GOVERNANCE AND THE EFFICIENCY OF ECONOMIC SYSTEMS
GESY

Discussion Paper No. 371

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January 2012

Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

Sonderforschungsbereich/Transregio 15 · www.sfbtr15.de
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January 25, 2012

Abstract

A principal uses security bid auctions to award an incentive contract to one among several agents, in the presence of hidden action and hidden information. Securities range from cash to equity and call options. "Steeper" securities are better surplus extractors that narrow the gap between the two highest valuations, yet reduce effort incentives. In view of this trade-off, the generalized equity auction that includes a (possibly negative) cash reward to the winner tends to outperform all other auctions, although it cannot extract the entire surplus *and* implement efficient effort. Hence, profit sharing emerges without risk aversion or limited liability.

KEYWORDS: Auctions, agency problems, licensing, innovation, mechanism design.
JEL CLASSIFICATIONS: D21, D43, D44, D45.

1 Introduction

If a risk neutral principal deals with a risk neutral and sufficiently wealthy agent whose effort is unobservable, the optimal contract is a franchising contract. There, the agent becomes full residual claimant in exchange for a take-it-or-leave-it cash payment.[1] This is true even if the agent's ability is unknown.

If the principal can recruit the agent from a pool of agents whose abilities are unknown, he can do better by replacing the take-it-or-leave-it offer by a standard auction that awards the contract to the highest bidder.

If output is contractible, the principal can do even better by using a contingent payment auction in which cash bids are replaced by bidding with securities. However, such auction formats may adversely affect effort incentives.

The present paper explores the profitability of cash versus security bid auctions ranging from equity (share) to call option auctions for agency contracts in the presence of hidden action and hidden

*Research support by the *National Research Foundation of Korea* funded by the Korean Government (NRF-2010-330-B00085) and the *Deutsche Forschungsgemeinschaft (DFG), SFB Transregio 15,* "Governance and Efficiency of Economic Systems" is gratefully acknowledged.

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[1] If the agent is risk averse, the optimal contract exhibits profit sharing (the classical reference is Holmstrœm, 1979), and if the agent cannot make advance payments and is subject to limited liability, it is a bonus contract (Innes, 1990).

information. These auction formats differ in their ability to extract surplus and implement efficient effort. For example, cash auctions give the agent the full profit, and thus exhibit the strongest possible effort incentives. However, they are weak in extracting surplus, because the principal can extract only the second highest in expected profit. Whereas a call option auction, that gives the principal the right to become residual claimant in exchange for a fixed strike price, extracts the full surplus, yet completely lacks effort incentives.

Our main finding is that an equity or share auction, supplemented by a cash transfer to the winner, tends to be the most profitable auction format for the principal, because after fine-tuning it with cash rewards it manages to strike the best balance between effort incentives and surplus extraction.

Share auctions have been introduced by Hansen (1985) who showed that share auctions (which are feasible if the winner's valuation is verifiable *ex post*) can extract more surplus than standard cash auctions. One may interpret this result as an implication of the "linkage principle" according to which linking the price to a variable that is correlated with bidders' private information tends to lower bidders' information rents (see Milgrom, 1987). Later, Crémer (1987) pointed out that by adding cash transfers, share auctions can, paradoxically, achieve full surplus extraction.[2]

In the recent literature, contingent payment auctions have been revived and extended by DeMarzo, Kremer, and Skrzypacz (2005).[3] They consider a larger class of security bid auctions and rank them according to their capacity to extract surplus. In particular, they introduce the concept of "steepness", which partially orders securities, and show that surplus extraction increases in steepness. Essentially, the equilibrium price of a security bid auction with a steeper security responds more strongly to the winner's valuation. The standard cash auction has the lowest steepness, because the equilibrium price reflects only the second highest valuation and thus cannot extract the gap between the highest and the second highest valuation. In turn, the steepest security is the call option, which entitles the auctioneer to the valuation of the winner in exchange for a fixed price.

Che and Kim (2010) commented on DeMarzo, Kremer, and Skrzypacz (2005), pointing out that steepness of securities is only indicative of profitability if the considered security bid auctions make the same selection of the winner. In the presence of adverse selection, this is not assured, and the cash auction can make a better selection and yield higher expected profits than the steeper equity (share) or call option auctions.

In the present paper we analyze security bid auctions in the presence of both hidden information *and* hidden action. We show that the performance of security bid auctions hinges upon their power of surplus extraction and the implied effort incentives. As in DeMarzo, Kremer, and Skrzypacz (2005), the steeper securities are better surplus extractors, yet this applies only contingent on a given choice of effort. The flip side is that the steeper securities dilute effort incentives. In the face of this trade-off, the generalized share or equity auction, that includes a cash reward to the winner, tends to outperform the less "steep" cash/debt and the "steeper" call option auctions.

Our model applies to a large range of agency problems with competition, ranging from the sale of a product innovation to entrepreneurs who compete for the exclusive use of this innovation, government licensing, franchising and other forms of subcontracting to mergers and acquisitions.

Our analysis also sheds light on the full surplus extraction paradox posed by Crémer (1987) because we show that a share auction can only implement low effort and full full surplus extraction, but *cannot* implement high effort and full surplus extraction.

[2] Samuelson (1987) commented that adverse selection or moral hazard may interfere with surplus extraction.

[3] See also Board (2007) and Rhodes-Kropf and Viswanathan (2005).

The plan of the paper is as follows: In Section 2 we state the model and basic assumptions. In Section 3 we solve the bidding games for different security bid auctions. In Section 4 we generalize the share or equity auction by adding a cash reward or payment to the winner and show that this makes it possible to either increase or lower share bids, which affects effort incentives, and characterize the optimal generalized share auction. In Section 5 we characterize the optimal mechanism, using the revelation principle. In Section 6 we rank the different security bid auctions according to their profitability for the principal, and in Section 7 we discuss possible extensions.

2 Model

A principal wants to award an incentive contract to one of $n \geq 2$ potential agents, using a security bid auction.

Agents differ in their ability to generate revenue. Agents know their own ability, measured by a productivity index $x \in [0,1]$, but not that of others. They are subject to a production function that maps their effort and productivity into their output.

Output is observable and verifiable, but effort is not observable ("hidden action"), which gives rise to a principal-agent problem.

The principal awards incentive contracts – ranging from franchising, to fixed-wage and standard principal-agent sharing contracts – and employs one of the following Vickrey style auction rules in which bids are financial securities:

1. Standard (cash) auction: bidders offer cash payments in exchange for becoming the residual claimant; the highest bidder wins, and the winner pays the second highest bid.
2. Debt auction: bidders offer *IOU*'s in exchange for becoming the residual claimant; the bidder with the highest offer wins and has to pay off the second highest debt, subject to limited liability.
3. Call option auction: bidders offer strike prices for a call option on their output; the bidder with the lowest strike price wins; the call can be exercised at the second lowest strike price.
4. Equity (share) auction: bidders offer output shares; the bidder with the highest share offer wins and pays the second highest share of his output (and possibly collects a cash reward for the winner which may be positive or negative).

The timing of the game is as follows: First, bidders (agents) draw their productivity index, which is their private information, and then simultaneously make their bids; second, the auctioneer (principal) selects the winner; third, the winning bidder chooses his effort e; and finally, output is observed, and the auctioneer collects payments, if any. The equilibrium concept is that of a subgame perfect Nash equilibrium.

The following simplifying assumptions are made.

All parties are risk neutral. The production function, $\phi(x,e) := x + e$, is additive in the productivity index, x, and effort, e. Effort is either high (H) or low (L), $e_H > e_L > 0$, and the corresponding cost of effort is $c_H > c_L$, with $\Delta e := e_H - e_L > c_H - c_L =: \Delta c$, assuring that e_H is the efficient choice for all x.[4] Bidders' productivity index (to which we also refer as bidders' type) is an *i.i.d.* random

[4]Our analysis easily generalizes to continuum of effort choices combined with a quadratic cost function. This indicates that our assumption of binary effort is not restrictive.

variable, drawn from the continuously differentiable distribution function $F : [0,1] \to [0,1]$ with positive p.d.f. f everywhere (symmetric independent private values model).

We denote the k-th largest order statistic of a sample of *i.i.d.* random variables with sample size n by $X_{(k:n)}$, its c.d.f. by $F_{(k:n)}$, and the joint p.d.f. of the highest and the second highest order statistic by $f_{(12:n)}(x,y)$ (for $x > y$).

3 Equilibrium of standard security bid auctions

We now characterize the equilibrium strategies and payoffs of the above stated security bid auctions.

Proposition 1 (Cash/debt auction)**.** *In equilibrium the winner chooses high effort $e_i = e_H$, and each bidder bids the net profit $x_i + e_H - c_H$. The principal's equilibrium expected revenue is*

$$\Pi^c = E\left(X_{(2:n)}\right) + e_H - c_H. \tag{1}$$

Similarly, in the debt auction, the winner chooses e_H and each bidder offers an IOU *that promises to pay $d_i := x_i + e_H - c_H$, which yields the same Π^c.*

Proof. Because the winner is residual claimant he has undiluted incentives and chooses the efficient effort level, e_H. Given this effort strategy, the asserted bid strategy is obviously a (weakly) dominant strategy.

The difference between cash and debt auctions is that cash bids are paid in advance, while debt is payed after output has been observed. This makes a difference only in the event of bankruptcy; however, bankruptcy cannot occur because the output always exceeds the requested debt payment. In order to show this, let d' denote the second highest *IOU*; then, the winner's profit is $x_i + e_H - d' > x_i + e_H - d_i = c_H > 0$, as asserted. □

Proposition 2 (Call option)**.** *In equilibrium the winner plays the effort strategy $e_i(k')$, as a function of the second highest strike price k', with $e_i(k') = e_H$ if $k' > x + e_L + \Delta c$ and $e_i(k') = e_L$ otherwise. Each bidder bids the strike price $k_i = c_L$. On the equilibrium path the winner is chosen at random among all bidders and then chooses low effort. The principal's expected revenue is equal to*

$$\Pi^k = E(X) + e_L - c_L. \tag{2}$$

Proof. The principal will exercise his call option if and only if the strike price he has to pay, k', is less than the observed output of the winner, $x + e_i$. Taking this into account, in the effort subgame the winner chooses e_H if and only if

$$\min\left\{x + e_H, k'\right\} - c_H > \min\left\{x + e_L, k'\right\} - c_L. \tag{3}$$

By checking all possible cases[5] one finds that this is true if and only if $k' > x + e_L + \Delta c$.

Given the equilibrium effort strategy, it is, again, a weakly dominant strategy to bid a strike price equal to c_L. And on the equilibrium path, the winner chooses e_L.

Because in equilibrium all bids are the same, the principal selects the winner at random and exercises the option. Therefore, his expected payoff is as asserted. □

[5] Altogether, there are three cases: 1) $k' > x + e_H$, 2) $k' \in (x + e_L, x + e_H)$, 3) $k' < x + e_L$. In each case, the stated condition must hold.

We mention that the call option can be interpreted as a standard fixed-wage contract, where the winning bidder is paid a fixed wage equal to k. This is due to the fact that in equilibrium the option is exercised with probability one, and bankruptcy never occurs. Not surprisingly, a fixed-wage contract lacks effort incentives, which is why the winner exerts low effort, $e = e_L$.

Lemma 1 (Share auction)**.** *1) Bidders' equilibrium effort strategy is a function of the second highest share, s', where $\mathbb{1}_A$ is the indicator function of set A:*

$$e(s') = \mathbb{1}_{s' \leq s_0} e_H + \mathbb{1}_{s' > s_0} e_L, \quad \text{where} \quad s_0 := \frac{\Delta e - \Delta c}{\Delta e}. \tag{4}$$

2) Conditional on choosing effort e_i, the equilibrium share function is

$$s_i(x) = 1 - \frac{c_i}{x + e_i}, \quad i \in \{L, H\}. \tag{5}$$

Proof. 1) Given s', the winner chooses high effort if and only $(1-s')(x+e_H) - c_H > (1-s')(x+e_L) - c_L$.

2) The equilibrium bid must be such that the bidder never regrets losing, independent of the rivals' bids, i.e., $(1 - s_i(x))(x_i + e_i) = c_i$. □

Lemma 2 (Share auction)**.** *The equilibrium share functions s_H, s_L are single-crossing:*

$$s_H(x) \gtreqless s_L(x) \iff s_H(x) \lesseqgtr s_0. \tag{6}$$

At the crossing point x_0:

$$x_0 := \frac{c_L \Delta e - e_L \Delta c}{\Delta c} \tag{7}$$

one has $s_H(x_0) = s_L(x_0) = s_0$, provided $x_0 \in [0, 1]$.

The proof follows immediately from Lemma 1.

Proposition 3 (Share auction)**.** *In equilibrium each bidder plays the bid strategy:*

$$s(x) = \max\{s_L(x), s_H(x)\}, \tag{8}$$

together with the effort strategy (4). *The principal's expected revenue is equal to*

$$\Pi^s = \int_0^1 \int_0^x s(y)\big(x + e(s(y))\big) f_{(12:n)}(x,y)\,dy\,dx. \tag{9}$$

Proof. Consider a bidder with $x > x_0$. Suppose that bidder deviates from the candidate equilibrium strategy and bids a share $s > s_L(x)$. If that makes a difference, the bidder wins with the deviating bid, s, but would have lost if he had bid $s_L(x)$. In that event, he must pay a share that is at least as high as $s_L(x)$. Because by definition of s_L his profit is non-positive if he pays s_L (no matter which effort he then chooses), this deviation is not profitable. The argument for the case when $s < s_L(x)$ is similar. The same applies to the case $x < x_0$. □

The equilibrium of the share auction is illustrated in Figure 1 for the case $x_0 \in (0, 1)$.[6] The equilibrium bid function is the upper envelope $s(x) := \max\{s_L(x), s_H(x)\}$, and, depending upon the second highest share, s', the winner chooses $e(s') = e_H$ for all $s' < s_0$ and $e(s') = e_L$ for all $s' > s_0$.

[6]This plot assumes uniformly distributed productivity parameters and $(e_H, e_L, c_H, c_L) = (3.3, 2, 1.5, 1)$.

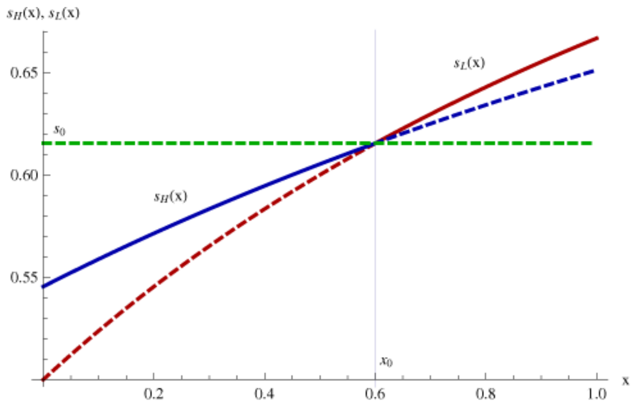

Figure 1: Share auction: (upper envelope) equilibrium bid function for $x_0 \in (0,1)$

On the equilibrium path, the share auction yields a high output share (high extraction) combined with low effort if the two highest productivity parameters, $x > x'$, are both higher than x_0, and it yields low extraction combined with high effort if $x' < x_0$. This is in sharp contrast to the cash/debt and the call auction where we have either full incentives combined with low extraction (cash/debt auction) or low incentives combined with full extraction (call auction). However, as we show in the next section, one can fine tune the share auction and improve its performance by adding a cash transfer component that yields stronger effort incentives without reducing overall revenue.

4 Generalized share auction

In his comment to Hansen (1985), who had introduced share auctions to the literature, Crémer (1987) claimed that the auctioneer can extract the full surplus by adding a positive cash reward to the winner. Following this suggestion, we now add a lump-sum transfer, r, to be paid in cash to the winner of the share auction. This reward may be either positive or negative. We will show that the performance of the share auction can thereby be improved, although full surplus extraction cannot be achieved, unless one is satisfied with implementing low effort.

The cash reward can be regarded as reducing the cost of the winner from c to $c-r$. Therefore, by (5), equilibrium share functions s_L, s_H are increasing while the threshold level x_0 is decreasing in r:

$$s_i(x_i; r) = 1 - \frac{c_i - r}{x_i + e_i}, i \in \{L, H\}, \quad x_0(r) = \frac{(c_L - r)\Delta e - e_L \Delta c}{\Delta c}. \tag{10}$$

It follows that adding a positive reward increases the share of the auctioneer, which contributes to increase his revenue, yet may adversely affect effort incentives; whereas, adding a negative reward reduces his profit share, yet may strengthen effort incentives. The optimal generalized share auction balances this trade-off between surplus extraction and effort incentives.

We denote the expected revenue of the principal in the generalized share auction by $\Pi^s(r)$, and the maximizer of $\Pi^s(r)$ by r^*.

Proposition 4 (Generalized Share Auction)**.** *The generalized share auction: 1) can implement low effort and full surplus extraction, but 2) cannot implement high effort and full surplus extraction. 3) The optimal generalized share auction exhibits an effort "distortion at the top".*

Proof. Let x, x' denote the highest and second highest productivity parameters.

1) To prove this, let $r \to c_L$. This implies $x_0 < 0$, by (10), hence $s(x) = s_L(x)$ for all x and the winner chooses e_L, and the share paid by the winner approaches 1, by (10). Therefore, the principal's revenue is:

$$\begin{aligned}\pi &= s_L(x')(x+e_L) - r \\ &= s_L(x)(x+e_L) - r - \left(s_L(x) - s_L(x')\right)(x+e_L) \\ &= x + e_L - (c_L - r) - r - \left(s_L(x) - s_L(x')\right)(x+e_L).\end{aligned}$$

Because both $s_L(x)$ and $s_L(x')$ converge to 1, π converges to $x + e_L - c_L$.

2) The principal's revenue is:

$$\begin{aligned}\pi &= s_H(x')(x+e_H) - r \\ &= s_H(x)(x+e_H) - r - \left(s_H(x) - s_H(x')\right)(x+e_H) \\ &= x + e_H - (c_H - r) - r - \left(s_H(x) - s_H(x')\right)(x+e_H).\end{aligned} \tag{11}$$

According to the equilibrium bidding strategy,

$$(1 - s_H(x'))(x' + e_H) - c_H = (1 - s_H(x))(x + e_H) - c_H = 0.$$

Hence,

$$(s_H(x) - s_H(x'))(x + e_H) = (1 - s_H(x'))(x - x'). \tag{12}$$

Combining (11) and (12), we get

$$\begin{aligned}x + e_H - c_H - \pi &= (1 - s_H(x'))(x - x') \\ &\geq (1 - s_0)(x - x').\end{aligned}$$

Therefore, the principal's expected revenue is bounded away from the expected surplus.

3) Suppose, *per absurdum*, that $x_0(r^*) > 1$. Then, by slightly increasing the cash reward, $dr > 0$, in such a way that e_H remains optimal for all x (i.e., $x_0(r^* + dr) > 1$), the principal's expected revenue can be increased for the following reason. Denote the increment in the second highest share that is induced by dr by ds'. In the equilibrium of the Vickrey auction $(x' + e_H)(1 - s') - c_H + r \equiv 0$. Therefore, $(x' + e_H)ds' - dr \equiv 0$, which implies that by increasing r the principal's revenue increases by the amount:

$$(x + e_H)ds' - dr > (x' + e_H)ds' - dr \equiv 0.$$

Finally, denote the r that induces $x_0(r) = 1$ by r_1. Evidently,

$$\partial_r \Pi^s(r)|_{r=r_1} = \int_0^1 \int_0^x \frac{x + e_H}{y + e_H} f_{(12:n)}(x, y)dydx > 0;$$

therefore, the optimal cash reward r^* is greater than r_1, and r^* induces $x_0(r^*) < 1$, as asserted. □

5 The optimal mechanism

In similar models Laffont and Tirole (1987) and McAfee and McMillan (1987) characterized the optimal mechanism for awarding incentive contracts. In order to compare the performance of the above security bid auctions with that of the optimal mechanism we adapt their analysis to the present framework.

By the revelation principle, we can restrict attention to direct mechanisms (T, Q, ψ) that are truthfully implementable in Bayesian Nash equilibrium and assure voluntary participation. There, $T_i : [0,1]^n \to \mathbb{R}$ is the transfer to player i, and $Q_i : [0,1]^n \to [0,1]$ his probability of winning, as functions of the reported type profile. In addition, the mechanism stipulates the output that the winner is required to produce as a function of the winner's reported type, $\psi : [0,1] \to \mathbb{R}$.

The optimal mechanism, which is derived in detail in the Appendix, has the following main characteristics:

Proposition 5. *The optimal mechanism selects the highest type as winner (selection rule), requires the winner to satisfy the output target ψ and stipulates the transfer rule T, based on a threshold level $\hat{x} \in [0,1]$:*

$$Q_i(x_i, x_{-i}) = \begin{cases} 1 & \textit{if} \quad x_i > \max\{x_{-i}\} \\ 0 & \textit{if} \quad x_i < \max\{x_{-i}\}, \end{cases} \qquad \text{(selection rule)}$$

$$\psi(x) = \begin{cases} x + e_L & \textit{if} \quad x < \hat{x} \\ x + e_H & \textit{otherwise}, \end{cases} \qquad \text{(output target)}$$

$$T_i(x_i, x_{-i}) = \begin{cases} Q_i(x_i, x_{-i})c_L & \textit{if} \quad x < \hat{x} \\ Q_i(x_i, x_{-i})c_H + u(x_i) & \textit{otherwise}. \end{cases} \qquad \text{(transfer rule)}$$

There $u(x)$ is a supplementary transfer that is defined in the Appendix.

Corollary 1. *The optimal mechanism exhibits "no distortion at the top", i.e., it implements high effort whenever the winner's productivity is sufficiently high, and generally exhibits a "distortion at the bottom".*[7]

6 Performance ranking

To prepare the ranking of the above mechanisms, we introduce the following conditions:

$$\frac{e_H}{(e_H + 1)} > \frac{\Delta c}{\Delta e} \qquad \text{(condition A)}$$

$$\Delta e - \Delta c > E(X) - E(X_{(2:n)}). \qquad \text{(condition B)}$$

Condition A is slightly stronger than our assumption concerning the technology. Whereas our assumption requires $\Delta c/\Delta e < 1$, condition A requires that $\Delta c/\Delta e$ is smaller than the ratio $e_H/(e_H+1)$ which is itself smaller than one. Therefore, condition A is satisfied if the technology is sufficiently productive.

Condition B is a weak requirement concerning the number of bidders and their productivity. It can only be violated if $E(X) > E(X_{(2:n)})$; if F is the uniform distribution, this is possible if and only if $n = 2$.

Proposition 6 (Ranking). *The generalized share auction is not optimal. However, it is more profitable than the cash/debt auction if condition A holds, and the cash auction is more profitable than the call auction if condition B holds.*

[7] The distortion at the bottom does not occur if the technology is highly productive. A distortion at the bottom occurs, for example, if F is the uniform distribution, $\Delta e/\Delta c < 3$, $1/3 < \Delta e < 1/2$, and $n = 2$.

Proof. 1) The optimal generalized share auction exhibits an "effort distortion at the top" because it implies that the winner chooses low effort if the second highest productivity parameter is below $x_0(r^*)$, where $x_0(r^*) < 1$ by Proposition 4. Therefore, the optimal generalized share auction is not an optimal mechanism.

2) We will show that by choosing a reward $r \le c_L$ (either positive or negative) in such a way that $x_0(r) = 1$, the principal can induce bidders to choose high effort for all type profiles, and earn a higher expected revenue than in the cash auction. Therefore, in the optimal generalized share auction the principal's expected revenue must be higher than that of the cash auction.

From (10) it follows that one can induce $x_0 = 1$ by offering the reward $r_1 := c_L - \frac{\Delta c}{\Delta e}(1+e_L)$. If this is done, high effort is chosen by all types. Let x, x' be the highest and second highest productivity parameters. Then, the associated revenue of the principal can be assessed as follows:

$$\begin{aligned} s(x')(x+e_H) - r_1 &= \left(1 - \frac{c_H - r_1}{x' + e_H}\right)(x+e_H) - r_1 \\ &= \left(x' + e_H - c_H + r_1\right)\frac{x+e_H}{x'+e_H} - r_1 \\ &> \left(x' + e_H - c_H + r_1\right) - r_1 \\ &= x' + e_H - c_H \quad \text{(revenue in cash auction)}. \end{aligned}$$

There, the inequality follows from the fact that $(x+e_H)/(x'+e_H) > 1$ and $e_H - c_H + r_1 > 0$, which are implied by the assumption of the proposition.

3) By (1) and (2) one finds that $\Pi^c > \Pi^k$ if and only if condition B holds. □

Example 1. *To illustrate the performance ranking, suppose* $n = 2, F(x) = x, (e_H, e_L, c_H, c_L) = (4/5, 2/5, 1/2, 3/10)$. *Then, the different mechanisms give rise to expected profits of the principal that are summarized in the following table (which also includes the maximum surplus):*

Generalized Share Auction	Share Auction	Cash/Debt Auction	Call Auction	Optimal Mechanism	Full Surplus Extraction
0.7058	0.6810	0.6333	0.6000	0.8947	0.9667

Table 1: Principal's Expected Profit

7 Discussion

One may ask whether the above analysis would differ if we had considered first-price in lieu of Vickrey auctions. In order to answer this question, one may refer to a result by DeMarzo, Kremer, and Skrzypacz (2005). For this purpose, notice that in all considered auctions, with the exception of call auctions, the highest type wins. Therefore, these auctions are examples of so-called "general symmetric mechanisms". Moreover, the set of each type of securities is convex. Therefore, by DeMarzo, Kremer, and Skrzypacz (2005, Proposition 3), first-price and Vickrey auctions are revenue equivalent within each type of security bid auction.[8]

Revenue equivalence also extends to call auctions, even though they do not qualify as "general symmetric mechanisms", because there, the equilibrium strategy (i.e., to bid a strike price equal to

[8] Existence of a monotone equilibrium of the first-price auction is guaranteed by their Lemma 3.

the low effort cost) is the same in both the first-price and the Vickrey auction, and in equilibrium the winner is selected at random.

But, of course, revenue equivalence does not apply across different security bid auctions.

In future work one may pursue several natural extensions. One concerns replacing private by common or affiliated values. Another concerns the fact that licensing typically occurs in a context of oligopolistic competition, where agents interact with each other in an aftermarket, and the winner's type affects the payoff functions in the subsequent oligopoly subgame, which in turn affects bidding. This suggests that one should extend the model to include the strategic interaction in an aftermarket, which may give rise to signalling issues, as in Das Varma (2003) and Goeree (2003) and the subsequent literature.[9]

A Appendix: The optimal mechanism

Because the game is symmetric (players' productivity parameters are *i.i.d.* random variables), we restrict attention to symmetric mechanisms with respect to the permutation of type profiles.

For convenience we define (and omit the subscripts in T_i, Q_i):

$$t(x_i) := E_{x_{-i}}\big(T(x_i, x_{-i})\big), \quad q(x_i) := E_{x_{-i}}\big(Q(x_i, x_{-i})\big). \tag{13}$$

Define $\gamma(x_i, z_i)$ as the winner's cost of fulfilling his output requirement when the agent reports type z_i while his true type is x_i, i.e.

$$\gamma(x_i, z_i) = \begin{cases} c_L & \text{if} \quad \psi(z_i) \leq x_i + e_L \\ c_H & \text{if} \quad x_i + e_L < \psi(z_i) \leq x_i + e_H \\ \infty & \text{otherwise.} \end{cases}$$

We say that it is *feasible* for type x_i to report z_i if $\gamma(x_i, z_i) < \infty$. For convenience define $U(x_i, z_i) := t(z_i) - q(z_i)\gamma(x_i, z_i)$. A mechanism is *truthfully implementable* if

$$\begin{aligned} &\psi(x_i) \leq x_i + e_H && \text{for all } x_i \in [0,1] \\ &U(x_i, x_i) \geq U(x_i, z_i) && \text{for all } x_i, z_i \in [0,1] \quad \text{(IC)} \\ &U(x_i, x_i) \geq 0 && \text{for all } x_i \in [0,1] \quad \text{(IR)} \end{aligned}$$

Given two truthfully implementable mechanisms (T, Q, ψ) and $(\hat{T}, Q, \hat{\psi})$, we say that $(\hat{T}, Q, \hat{\psi})$ *improves upon* (T, Q, ψ) if $q(x)\hat{\psi}(x) - \hat{t}(x) \geq q(x)\psi(x) - t(x)$ for all $x \in [0,1]$ and $q(x)\hat{\psi}(x) - \hat{t}(x) > q(x)\psi(x) - t(x)$ for some $x \in [0,1]$. The binary relation "improves upon" defined on the set of truthfully implementable mechanisms defines a partial ordering. Because we are looking for the optimal mechanism, we will focus on the *maximal* truthfully implementable mechanisms. In particular, maximal truthfully implementable mechanisms have the property that $\psi(x)$ is either equal to $x + e_L$ or $x + e_H$. The following two lemmas show that there is no loss of generality when we restrict our search for the optimal mechanism to the ones where $\psi(x)$ is either equal to $x + e_L$ or $x + e_H$.

Lemma 3. *Suppose that (T, Q, ψ) is a truthfully implementable mechanism and $\psi(x') < x' + e_L$ for some x'. Define $\hat{\psi}$ by $\hat{\psi}(x) = \psi(x)$ for $x \neq x'$ and $\hat{\psi}(x') = x' + e_L$. Then $(T, Q, \hat{\psi})$ is truthfully implementable.*

[9] A step in that direction is Ding, Fan, and Wolfstetter (2010) who analyze takeover bidding in cash and equity (share) auctions, assuming bidders interact in a downstream oligopoly market, and bids signal firms' unknown synergy parameters.

Proof. We only need to check (IC).

$$\hat{U}(x',x') = t(x') - q(x')c_L = U(x',x') \geq U(x',x) = t(x) - q(x)\gamma(x',x) = \hat{U}(x',x),$$
$$\hat{U}(x,x) = t(x) - q(x)\gamma(x,x) = U(x,x) \geq U(x,x') = t(x') - q(x')\gamma(x,x') \geq \hat{U}(x,x')$$

The last inequality holds because $\hat{\gamma}(x,x') \geq \gamma(x,x')$. □

Lemma 4. *Suppose that (T,Q,ψ) is a truthfully implementable mechanism and $x' + e_L < \psi(x') < x' + e_H$ for some x'. Define $\hat{\psi}$ by $\hat{\psi}(x) = \psi(x)$ for $x \neq x'$ and $\hat{\psi}(x') = x' + e_H$. Then $(T,Q,\hat{\psi})$ is truthfully implementable.*

The proof is the same as that of the previous lemma. Hence we will focus on truthfully implementable mechanisms where $\psi(x)$ is either equal to $x + e_L$ or $x + e_H$. Define $X_H := \{x \mid \psi(x) = x + e_H\}$ and $X_L := \{x \mid \psi(x) = x + e_L\}$.

Lemma 5. *A maximal truthfully implementable mechanism (T,Q,ψ) satisfies $\psi(x) = x + e_H$ for all $x \in (1 - \Delta e, 1]$.*

Proof. Suppose to the contrary that $\psi(x') = x' + e_L$ for some $x' \in (1 - \Delta e, 1]$. Define $\hat{\psi}$ by $\hat{\psi}(x) = \psi(x)$ for $x \neq x'$ and $\hat{\psi}(x') = x' + e_H$ and $\hat{T}$ by $\hat{T}_i(x_i, x_{-i}) = T_i(x_i, x_{-i})$ for $x_i \neq x'$ and $\hat{T}_i(x', x_{-i}) = T_i(x', x_{-i}) + q(x')\Delta c$. Then $\hat{t}(x') = t(x') + q(x')\Delta c$ and $(\hat{T},Q,\hat{\psi})$ is truthfully implementable as shown below. This contradicts the fact that (T,Q,ψ) is a maximal truthfully implementable mechanism, because

$$q(x)\hat{\psi}(x) - \hat{t}(x) = q(x)(\psi(x) + \Delta e) - (t(x) + q(x)\Delta c) > q(x)\psi(x) - t(x).$$

We now show that $(\hat{T},Q,\hat{\psi})$ is truthfully implementable. It is obvious that (IR) holds for $(\hat{T},Q,\hat{\psi})$. In checking the condition (IC), it is sufficient to check the condition for the types above x', because the types below x' cannot fulfill the output requirement.

$$\hat{U}(x,x') = t(x') + q(x')\Delta c - q(x')c_H = t(x') - q(x')c_L = U(x,x') \leq U(x,x) = \hat{U}(x,x).$$

□

Lemma 6. *A mechanism (T,Q,ψ) satisfies (IC) if and only if the following conditions hold:*
i) U(x,x) is non-decreasing in x
ii) $x \in X_H$ and $x' \geq x + \Delta e \Rightarrow U(x',x') \geq U(x,x) + q(x)\Delta c$
iii) $x' \in X_L$ and $x' \in (x, x + \Delta e] \Rightarrow U(x',x') \leq U(x,x) + q(x')\Delta c$.

Proof. (1) Necessity: i) Due to the transitivity of the inequality, it is sufficient to show $U(x',x') \geq U(x,x)$ for x,x' with $x' \in (x, x+\Delta e]$. If $x \in X_H$, then $U(x',x') \geq U(x',x) = t(x) - q(x)c_H = U(x,x)$. If $x \in X_L$, then $U(x',x') \geq U(x',x) = t(x) - q(x)c_L = U(x,x)$.

ii) Suppose $x \in X_H$ and $x' \geq x + \Delta e$. Then (IC) implies

$$U(x',x') \geq U(x',x) = t(x) - q(x)c_L = U(x,x) + q(x)\Delta c.$$

iii) Suppose $x' \in X_L$ and $x' \in (x, x + \Delta e)$. Then (IC) implies

$$U(x,x) \geq U(x,x') = t(x') - q(x')c_H = U(x',x') - q(x')\Delta c.$$

(2) Sufficiency: If $z \in X_H$ and $z \leq x - \Delta e$, then

$$U(x,z) = t(z) - q(z)c_L = U(z,z) + q(z)\Delta c \leq U(x,x) \text{ by ii).}$$

If $z \in X_H$ and $z \in (x - \Delta e, x)$, then

$$U(x,z) = t(z) - q(z)c_H = U(z,z) \leq U(x,x) \text{ by i).}$$

If $z \in X_L$ and $z < x$, then

$$U(x,z) = t(z) - q(z)c_L = U(z,z) \leq U(x,x) \text{ by i).}$$

If $z \in X_L$ and $z \in (x, x + \Delta e]$, then

$$U(x,z) = t(z) - q(z)c_H = U(z,z) - q(z)\Delta c \leq U(x,x) \text{ by iii).}$$

If either $z \in X_L$ and $z > x + \Delta e$ or $z \in X_H$ and $z > x$, then $U(x,z) = -\infty < U(x,x)$. □

As long as the principal can extract the entire surplus, he prefers to assign high output, $x + e_H$, because $e_H - c_H \geq e_L - c_L$. Lemma 5 shows that $(1 - \Delta e, 1] \subset X_H$. The following lemma shows that the principal can fully extract the surplus when $X_H = (1 - \Delta e, 1]$.

Lemma 7. *The principal can extract the entire surplus if $\psi(x) = x + e_H$ for $x \in (1 - \Delta e, 1]$ and $\psi(x) = x + e_L$ for $x \in [0, 1 - \Delta e]$.*

Proof. Consider the following mechanism:

$$\psi(x) = \begin{cases} x + c_L & \text{if} \quad x \leq 1 - \Delta e \\ x + c_H & \text{otherwise,} \end{cases}$$

$$Q_i(x_i, x_{-i}) = \begin{cases} 1 & \text{if} \quad x_i > \max\{x_{-i}\} \\ 0 & \text{if} \quad x_i < \max\{x_{-i}\}, \end{cases}$$

$$T_i(x_i, x_{-i}) = \begin{cases} Q_i(x_i, x_{-i})c_L & \text{if} \quad x \leq 1 - \Delta e \\ Q_i(x_i, x_{-i})c_H & \text{otherwise.} \end{cases}$$

Under this mechanism, $q(x) = G(x) := F(x)^{n-1}$ and $t(x) = q(x)\gamma(x,x)$. Hence $U(x,x) = t(x) - q(x)\gamma(x,x) = 0$ for all x, and thus (IR) is satisfied. Furthermore, (IC) is satisfied by Lemma 6. Since it always selects the highest type, it maximizes the revenue among all truthful mechanisms with $X_H = (1 - \Delta e, 1]$. □

Note that the principal can extract the entire surplus with $X_H = [0, 1]$ if $\Delta e > 1$. Given $X_H = (1 - \Delta e, 1]$, the optimal selection rule is Q, because the output $\psi(x)$ is increasing in winner's type x, whereas the payment is already determined when agents report their types.

For an arbitrary X_H define $\hat{x} := \inf\{x \mid x \in X_H\}$. Define a function $u : [0,1] \to \mathbb{R}$ as follows: [10]

$$u(x) = \begin{cases} 0 & \text{if} \quad x < \hat{x} + \Delta e \\ \sup\{u(y) + q(y)\Delta c \mathbb{1}_{X_H} \mid y \leq x - \Delta e,\ y \in X_H\} & \text{otherwise.} \end{cases}$$

[10]The value of u for larger x is defined by that for smaller x.

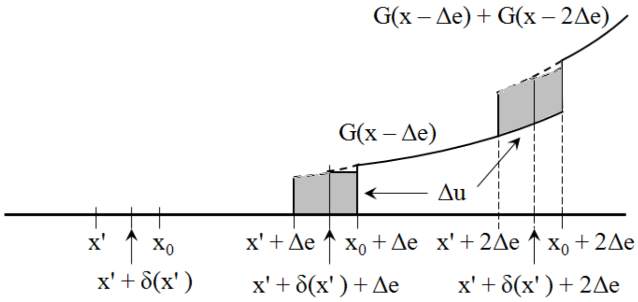

Figure 2: Increase in payment due to increase in X_H

Also define $t(x) = u(x) + G(x)\gamma(x,x)$. Then, $U(x,x) = u(x)$ and the conditions in Lemma 6 are satisfied. Furthermore, the payment is minimized for the given X_H.

Suppose $X_H = (x_0, 1]$ and consider increasing the size of the high effort region X_H by a small interval so that the probability of high effort is increased by $\varepsilon > 0$. In other words consider choosing x' and $\delta(x')$ so that $n\int_{x'}^{x'+\delta(x')} G(t)f(t)dt = F(x'+\delta(x'))^n - F(x')^n = \varepsilon$ and that the new high effort region becomes $X'_H = [x', x'+\delta(x')] \bigcup X_H$. We now show that it is most profitable to increase X_H from the top so that $X'_H = [x', 1]$.

Since the principal's expected revenue is $\Pi = n\int_0^1 \Big(G(x)\psi(x) - t(x)\Big) f(x)dx$, the change in the principal's expected revenue due to change in X_H is:

$$\begin{aligned}\Delta\Pi &= n\int_{x'}^{x'+\delta(x')} G(x)(\Delta e - \Delta c) f(x)dx - n\int_0^1 \Delta u(x) f(x)dx \\ &= \varepsilon(\Delta e - \Delta c) - n\int_0^1 \Delta u(x) f(x)dx.\end{aligned}$$

Hence, x' and $\delta(x')$ should be chosen to minimize $\int_0^1 \Delta u(x) f(x)dx$. A particular choice of x' (and $\delta(x')$) is drawn in Figure 2. In this graph x' is chosen slightly away from the top. Since $(x'+\delta(x'), x_0] \subset X_L$, the graph of $u(x)$ on the interval $(x'+\delta(x')+\Delta e, x_0+\Delta e]$ is flat. On this interval, $\Delta u(x)$ is smaller compared to the case where x' is chosen at the top. On the other hand, $\Delta u(x)$ is larger on the interval starting from x' compared to the case where x' is chosen at the top. In fact, $\Delta u(x) = G(x)$ on this interval, and the interval on which the difference between the two $\Delta u(x)$'s is $G(x)$ is larger when $x'+\delta(x')$ is farther away from x_0. Hence, $\int_0^1 \Delta u(x) f(x)dx$ is minimized when x' is chosen at the top.[11]

Therefore, the optimal mechanism is characterized by Proposition 5.

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[11] Cases not explained by the picture is the cases where x' is close to 0. In these cases $u(x) \geq G(x'+\delta(x'))\Delta c \geq \varepsilon/\varepsilon^{1/n}$ for $x \geq x'+\delta(x')$, which is larger relative to ε when ε is small. Thus these cases are also dominated by x' chosen at the top.

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