beta)\theta-\beta E\{\theta_I|\theta_I>\theta\}}{2(2-\beta)(\alpha-\theta)+\beta N(\overline{\theta}-\theta)}\Bigg)<0.\end{aligned}$$

B.2 Bertrand Competition

Consider any given set $\mathcal{S}\subseteq[\underline{\theta},\overline{\theta}]$. First, it is useful to show that $\Phi^b(\theta;\mathcal{S})$ in (A.3) is increasing in $E\{\theta_I|\theta_I\in\mathcal{S}\}$:

$$\begin{aligned}\frac{\partial\Phi^b(\theta;\mathcal{S})}{\partial E\{\theta_I|\theta_I\in\mathcal{S}\}} &= \frac{\frac{\beta}{2}}{[1+(N-1)\beta](\overline{\theta}-\theta)}\cdot\frac{m_I^b(\theta,N\overline{\theta};\{\theta\})+m_I^b(\theta,N\overline{\theta};\mathcal{S})}{m_I^b(\theta,N\overline{\theta};\{\theta\})+m_I^b(\theta,N\theta;\{\theta\})} \\ &\quad +\frac{\frac{\beta}{2}}{[1+(N-1)\beta](\overline{\theta}-\theta)}\cdot\frac{\frac{\beta}{2}\cdot\beta N\left(E\{\theta_I|\theta_I\in\mathcal{S}\}-\theta\right)}{m_I^b(\theta,N\overline{\theta};\{\theta\})+m_I^b(\theta,N\theta;\{\theta\})} \\ &= \frac{\beta}{[1+(N-1)\beta](\overline{\theta}-\theta)}\cdot\frac{m_I^b(\theta,N\overline{\theta};\mathcal{S})}{m_I^b(\theta,N\overline{\theta};\{\theta\})+m_I^b(\theta,N\theta;\{\theta\})}>0\end{aligned}$$

Differentiating this expression with respect to θ gives:

$$\begin{aligned}\frac{\partial^2\Phi^b(\theta;\mathcal{S})}{\partial E\{\theta_I|\theta_I\in\mathcal{S}\}\partial\theta} &= \beta\frac{m_I^b(\theta,N\overline{\theta};\mathcal{S})}{[1+(N-1)\beta](\overline{\theta}-\theta)^2\left[m_I^b(\theta,N\overline{\theta};\{\theta\})+m_I^b(\theta,N\theta;\{\theta\})\right]} \\ &\quad -\beta\frac{(1-\beta)[2+(2N-1)\beta]+[1+(N-1)\beta]\beta N+\frac{\beta}{2}\cdot\beta N}{[1+(N-1)\beta](\overline{\theta}-\theta)\left[m_I^b(\theta,N\overline{\theta};\{\theta\})+m_I^b(\theta,N\theta;\{\theta\})\right]} \\ &\quad +\beta\frac{m_I^b(\theta,N\overline{\theta};\mathcal{S})\left(2(1-\beta)[2+(2N-1)\beta]+[1+(N-1)\beta]\beta N\right)}{[1+(N-1)\beta](\overline{\theta}-\theta)\left[m_I^b(\theta,N\overline{\theta};\{\theta\})+m_I^b(\theta,N\theta;\{\theta\})\right]^2} \\ &> \beta\frac{m_I^b(\overline{\theta},N\overline{\theta};\mathcal{S})}{[1+(N-1)\beta](\overline{\theta}-\theta)^2\left[m_I^b(\theta,N\overline{\theta};\{\theta\})+m_I^b(\theta,N\theta;\{\theta\})\right]}>0\end{aligned}$$

This inequality implies $\partial\Phi^b(\theta;\mathcal{S})/\partial\theta<\partial\Phi^b(\theta;\{\overline{\theta}\})/\partial\theta$, where:

$$\frac{\partial\Phi^b(\theta;\{\overline{\theta}\})}{\partial\theta}=\frac{\beta^2N[2+(2N-1)\beta]}{2[1+(N-1)\beta]}\cdot\frac{-m_I^b(\overline{\theta},N\overline{\theta};\{\overline{\theta}\})}{m_I^b(\theta,N\overline{\theta};\{\theta\})+m_I^b(\theta,N\theta;\{\theta\})}<0$$

Hence, $\Phi^b(\theta;\mathcal{S})$ is decreasing in θ (i.e., $\partial\Phi^b(\theta;\mathcal{S})/\partial\theta < 0$).

B.2.1 Number of Firms

Consider any given technology $\underline{\theta} < \theta < \overline{\theta}$. The function $\widetilde{\Phi}^b(\theta)$ in (A.4) can be written as:

$$\begin{aligned}
\widetilde{\Phi}^b(\theta) &= 1-\gamma-\frac{\frac{\beta}{2}\left(\theta-E\{\theta_I|\theta_I\le\theta\}\right)}{[1+(N-1)\beta](\overline{\theta}-\theta)}\cdot\left(1+\frac{m_I^b(\theta,N\overline{\theta};[\underline{\theta},\theta])-m_I^b(\theta,N\theta;\{\theta\})}{m_I^b(\theta,N\overline{\theta};\{\theta\})+m_I^b(\theta,N\theta;\{\theta\})}\right)\\
&= 1-\gamma-\frac{\frac{\beta}{2}\left(\theta-E\{\theta_I|\theta_I\le\theta\}\right)}{\overline{\theta}-\theta}\left(\frac{1}{1+(N-1)\beta}+\frac{\beta N(\overline{\theta}-\theta)}{m_I^b(\theta,N\overline{\theta};\{\theta\})+m_I^b(\theta,N\theta;\{\theta\})}\right.\\
&\qquad\left.-\frac{1}{1+(N-1)\beta}\cdot\frac{\beta N\frac{\beta}{2}(\theta-E\{\theta_I|\theta_I\le\theta\})}{m_I^b(\theta,N\overline{\theta};\{\theta\})+m_I^b(\theta,N\theta;\{\theta\})}\right)
\end{aligned}$$

Differentiating $\widetilde{\Phi}^b(\theta)$ with respect to N gives:

$$\begin{aligned}
\frac{\partial\widetilde{\Phi}^b(\theta)}{\partial N} &= \beta\frac{\frac{\beta}{2}\left(\theta-E\{\theta_I|\theta_I\le\theta\}\right)}{\overline{\theta}-\theta}\left(\frac{1}{[1+(N-1)\beta]^2}\left[1-\frac{\beta N\frac{\beta}{2}(\theta-E\{\theta_I|\theta_I\le\theta\})}{m_I^b(\theta,N\overline{\theta};\{\theta\})+m_I^b(\theta,N\theta;\{\theta\})}\right]\right.\\
&\qquad\left.-\frac{\left[2(1-\beta)(2-\beta)(\alpha-\theta)-(\beta N)^2(\overline{\theta}-\theta)\right]}{\left[m_I^b(\theta,N\overline{\theta};\{\theta\})+m_I^b(\theta,N\theta;\{\theta\})\right]^2}\left[(\overline{\theta}-\theta)-\frac{\frac{\beta}{2}(\theta-E\{\theta_I|\theta_I\le\theta\})}{1+(N-1)\beta}\right]\right)\\
&= \beta\frac{\frac{\beta}{2}\left(\theta-E\{\theta_I|\theta_I\le\theta\}\right)}{\overline{\theta}-\theta}\left(\frac{m_I^b(\theta,N\overline{\theta};[\underline{\theta},\theta])+m_I^b(\theta,N\theta;\{\theta\})}{[1+(N-1)\beta]^2\left[m_I^b(\theta,N\overline{\theta};\{\theta\})+m_I^b(\theta,N\theta;\{\theta\})\right]}\right.\\
&\qquad\left.-\frac{\left[2(1-\beta)(2-\beta)(\alpha-\theta)-(\beta N)^2(\overline{\theta}-\theta)\right]\left[m_I^b(\theta,N\overline{\theta};[\underline{\theta},\theta])-m_I^b(\theta,N\theta;\{\theta\})\right]}{[1+(N-1)\beta]\left[m_I^b(\theta,N\overline{\theta};\{\theta\})+m_I^b(\theta,N\theta;\{\theta\})\right]^2}\right)\\
&= \beta\frac{\frac{\beta}{2}\left(\theta-E\{\theta_I|\theta_I\le\theta\}\right)}{\left(\overline{\theta}-\theta\right)[1+(N-1)\beta]^2\left[m_I^b(\theta,N\overline{\theta};\{\theta\})+m_I^b(\theta,N\theta;\{\theta\})\right]^2}\phi_N^b
\end{aligned}$$

with

$$\begin{aligned}
\phi_N^b \equiv\ &\left[m_I^b(\theta,N\overline{\theta};[\underline{\theta},\theta])+m_I^b(\theta,N\theta;\{\theta\})\right]\cdot\left[m_I^b(\theta,N\overline{\theta};\{\theta\})+m_I^b(\theta,N\theta;\{\theta\})\right]\\
&-\left[m_I^b(\theta,N\overline{\theta};[\underline{\theta},\theta])-m_I^b(\theta,N\theta;\{\theta\})\right]\\
&\qquad\qquad * \frac{1+(N-1)\beta}{\beta N}\left[2(1-\beta)(2-\beta)(\alpha-\theta)-(\beta N)^2(\overline{\theta}-\theta)\right]
\end{aligned}$$

First, it is obvious that:

$$m_I^b(\theta,N\overline{\theta};[\underline{\theta},\theta])+m_I^b(\theta,N\theta;\{\theta\}) > \left|m_I^b(\theta,N\overline{\theta};[\underline{\theta},\theta])-m_I^b(\theta,N\theta;\{\theta\})\right|. \qquad \text{(B.4)}$$

Second, for the comparison between the terms $m_I^b(\theta, N\overline{\theta}; \{\theta\}) + m_I^b(\theta, N\theta; \{\theta\})$ and $\frac{1+(N-1)\beta}{\beta N} \left|2(1-\beta)(2-\beta)(\alpha-\theta) - (\beta N)^2(\overline{\theta}-\theta)\right|$, I distinguish the following cases. **(i)** If $2(1-\beta)(2-\beta)(\alpha-\theta) \le (\beta N)^2(\overline{\theta}-\theta)$, then (3.9) gives:

$$m_I^b(\theta, N\overline{\theta}; \{\theta\}) + m_I^b(\theta, N\theta; \{\theta\}) > \frac{1+(N-1)\beta}{\beta N}\left| 2(1-\beta)(2-\beta)(\alpha-\theta) - (\beta N)^2(\overline{\theta}-\theta)\right|. \quad \text{(B.5)}$$

Inequalities (B.4) and (B.5) imply that $\phi_N^b > 0$. **(ii)** If $m_I^b(\theta, N\overline{\theta}; [\underline{\theta}, \theta]) \le m_I^b(\theta, N\theta; \{\theta\})$ and $2(1-\beta)(2-\beta)(\alpha-\theta) > (\beta N)^2(\overline{\theta}-\theta)$, then $\phi_N^b > 0$ holds obviously. **(iii)** Finally, if $m_I^b(\theta, N\overline{\theta}; [\underline{\theta}, \theta]) > m_I^b(\theta, N\theta; \{\theta\})$ and $2(1-\beta)(2-\beta)(\alpha-\theta) > (\beta N)^2(\overline{\theta}-\theta))$, the following holds:

$$\begin{aligned}
\phi_N^b &> 4m_I^b(\theta, N\theta; \{\theta\})^2 \\
&\quad - \frac{m_I^b(\theta, N\overline{\theta}; [\underline{\theta}, \theta]) - m_I^b(\theta, N\theta; \{\theta\})}{\beta N}\left[1+(N-1)\beta\right] 2(1-\beta)(2-\beta)(\alpha-\theta) \\
&> 4m_I^b(\theta, N\theta; \{\theta\})^2 - 2(1-\beta)(2-\beta)\left[1+(N-1)\beta\right]^2(\alpha-\theta)^2 \\
&= 2(1-\beta)\left(2(1-\beta)[2+(2N-1)\beta]^2 - (2-\beta)\left[1+(N-1)\beta\right]^2\right)(\alpha-\theta)^2 > 0
\end{aligned}$$

Hence, $\phi_N^b > 0$ in any case, which gives $\partial\widetilde{\Phi}^b(\theta)/\partial N > 0$, i.e., $\widetilde{\Phi}^b(\theta)$ is increasing in N.

B.2.2 Degree of Substitutability

For any given technology $\underline{\theta} < \theta < \overline{\theta}$, the function $\widetilde{\Phi}^b(\theta)$ in (A.4) can be written as:

$$\widetilde{\Phi}^b(\theta) = 1-\gamma-\frac{\theta - E\{\theta_I|\theta_I \le \theta\}}{2\left(\overline{\theta}-\theta\right)}\cdot\frac{\beta}{1+(N-1)\beta}\left(1+\frac{m_I^b(\theta, N\overline{\theta}; [\underline{\theta}, \theta]) - m_I^b(\theta, N\theta; \{\theta\})}{m_I^b(\theta, N\overline{\theta}; \{\theta\}) + m_I^b(\theta, N\theta; \{\theta\})}\right)$$

Differentiating $\widetilde{\Phi}^b(\theta)$ with respect to β gives:

$$\begin{aligned}
\frac{\partial\widetilde{\Phi}^b}{\partial\beta} &= -\frac{\theta - E\{\theta_I|\theta_I \le \theta\}}{2\left(\overline{\theta}-\theta\right)}\cdot\left[\frac{1}{[1+(N-1)\beta]^2}\left(1+\frac{m_I^b(\theta, N\overline{\theta}; [\underline{\theta}, \theta]) - m_I^b(\theta, N\theta; \{\theta\})}{m_I^b(\theta, N\overline{\theta}; \{\theta\}) + m_I^b(\theta, N\theta; \{\theta\})}\right)\right. \\
&\quad \left. + \frac{\beta}{1+(N-1)\beta}\cdot\frac{\partial}{\partial\beta}\left(\frac{m_I^b(\theta, N\overline{\theta}; [\underline{\theta}, \theta]) - m_I^b(\theta, N\theta; \{\theta\})}{m_I^b(\theta, N\overline{\theta}; \{\theta\}) + m_I^b(\theta, N\theta; \{\theta\})}\right)\right]
\end{aligned}$$

It is straightforward to show that taking the limit for $\beta \to 0$ gives:

$$\lim_{\beta\to 0}\frac{\partial\widetilde{\Phi}^b}{\partial\beta} = -\frac{\theta - E\{\theta_I|\theta_I \le \theta\}}{2\left(\overline{\theta}-\theta\right)} < 0$$

since $\lim_{\beta\to 0}\frac{m_I^b(\theta, N\overline{\theta}; [\underline{\theta}, \theta]) - m_I^b(\theta, N\theta; \{\theta\})}{m_I^b(\theta, N\overline{\theta}; \{\theta\}) + m_I^b(\theta, N\theta; \{\theta\})} = 0$, and $\lim_{\beta\to 0}\frac{\partial}{\partial\beta}\left(\frac{m_I^b(\theta, N\overline{\theta}; [\underline{\theta}, \theta]) - m_I^b(\theta, N\theta; \{\theta\})}{m_I^b(\theta, N\overline{\theta}; \{\theta\}) + m_I^b(\theta, N\theta; \{\theta\})}\right)$ is finite.

C Extensions

Here I analyze the following model extensions. First, I consider a divisible innovation. Second, I reverse the timing. Finally, I analyze a model with two innovative firms.

C.1 Divisible Innovation

Consider a similar setting as in Anton and Yao (2003). That is, the innovative firm with innovation θ_I chooses to apply for a patent of technology t_I with $t_I \geq \theta_I$. As in Anton and Yao (2003), I focus on an equilibrium in which firm I patents its innovation, and the firm's patenting strategy is fully revealing. In other words, I suppose that firm I patents according to the monotonic strategy $\varphi(\theta_I)$, such that $\varphi(\theta_I) \geq \theta_I$ and $\varphi(\overline{\theta}) = \overline{\theta}$. Hence, the non-innovative firms infer from observing patented technology t_I that firm I has technology $\varphi^{-1}(t_I)$. Whereas Anton and Yao (2003) analyze the incentives to patent a drastic innovation, I analyze a model with a non-drastic innovation here.

C.1.1 Equilibrium outputs

Given equilibrium inferences, a non-innovative firm with technology $\theta_n \in \{t_I, \overline{\theta}\}$ sets the output $q_n^c(\theta_n, \varphi^{-1}(t_I) + (N-1)\theta_n; \{\theta_n\})$ in equilibrium, where q_n^c is defined in (3.2) for $n = 1, .., N$. Firm I plays a best response against these output levels, i.e.,

$$\begin{aligned} q_I^*(\theta_I, \theta_n; t_I) &= r_I^c(\mathbf{q}_n^c(\bullet); \theta_I) \\ &= \frac{1}{2}\Big(\alpha - \theta_I - \beta N q_n^c(\theta_n, \varphi^{-1}(t_I) + (N-1)\theta_n; \{\theta_n\})\Big) \\ &= \frac{1}{2+N\beta}\left(\alpha - \theta_I + \frac{\beta N}{2}(\theta_n - \theta_I) + \frac{\beta^2 N}{2(2-\beta)}\left[\theta_n - \varphi^{-1}(t_I)\right]\right) \quad \text{(C.1)} \end{aligned}$$

In equilibrium, firm I's product market profit equals: $\pi_I^*(\theta_I, \theta_n; t_I) = q_I^*(\theta_I, \theta_n; t_I)^2$.

C.1.2 Equilibrium patenting

The expected profit of firm I with technology θ_I from patenting t_I, given beliefs consistent with strategy φ, is:

$$\Pi_I(\theta_I, t_I) \equiv \gamma \pi_I^*(\theta_I, \overline{\theta}; t_I) + (1-\gamma)\pi_I^*(\theta_I, t_I; t_I)$$

Hence, the optimal patenting strategy satisfies $\partial \Pi_I(\theta_I, t_I)/\partial t_I = 0$, which is equivalent to:

$$\gamma q_I^*(\theta_I, \overline{\theta}; t_I)\frac{\partial q_I^*(\theta_I, \overline{\theta}; t_I)}{\partial t_I} + (1-\gamma) q_I^*(\theta_I, t_I; t_I)\frac{\partial q_I^*(\theta_I, t_I; t_I)}{\partial t_I} = 0 \quad \text{(C.2)}$$

where
$$\frac{\partial q_I^*(\theta_I,\overline{\theta};t_I)}{\partial t_I}=\frac{-\beta N}{2(2-\beta)(2+N\beta)}\cdot\beta\frac{d\varphi^{-1}(t_I)}{dt_I}$$
and
$$\frac{\partial q_I^*(\theta_I,t_I;t_I)}{\partial t_I}=\frac{\beta N}{2(2-\beta)(2+N\beta)}\left(2-\beta\frac{d\varphi^{-1}(t_I)}{dt_I}\right)$$
Substituting these expressions in the first order condition (C.2) gives:
$$\left[\gamma q_I^*(\theta_I,\overline{\theta};t_I)+(1-\gamma)q_I^*(\theta_I,t_I;t_I)\right]\beta\frac{d\varphi^{-1}(t_I)}{dt_I}=2(1-\gamma)q_I^*(\theta_I,t_I;t_I)$$
or
$$\frac{d\varphi^{-1}(t_I)}{dt_I}=\frac{2(1-\gamma)q_I^*(\theta_I,t_I;t_I)}{\beta\left[\gamma q_I^*(\theta_I,\overline{\theta};t_I)+(1-\gamma)q_I^*(\theta_I,t_I;t_I)\right]}$$
By using $\varphi(\theta_I)=t_I$, this equality is equivalent to:
$$\begin{aligned}\frac{d\varphi(\theta_I)}{d\theta_I}&=\frac{\beta}{2}\left(1+\frac{\gamma}{1-\gamma}\cdot\frac{q_I^*(\theta_I,\overline{\theta};\varphi(\theta_I))}{q_I^*(\theta_I,\varphi(\theta_I);\varphi(\theta_I))}\right)\Leftrightarrow\\ \frac{d\varphi(\theta_I)}{d\theta_I}&=\frac{\beta}{2}\left(1+\frac{\gamma}{1-\gamma}\left[1+\frac{q_I^*(\theta_I,\overline{\theta};\varphi(\theta_I))-q_I^*(\theta_I,\varphi(\theta_I);\varphi(\theta_I))}{q_I^*(\theta_I,\varphi(\theta_I);\varphi(\theta_I))}\right]\right)\end{aligned}$$
Using (C.1), this can be written as:
$$\frac{d\varphi(\theta_I)}{d\theta_I}=\frac{\beta}{2}\left(\frac{1}{1-\gamma}+\frac{\gamma}{1-\gamma}\cdot\frac{\beta N\left[\overline{\theta}-\varphi(\theta_I)\right]}{(2-\beta)(\alpha-\theta_I)+\beta N\left[\varphi(\theta_I)-\theta_I\right]}\right)\tag{C.3}$$
A solution to differential equation (C.3), which satisfies $\varphi(\theta_I)\geq\theta_I$ and $\varphi(\overline{\theta})=\overline{\theta}$, is an equilibrium patenting strategy. I denote the equilibrium strategy by $\widehat{\varphi}$.

First, in the absence of protection (i.e., $\gamma=0$) the differential equation (C.3) reduces to: $\varphi'(\theta_I)=\frac{\beta}{2}$. By using the condition $\varphi(\overline{\theta})=\overline{\theta}$, this gives the equilibrium strategy $\widehat{\varphi}(\theta_I)=\frac{\beta}{2}\theta_I+(1-\frac{\beta}{2})\overline{\theta}$, which is similar to (14) in Anton and Yao (2003). In equilibrium, the innovative firm patents technologies of relatively low efficiency (i.e., $\theta_I\geq\frac{\beta}{2}\underline{\theta}+(1-\frac{\beta}{2})\overline{\theta}$), while it does not patent technologies that are more efficient.

Second, if protection is strong, it is possible to obtain an explicit solution too. Clearly, it follows from applying the constraints $\varphi(\theta_I)\geq\theta_I$ and $\varphi(\theta_I)\leq\overline{\theta}$ to equation (C.3), that $\varphi'(\theta_I)\geq\frac{\beta}{2}\cdot\frac{1}{1-\gamma}$ for any θ_I. If $\gamma\geq1-\frac{\beta}{2}$, then the inequality becomes $\varphi'(\theta_I)\geq1$ for any θ_I, which implies that the constraint $\varphi(\theta_I)\geq\theta_I$ becomes binding. Therefore, the equilibrium strategy gives full patenting (i.e., $\widehat{\varphi}(\theta_I)=\theta_I$ for any θ_I) if $\gamma\geq1-\frac{\beta}{2}$. For sufficiently strong protection, the signaling effect dominates, which gives firm I an incentive to patent its innovation completely.

Finally, for intermediate values of the protection parameter (i.e., $0 < \gamma < 1 - \frac{\beta}{2}$), differential equation (C.3) is difficult to solve analytically. For a numerical example (i.e., $\alpha = 4$, $\beta = 1$, $N = 1$, $\underline{\theta} = 0$, and $\overline{\theta} = 1$), I approximated some solutions of (C.3) numerically for different intermediate values of γ.[31] These solutions are sketched in Figure 5. The figure suggests that for sufficiently weak protection parameters

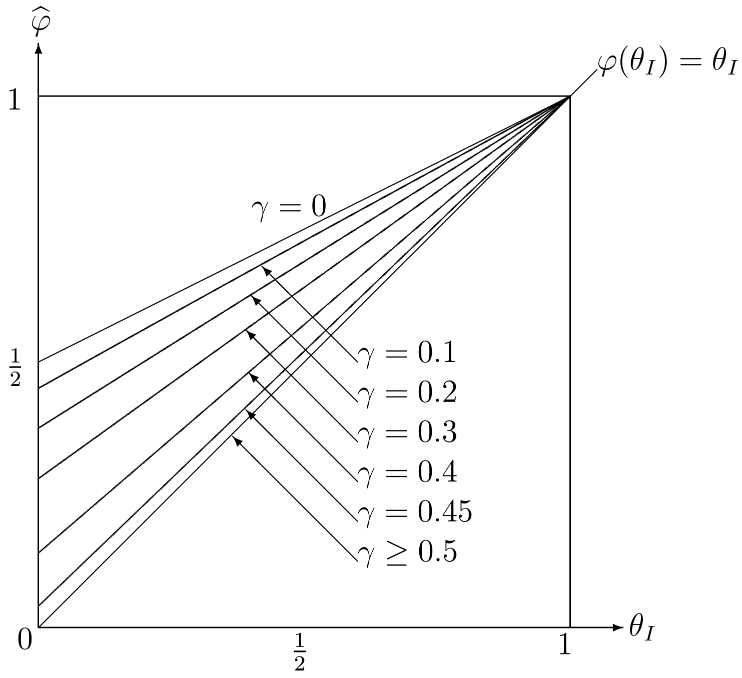

Figure 5: Equilibrium patenting (divisible innovation)

(e.g., $\gamma \leq 0.45$), the innovative firm keeps its most efficient technologies secret, and signals by patenting only lesser efficient technologies (i.e., $\widehat{\varphi}(\theta_I) > \theta_I$). Moreover, the numerical examples suggest that the equilibrium strategies are concave in θ_I. Also this means that firm I tends to skew its patenting strategy in the direction of inefficient technologies. For protection parameter values close to $\frac{1}{2}$ (e.g., $\gamma = 0.475$), concavity of the equilibrium strategy gives full patenting of efficient technologies, and partial patenting for less efficient technologies (i.e., $\widehat{\varphi}(\theta_I) = \theta_I$ if $\theta_I \leq \widehat{\theta}$, and $\widehat{\varphi}(\theta_I) > \theta_I$ if $\widehat{\theta} < \theta_I < \overline{\theta}$, for some $\widehat{\theta}$ with $\underline{\theta} < \widehat{\theta} < \overline{\theta}$). Finally, Figure 5 suggests that stronger patent protection gives the innovative firm an incentive to patent a greater part of its innovation (i.e., $\partial\widehat{\varphi}(\theta_I)/\partial\gamma < 0$ for any θ_I). This is consistent with Proposition 3.

If protection is weak, the description of the equilibrium strategy $\widehat{\varphi}$ suggests that it is an increasing, concave transformation of θ_I, i.e., $\widehat{\varphi} : [\underline{\theta}, \overline{\theta}] \to [\delta\underline{\theta} + (1-\delta)\overline{\theta}, \overline{\theta}]$ for some $0 < \delta < 1$, where $\widehat{\varphi}(\underline{\theta}) = \delta\underline{\theta} + (1-\delta)\overline{\theta}$ and $\widehat{\varphi}(\overline{\theta}) = \overline{\theta}$. The properties of the

[31] I used Wolfram Mathematica 6 to solve the differential equation numerically.

equilibrium strategy give the following inequality: $\widehat{\varphi}(\theta_I) \geq \delta\theta_I + (1-\delta)\overline{\theta} \geq \theta$ for any $\theta_I \in [\underline{\theta}, \overline{\theta}]$. Then, for any $y \in [\delta\underline{\theta} + (1-\delta)\overline{\theta}, \overline{\theta}]$, the distribution of $\widehat{\varphi}(\theta)$ relates as follows to the distribution of θ:

$$F_{\widehat{\varphi}(\theta)}(y) = \Pr[\widehat{\varphi}(\theta) \leq y] = \Pr[\theta \leq \widehat{\varphi}^{-1}(y)] = F_\theta(\widehat{\varphi}^{-1}(y)).$$

Clearly, if $y \in [\underline{\theta}, \delta\underline{\theta} + (1-\delta)\overline{\theta}]$, then $F_{\widehat{\varphi}(\theta)}(y) = 0$. The distribution of patented technologies $\widehat{\varphi}(\theta)$ is therefore:

$$F_{\widehat{\varphi}(\theta)}(y) = \begin{cases} 0, & \text{if } \underline{\theta} \leq y < \delta\underline{\theta} + (1-\delta)\overline{\theta}, \\ F_\theta(\widehat{\varphi}^{-1}(y)), & \text{if } \delta\underline{\theta} + (1-\delta)\overline{\theta} \leq y \leq \overline{\theta}. \end{cases}$$

Clearly, if $\underline{\theta} \leq y < \delta\underline{\theta} + (1-\delta)\overline{\theta}$, then $F_{\widehat{\varphi}(\theta)}(y) = 0 \leq F_\theta(y)$. The inverse transformation $\widehat{\varphi}^{-1} : [\delta\underline{\theta} + (1-\delta)\overline{\theta}, \overline{\theta}] \rightarrow [\underline{\theta}, \overline{\theta}]$ is an increasing, convex function, which satisfies the inequality $\widehat{\varphi}^{-1}(y) \leq \overline{\theta} - \frac{1}{\delta}\left(\overline{\theta} - y\right) \leq y$ for any $y \in [\delta\underline{\theta} + (1-\delta)\overline{\theta}, \overline{\theta}]$. Hence, if $\delta\underline{\theta} + (1-\delta)\overline{\theta} \leq y \leq \overline{\theta}$, then $F_{\widehat{\varphi}(\theta)}(y) = F_\theta(\widehat{\varphi}^{-1}(y)) \leq F_\theta(y)$. In short, $F_{\widehat{\varphi}(\theta)}(y) \leq F_\theta(y)$ for any $y \in [\underline{\theta}, \overline{\theta}]$, i.e., the distribution of $\widehat{\varphi}(\theta_I)$ first-order stochastically dominates the distribution of θ_I. The equilibrium strategy $\widehat{\varphi}$ skews the technology distribution towards inefficient technologies.

Similarly, if the protection parameter is close to $1 - \frac{\beta}{2}$, then the numerical analysis suggests that there exists a threshold level $\widehat{\theta}$, with $\underline{\theta} < \widehat{\theta} < \overline{\theta}$, such that the equilibrium strategy is:

$$\widehat{\varphi}(\theta) = \begin{cases} \theta, & \text{if } \underline{\theta} \leq \theta < \widehat{\theta}, \\ g(\theta), & \text{if } \widehat{\theta} \leq \theta \leq \overline{\theta}, \end{cases}$$

where $g : [\widehat{\theta}, \overline{\theta}] \rightarrow [\widehat{\theta}, \overline{\theta}]$ is an increasing, concave function with $g(\widehat{\theta}) = \widehat{\theta}$ and $g(\overline{\theta}) = \overline{\theta}$. Using similar arguments as before, the distribution of $\widehat{\varphi}$ becomes:

$$F_{\widehat{\varphi}(\theta)}(y) = \begin{cases} F_\theta(y), & \text{if } \underline{\theta} \leq y < \widehat{\theta}, \\ F_\theta(g^{-1}(y)), & \text{if } \widehat{\theta} \leq y \leq \overline{\theta}. \end{cases}$$

Convexity of g^{-1} in combination with $g^{-1}(\widehat{\theta}) = \widehat{\theta}$ and $g^{-1}(\overline{\theta}) = \overline{\theta}$ yields: $g^{-1}(y) \leq y$ for any $\widehat{\theta} \leq y \leq \overline{\theta}$. This implies that $F_{\widehat{\varphi}(\theta)}(y) \leq F_\theta(y)$ for any $y \in [\underline{\theta}, \overline{\theta}]$. In summary, the equilibrium patenting strategy appears to be such that the distribution of the patented technologies first-order stochastically dominates the distribution of technologies.

In both cases, the equilibrium patenting strategy skews the technology distribution towards inefficient technologies. That is, when the innovation is divisible, then an innovative firm tends to patent small innovations to a greater extent than big innovations. By contrast, Proposition 1 shows that the firm has an incentive to do the opposite (i.e., patent big innovations to a greater extent than small innovations), when its innovation is non-divisible.

C.2 Timing

Consider the model where the patent validity is determined after the firms set their product market variables. In the subgame that starts after firm I patents its technology, a non-innovative firm chooses its product market variable that maximizes its expected profit $\pi_n(\bullet;\gamma\overline{\theta}+(1-\gamma)\theta_I)$ for $n \in \{1,..,N\}$, and firm I expects to earn the profit $\pi_I^r(\theta_I, N[\gamma\overline{\theta}+(1-\gamma)\theta_I];\{\theta_I\})$ in equilibrium for $r \in \{c,b\}$.

C.2.1 Cournot competition

For any given $\mathcal{S} \subseteq [\underline{\theta},\overline{\theta}]$ and $\theta_I \in [\underline{\theta},\overline{\theta}]$, firm I prefers secrecy if $\pi_I^c(\theta_I, N\overline{\theta};\mathcal{S}) \geq \pi_I^c(\theta_I, N[\gamma\overline{\theta}+(1-\gamma)\theta_I];\{\theta_I\})$, which is equivalent to $T^c(\theta_I;\mathcal{S}) \geq 0$, where:

$$T^c(\theta;\mathcal{S}) \equiv 1-\gamma-\frac{\beta}{2}\cdot\frac{E\{\theta_I|\theta_I \in \mathcal{S}\}-\theta}{\overline{\theta}-\theta}.$$

Clearly, $T^c(\theta;\mathcal{S})$ is increasing in θ, and $T^c(\underline{\theta};[\underline{\theta},\overline{\theta}]) \geq 0 \Leftrightarrow \gamma \leq 1-\frac{\beta}{2}\cdot\frac{E\{\theta_I\}-\underline{\theta}}{\overline{\theta}-\underline{\theta}}$, while $T^c(\overline{\theta};\{\overline{\theta}\}) \leq 0 \Leftrightarrow \gamma \geq 1-\frac{\beta}{2}$. Further, for the continuous function $\widetilde{T}^c(\theta) \equiv T^c(\theta;[\theta,\overline{\theta}])$ and for $1-\frac{\beta}{2}\cdot\frac{E\{\theta_I\}-\underline{\theta}}{\overline{\theta}-\underline{\theta}} < \gamma < 1-\frac{\beta}{4}$, it is easily verified that $\widetilde{T}^c(\underline{\theta}) < 0 < \lim_{\theta\uparrow\overline{\theta}} T^c(\theta)$. These basic properties are similar to those in the proof of Proposition 1.

For the proofs of Propositions 3-6 related to Cournot competition, it is sufficient to verify that $\widetilde{T}^c(\theta)$ is decreasing in γ, decreasing in $E\{\theta_I|\theta_I > \theta\}$, non-increasing in N, and decreasing in β.

C.2.2 Bertrand competition

For any given $\mathcal{S} \subseteq [\underline{\theta},\overline{\theta}]$ and $\theta_I \in [\underline{\theta},\overline{\theta}]$, firm I prefers secrecy if $\pi_I^b(\theta_I, N\overline{\theta};\mathcal{S}) \geq \pi_I^b(\theta_I, N[\gamma\overline{\theta}+(1-\gamma)\theta_I];\{\theta_I\})$, which is equivalent to $T^b(\theta_I;\mathcal{S}) \geq 0$, where:

$$T^b(\theta;\mathcal{S}) \equiv 1-\gamma-\frac{\frac{\beta}{2}(\theta-E\{\theta_I|\theta_I \in \mathcal{S}\})}{[1+(N-1)\beta](\overline{\theta}-\theta)}.$$

Clearly, $T^c(\theta;\mathcal{S})$ is decreasing in θ. Further, for the continuous function $\widetilde{T}^c(\theta) \equiv T^c(\theta;[\underline{\theta},\theta])$ it is easily verified that $\widetilde{T}^c(\underline{\theta}) > 0 > T^c(\overline{\theta})$. These basic properties are similar to those in the proof of Proposition 2.

For the proofs of Propositions 3-6 related to Bertrand competition, it is sufficient to verify that $\widetilde{T}^b(\theta)$ is decreasing in γ, increasing in $E\{\theta_I|\theta_I \leq \theta\}$, increasing in N, and decreasing in β.

C.3 Two-Sided Asymmetric Information

Consider the model where there are two innovative firms, I_1 and I_2, and no non-innovative firms ($N = 0$). At the beginning of the game each firm receives a draw from the interval $[\underline{\theta}, \overline{\theta}]$. Firm ℓ's technology θ_ℓ has the distribution $F_\ell : [\underline{\theta}, \overline{\theta}] \to [0, 1]$ with $\ell \in \{I_1, I_2\}$. The draws θ_{I_1} and θ_{I_2} are independent. Subsequently, the firms choose simultaneously whether to patent the innovation or keep it secret. To simplify the analysis, I assume that patents are invalid, i.e., $\gamma = 0$, and firms choose accommodating pricing strategies.

First, I present the equilibrium pricing strategies. Second, I characterize the patenting strategies.

C.3.1 Pricing strategies

Take any subset $\mathcal{S}_k \subseteq [\underline{\theta}, \overline{\theta}]$ and $\mathcal{P}_k \equiv [\underline{\theta}, \overline{\theta}] \backslash \mathcal{S}_k$, and assume that firm ℓ has beliefs consistent with the adoption of the following generic patenting strategy by firm k (for $\ell, k \in \{I_1, I_2\}$ and $\ell \neq k$):

$$\widehat{s}_k(\theta_k) = \begin{cases} \varnothing, & \text{if } \theta_k \in \mathcal{S}_k \\ \theta_k, & \text{if } \theta_k \in \mathcal{P}_k \end{cases} \tag{C.4}$$

That is, the expected cost of firm k after adoption of secrecy is $E\{\theta_k | \theta_k \in \mathcal{S}_k\}$.

If both firms share their technologies, then they set equilibrium prices which yield the following price-cost margins (for $\ell, k \in \{I_1, I_2\}$ and $\ell \neq k$):

$$m_\ell^{PP}(\theta_\ell; \theta_\ell, \theta_k) \equiv p_\ell^{PP}(\theta_\ell; \theta_\ell, \theta_k) - \min\{\theta_\ell, \theta_k\} = \frac{1-\beta}{2-\beta}\left(\alpha - \min\{\theta_\ell, \theta_k\}\right). \tag{C.5}$$

If both firms keep their technologies secret, firm ℓ chooses the following price-cost margin in equilibrium (for $\ell, k \in \{I_1, I_2\}$ and $\ell \neq k$):

$$\begin{aligned} m_\ell^{SS}(\theta_\ell; \mathcal{S}_\ell, \mathcal{S}_k) &= p_\ell^{SS}(\theta_\ell; \mathcal{S}_\ell, \mathcal{S}_k) - \theta_\ell = \frac{1-\beta}{2-\beta}\left(\alpha - \theta_\ell\right) \\ &\quad + \frac{\beta}{4-\beta^2}\left(E\{\theta_k | \theta_k \in \mathcal{S}_k\} - \theta_\ell + \frac{\beta}{2}\left[E\{\theta_\ell | \theta_\ell \in \mathcal{S}_\ell\} - \theta_\ell\right]\right). \end{aligned} \tag{C.6}$$

If firm ℓ shares technology θ_ℓ and firm k conceals, the firms' first-order conditions are as follows (for $\ell, k \in \{I_1, I_2\}$ and $\ell \neq k$):

$$\begin{aligned} 2p_\ell(\theta_\ell) &= (1-\beta)\alpha + \theta_\ell + \beta \int_{\theta \in \mathcal{S}_k} f_k(\theta | \theta_k \in \mathcal{S}_k) p_k(\theta, \theta_\ell) d\theta \\ &\text{and} \quad 2p_k(\theta_k, \theta_\ell) = (1-\beta)\alpha + \min\{\theta_\ell, \theta_k\} + \beta p_\ell(\theta_\ell). \end{aligned}$$

In this case (firm ℓ shares, firm k conceals) firm ℓ sets the following equilibrium margin:

$$\begin{aligned} m_\ell^{PS}(\theta_\ell;\theta_\ell,\mathcal{S}_k) &= p_\ell^{PS}(\theta_\ell;\theta_\ell,\mathcal{S}_k)-\theta_\ell \qquad \text{(C.7)}\\ &= \frac{1-\beta}{2-\beta}\Big(\alpha-\theta_\ell\Big)+\frac{\beta}{4-\beta^2}\Big(E\left(\min\{\theta_\ell,\theta_k\}|\theta_k\in\mathcal{S}_k\right)-\theta_\ell\Big), \end{aligned}$$

with $E\left(\min\{\theta_\ell,\theta_k\}|\theta_k\in\mathcal{S}_k\right)=F_k(\theta_k|\theta_k\in\mathcal{S}_k)E\{\theta_k|\theta_k\leq\theta_\ell,\theta_k\in\mathcal{S}_k\}+[1-F_k(\theta_k|\theta_k\in\mathcal{S}_k)]\,\theta_\ell$. Similarly, if firm ℓ hides θ_ℓ and firm k shares, firm ℓ sets the following price-cost margin in equilibrium (for $\ell,k\in\{I_1,I_2\}$ and $\ell\neq k$):

$$\begin{aligned} m_\ell^{SP}(\theta_\ell;\mathcal{S}_\ell,\theta_k) &= p_\ell^{SP}(\theta_\ell;\mathcal{S}_\ell,\theta_k)-\min\{\theta_\ell,\theta_k\} \qquad \text{(C.8)}\\ &= \frac{1-\beta}{2-\beta}\Big(\alpha-\min\{\theta_\ell,\theta_k\}\Big)+\frac{\beta}{4-\beta^2}\Big(\theta_k-\min\{\theta_\ell,\theta_k\}\\ &\quad+\frac{\beta}{2}\left[E\left(\min\{\theta_\ell,\theta_k\}|\theta_\ell\in\mathcal{S}_\ell\right)-\min\{\theta_\ell,\theta_k\}\right]\Big). \end{aligned}$$

Firm ℓ's expected equilibrium product market profit is (for any t_ℓ and t_k):

$$\pi_\ell^{t_\ell t_k}(\theta_\ell;\bullet)=\frac{1}{1-\beta^2}m_\ell^{t_\ell t_k}(\theta_\ell;\bullet)^2 \qquad \text{(C.9)}$$

C.3.2 Patenting strategies

Proposition 7 *If $\gamma=0$, then in any equilibrium, and for any $i\in\{I_1,I_2\}$, firm i chooses the patenting rule s_i^b in (C.4) with $\mathcal{S}_i=[\underline{\theta},\theta_i^b]$ for some $\underline{\theta}<\theta_i^b<\overline{\theta}$.*

Proof. Suppose that firm k chooses the technology sharing rule $\widehat{s}_k$ in (C.4). Further, suppose that firm k has beliefs consistent with (C.4), with $k=\ell$, for some subsets $\mathcal{S}_\ell\subseteq[\underline{\theta},\overline{\theta}]$ and $\mathcal{P}_\ell=[\underline{\theta},\overline{\theta}]\backslash\mathcal{S}_\ell$. Given these assumptions, the difference of the expected profit from technology sharing and secrecy for firm ℓ is:

$$\begin{aligned} \Psi(\theta_\ell;\mathcal{S}_\ell,\mathcal{S}_k) &\equiv \int_{\theta_k\in\mathcal{P}_k}\left[\pi_\ell^{PP}(\theta_\ell;\theta_\ell,\theta_k)-\pi_\ell^{SP}(\theta_\ell;\mathcal{S}_\ell,\theta_k)\right]f_k(\theta_k)d\theta_k\\ &\quad+\int_{\theta_k\in\mathcal{S}_k}\left[\pi_\ell^{PS}(\theta_\ell;\theta_\ell,\mathcal{S}_k)-\pi_\ell^{SS}(\theta_\ell;\mathcal{S}_\ell,\mathcal{S}_k)\right]f_k(\theta_k)d\theta_k \end{aligned}$$

where

$$\begin{aligned} \pi_\ell^{PP}(\theta_\ell;\theta_\ell,\theta_k)-\pi_\ell^{SP}(\theta_\ell;\mathcal{S}_\ell,\theta_k) &= \frac{1}{1-\beta^2}\Big(m_\ell^{PP}(\theta_\ell;\theta_\ell,\theta_k)^2-m_\ell^{SP}(\theta_\ell;\mathcal{S}_\ell,\theta_k)^2\Big)\\ &= \frac{1}{1-\beta^2}[m_\ell^{PP}(\theta_\ell;\theta_\ell,\theta_k)-m_\ell^{SP}(\theta_\ell;\mathcal{S}_\ell,\theta_k)]\\ &\quad\cdot[m_\ell^{PP}(\theta_\ell;\theta_\ell,\theta_k)+m_\ell^{SP}(\theta_\ell;\mathcal{S}_\ell,\theta_k)] \end{aligned}$$

and a similar expression for $\pi_\ell^{PS}(\theta_\ell;\theta_\ell,\mathcal{S}_k)-\pi_\ell^{SS}(\theta_\ell;\mathcal{S}_\ell,\mathcal{S}_k)$. The evaluation of Ψ at extreme values of θ_ℓ gives the following: $\Psi(\underline{\theta};\mathcal{S}_\ell,\mathcal{S}_k)<0\leq\Psi(\overline{\theta};\mathcal{S}_\ell,\mathcal{S}_k)$ for any $\mathcal{S}_\ell$ and $\mathcal{S}_k$. The second derivative of Ψ equals:

$$\begin{aligned}\frac{\partial^2\Psi(\theta_\ell;\mathcal{S}_\ell,\mathcal{S}_k)}{\partial\theta_\ell^2} &= \frac{1}{1-\beta^2}\int_{\theta_k\in\mathcal{P}_k}\left(\frac{\partial m_\ell^{PP}(\theta_\ell;\theta_\ell,\theta_k)}{\partial\theta_\ell}-\frac{\partial m_\ell^{SP}(\theta_\ell;\mathcal{S}_\ell,\theta_k)}{\partial\theta_\ell}\right)\\ &\quad\cdot\left(\frac{\partial m_\ell^{PP}(\theta_\ell;\theta_\ell,\theta_k)}{\partial\theta_\ell}+\frac{\partial m_\ell^{SP}(\theta_\ell;\mathcal{S}_\ell,\theta_k)}{\partial\theta_\ell}\right)f_k(\theta_k)d\theta_k\\ +\frac{1}{1-\beta^2}\Pr[\theta_k &\in \mathcal{S}_k]\left[\left(\frac{\partial m_\ell^{PS}(\theta_\ell;\theta_\ell,\mathcal{S}_k)}{\partial\theta_\ell}-\frac{\partial m_\ell^{SS}(\theta_\ell;\mathcal{S}_\ell,\mathcal{S}_k)}{\partial\theta_\ell}\right)\right.\\ &\quad\cdot\left(\frac{\partial m_\ell^{PS}(\theta_\ell;\theta_\ell,\mathcal{S}_k)}{\partial\theta_\ell}+\frac{\partial m_\ell^{SS}(\theta_\ell;\mathcal{S}_\ell,\mathcal{S}_k)}{\partial\theta_\ell}\right)\\ &\quad\left.+\,2m_\ell^{PS}(\theta_\ell;\theta_\ell,\mathcal{S}_k)\frac{\partial^2 m_\ell^{PS}(\theta_\ell;\theta_\ell,\mathcal{S}_k)}{\partial\theta_\ell^2}\right]\end{aligned}$$

since for any $\theta_\ell\in[\underline{\theta},\overline{\theta}]$

$$\frac{\partial^2 m_\ell^{PP}(\theta_\ell;\theta_\ell,\theta_k)}{\partial\theta_\ell^2}=\frac{\partial^2 m_\ell^{SP}(\theta_\ell;\mathcal{S}_\ell,\theta_k)}{\partial\theta_\ell^2}=\frac{\partial^2 m_\ell^{SS}(\theta_\ell;\mathcal{S}_\ell,\mathcal{S}_k)}{\partial\theta_\ell^2}=0$$

First, using (C.5) and (C.8), it is immediate that $\frac{\partial m_\ell^{PP}(\theta_\ell;\theta_\ell,\theta_k)}{\partial\theta_\ell}-\frac{\partial m_\ell^{SP}(\theta_\ell;\mathcal{S}_\ell,\theta_k)}{\partial\theta_\ell}\geq 0$ and $\frac{\partial m_\ell^{PP}(\theta_\ell;\theta_\ell,\theta_k)}{\partial\theta_\ell}+\frac{\partial m_\ell^{SP}(\theta_\ell;\mathcal{S}_\ell,\theta_k)}{\partial\theta_\ell}\leq 0$ for any θ_ℓ and θ_k, since $\partial\min\{\theta_\ell,\theta_k\}/\partial\theta_\ell\geq 0$. Second, using (C.6) and (C.7), gives $\frac{\partial m_\ell^{PS}(\theta_\ell;\theta_\ell,\mathcal{S}_k)}{\partial\theta_\ell}-\frac{\partial m_\ell^{SS}(\theta_\ell;\mathcal{S}_\ell,\mathcal{S}_k)}{\partial\theta_\ell}>0$ and $\frac{\partial m_\ell^{PS}(\theta_\ell;\theta_\ell,\mathcal{S}_k)}{\partial\theta_\ell}+\frac{\partial m_\ell^{SS}(\theta_\ell;\mathcal{S}_\ell,\mathcal{S}_k)}{\partial\theta_\ell}<0$, since $\partial E\left(\min\{\theta_\ell,\theta_k\}|\theta_k\in\mathcal{S}_k\right)/\partial\theta_\ell=\Pr[\theta_k\in\mathcal{S}_k\cap[\theta_\ell,\overline{\theta}]]/\Pr[\theta_k\in\mathcal{S}_k]\in[0,1]$. Finally,

$$\frac{\partial^2 m_\ell^{PS}(\theta_\ell;\theta_\ell,\mathcal{S}_k)}{\partial\theta_\ell^2}=\frac{\beta}{4-\beta^2}\cdot\frac{\partial^2 E\left(\min\{\theta_\ell,\theta_k\}|\theta_k\in\mathcal{S}_k\right)}{\partial\theta_\ell^2}\leq 0.$$

Hence, $\partial^2\Psi(\theta_\ell;\mathcal{S}_\ell,\mathcal{S}_k)/\partial\theta_\ell^2\leq 0$, i.e., $\Psi(\theta_\ell;\mathcal{S}_\ell,\mathcal{S}_k)$ is (weakly) concave in θ_ℓ. This fact, in combination with $\Psi(\underline{\theta};\bullet)<0\leq\Psi(\overline{\theta};\bullet)$, implies that firm ℓ's equilibrium patenting strategy is (C.4) for $k=\ell$, with $\mathcal{S}_\ell=[\underline{\theta},\theta_\ell^b]$ for some $\underline{\theta}\leq\theta_\ell^b\leq\overline{\theta}$. The evaluation of $\Psi(\theta;[\underline{\theta},\theta],\mathcal{S}_k)$ for extreme values of θ gives:

$$\Psi(\underline{\theta};[\underline{\theta},\underline{\theta}],\mathcal{S}_k)<0<\Psi(\overline{\theta};[\underline{\theta},\overline{\theta}],\mathcal{S}_k)$$

for any $\mathcal{S}_k\subseteq[\underline{\theta},\overline{\theta}]$, Hence, the intermediate value theorem implies that (for any $\mathcal{S}_k\subseteq[\underline{\theta},\overline{\theta}]$) there exists a θ_ℓ^b, with $\underline{\theta}<\theta_\ell^b<\overline{\theta}$, such that $\Psi(\theta_\ell^b;[\underline{\theta},\theta_\ell^b],\mathcal{S}_k)=0$. ■

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