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Signaling Competence in Elections

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Signaling Competence in Elections*

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Abstract

We analyze how political candidates can signal their competence and show that polarization might be a way of doing this. For this purpose, we study a unidimensional Hotelling-Downs model of electoral competition in which a fraction of candidates have the ability to correctly observe a policy-relevant state of the world. We show that candidates tend to polarize, even in the absence of policy bias. This is because proposing an extreme platform has a competence signaling effect and has a strictly higher probability of winning than proposing a median platform. The degree of polarization depends on how uncertain is the state of the world.

1 Introduction

In political statements, policy proposals and assessment of the policy-relevant situation are packaged. Politicians need to demonstrate that they have better knowledge about the situation than others and that their policy proposals adequately reflect such knowledge. For example, justifying monetary easing measures necessitates convincing the electorate that the easing is based on an accurate understanding of the source of a recession. Similarly, the use of military force may be justified only by convincing the electorate of the high chance of the targeting country holding weapons of mass destruction. This paper shows that polarization can be a way to signal the politician's better knowledge about the policy-relevant situation and his or her ability to propose an adequate policy, and that in elections extreme candidates actually tend to win more often.

Towards this end, this paper incorporates a dimension of politicians' "competence" within a standard electoral competition model. Our view of competence follows that of Stokes (1963). According to Stokes, the role of politicians includes identifying the electorate's concerns and trying to convey the message that their policy proposals effectively address those concerns. This perception defines a dimension of politicians' competence in our model: the

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ability to discover the most effective policies, under different circumstances. Under the assumption that only a portion of political candidates possess competence, the electorate tries to choose a competent person as their leader. This makes it necessary for the electoral candidates to appear competent and to reveal their competence to the electorate.

Our results show that proposing an extreme platform serves as a signal of competence and gives candidates a strictly higher probability of winning in comparison to proposing a median platform. This induces candidates to polarize, even in the absence of bias in policy preferences. This result stems from the fact that an extreme platform is a risky gamble for incompetent candidates,¹ which means proposing an extreme platform signals competence. Because of the signaling nature of our model, however, perfect separation of competent and incompetent candidates is not possible when the candidates care very much about winning the election; in order to make incompetent candidates choose extreme platforms, they must have a strictly higher probability of winning than they would have chosen the moderate platform.

We model electoral competition by adding a state space, which represents the policy-relevant situation, to a standard Hotelling-Downs one-dimensional policy location game. We focus solely on the effect on the “competence” dimension and abstract from heterogeneity of policy preferences across candidates and voters. The bliss policy depends on the realized state of the world and is probabilistic in nature. The probability distribution over states is such that the optimal policy, from an ex-ante point-of-view, is the median policy. Ex-post, however, the optimal policy is determined by the realized state of the world and can be different from the median policy. Competent candidates can observe the state of the world before the election and, hence, they are aware of the ex-post optimal policy. This implies that competent candidates’ strategies can be state-dependent, from which the electorate can deduce information about the true state of the world.

We characterize two different classes of equilibria. We first characterize the equilibrium in which competent candidates polarize more than do incompetent candidates. Because incompetent candidates are not informed of the state of the world and are risk averse, they avoid choosing an extreme policy. We also characterize the equilibrium in which incompetent candidates polarize more than do competent candidates. This equilibrium is interpreted as competent candidates implicitly coordinating amongst themselves to choose the ex-post best policy, by using information about the state. This is not possible for incompetent candidates, who have no clue about the opponent’s choice. Then they try to polarize, in order to take advantage of the extreme platforms’ competence signaling effect. The main insight of the paper, that proposing an extreme policy is advantageous for winning the election, holds in both types of equilibria.

The types of equilibria supported under different parameter specifications show a link between polarization and uncertainty. The behavior of candidates depends on how uncertain is the state of the world. Generally, the more uncertain the state of the world, the more political candidates polarize, which may yield a possible interpretation about what has been observed in practice; examples may include privatization in Thatcherite Britain in the wake of the Winter of Discontent, or polarization in Germany back in the time of the Great Depression, when there appeared political parties both the extreme left and the extreme

¹This effect is also examined by Majumdar and Mukand (2004), although they are not dealing with an electoral competition game.

right.

The basic setup of our model is based on Kartik and McAfee (2007). In their model, a fraction of candidates have “character”, which is unobserved by voters at the time of the election. Our model differs from their model in not treating competence as an attribute that voters intrinsically prefer. While in their model whether a candidate has a character or not enters directly in the voter’s payoff function, in our model the electorate cares only about what policy is implemented. This necessitates a very different construction of equilibrium from their model, which induces the result that each platform has a different probability of winning, in contrast to Kartik and McAfee (2007).

Our paper is related to two main strands of the literature. The first strand is the literature on career concerns, in which the decisions chosen reflect decision makers’ competence. Prendergast and Stole (1996) build a model in which a manager makes investment decisions over time, and show that the manager has an incentive to exaggerate his own information in order to appear to be a fast learner. While in their study competence is characterized by a variance of noise in information acquisition, we study competence as a binary variable that renders it possible for candidates to acquire a perfect signal about the state of the world. Majumdar and Mukand (2004) develop a dynamic model of policy choice in which competent politicians, who may have the ability to observe the policy-relevant state, care about future electoral prospects. They point out the possibility of inefficient persistence of previously enacted policies, since changing policies signals that a politician does not have the ability to observe the state.

The second strand of literature deals with political competition models in which political candidates possess private information. Schultz (1996) analyzes an election model in which ideologically biased candidates are informed about the policy-relevant state. He finds the relation between the degree to which candidates reveal their private information and their biases in policy preferences. Martinelli (2001) analyzes a model in which voters also have private information about the policy-relevant state and shows that the equilibrium does not result in policy convergence. Heidhues and Lagerlof (2003) show that the candidates have strong incentives to bias their platform choices toward the electorate’s prior beliefs, while, on the contrary, Kartik, Squintani, and Tinn (2012) show that candidates have an incentive to exaggerate their private information.²

Broadly, this study is one variation of the traditional electoral competition model (Downs, 1957 and Davis, Hinich, and Ordeshook, 1970). Most such models in the literature deal with the case in which the candidates are concerned solely with winning the election. In this study, we assume that candidates are also policy motivated in the usual sense of the term (Calvert, 1985 and Wittman, 1977), and that the degree of office motivation of candidates is known to the voter.³ For a recent study of such a hybrid preference model, see, for example, Saporiti (2010).

In trying to incorporate the competence or valence of candidates into Downsian election models, this paper is also related to Aragonés and Palfrey (2001) and Hummel (2010). They study electoral models in which one candidate enjoys an advantage in the sense that when his

²See also Jensen (2009), Laslier and Straeten (2004), and Loertscher (2012), for more on this strand of literature.

³Callander (2008) develops a simple model of electoral competitions in which candidates may be either office or policy motivated.

opponent candidate chooses the same platform, the electorate votes for him.⁴ Our perception of competence differs from theirs in that the voters do not intrinsically prefer a competent over an incompetent candidate as long as the candidate implements the optimal policy. In our model, the preference over competence is generated endogenously by the fact that competent candidates tend to choose appropriate policies.

This paper is organized as follows. Section 2 introduces the basic structure of the model. In Section 3, we characterize important equilibria of our game. We also discuss refinement issues in this section. Section 4 concludes. Formal proofs are given in the Appendix.

2 The Model

The basic element of the model is a standard Hotelling-Downs unidimensional policy location game augmented by an uncertain state of the world. There is a one-dimensional policy space $X = \{-1, 0, 1\}$, and a set of states of the world, $\Theta = \{-1, 0, 1\}$.⁵ There is a probability mass function over states $f : \Theta \rightarrow [0, 1]$, that satisfies

$$f(0) = m \in (0, 1) \text{ and } f(-1) = f(1) = (1 - m) / 2,$$

where m represents the degree of uncertainty about the state of the world. Elements of X and Θ are denoted by x and θ , respectively. There is a representative voter who has a policy preference defined on the product of X and Θ .⁶ Specifically, we assume that the voter's utility $u(x, \theta)$ takes the quadratic loss form $u(x, \theta) = -(x - \theta)^2$, which implies that the voter wants the implemented policy and the state of the world to be as close to each other as possible.⁷ It follows immediately that policy 0 maximizes the voter's expected utility, $\mathbb{E}_\theta[u(x, \theta)]$. Hereafter, we call platform 0 the median platform, and 1 and -1 extreme platforms.

There are two candidates, A and B . We introduce *competence* of candidates as a binary variable: candidate $i \in \{A, B\}$ either possesses competence ($c^i = C$) or does not ($c^i = I$). This is private information and is drawn independently from a Bernoulli distribution with $\Pr(c^i = C) = c > 0$. Before choosing his platform, a competent candidate observes the state of the world, θ , whereas an incompetent candidate does not.⁸

Our election game proceeds as follows. First, Nature chooses the types of the two candidates, determining whether they are competent or not, and this becomes private information. Second, Nature draws a state θ and competent candidates observe this. Then, the two candidates simultaneously choose their platforms. Since a competent candidate observes the state of the world, his platform choice can be state-dependent, while an incompetent candidate's

⁴Also, Ashworth and Mesquita (2009) study a game in which candidates invest in costly valences after choosing platforms.

⁵We discuss the choice of this discrete model setting in the conclusion.

⁶Alternatively, we can think that there are multiple voters and each voter is characterized by her preference parameter, b , and her preference over policy is represented as $-(x - \theta - b)^2$. The preference parameter of the median voter is 0.

⁷One way to understand this setting is to interpret x as a level of a government's fiscal spending and θ as a state of its economy. The people's preference about the level of fiscal spending swings with the state of the economy.

⁸Majumdar and Mukand (2004) define a high ability politician and a low ability politician in a similar way.

platform choice cannot be state-dependent. After observing the two candidates' platforms, the voter votes sincerely to maximize her expected utility, without knowing the true state of the world and without herself receiving any signal. After the election, the two candidates receive the same payoff from the chosen policy as does the voter. In addition to the payoff from the policy, the winning candidate obtains an office rent, $k \in (2, \infty)$.⁹ The value of k measures the degree to which a candidate is motivated by being elected regardless of his policy (i.e., the degree of "office motivation"). The value of k is common knowledge and is the same for all candidates.¹⁰

Since incompetent candidates do not observe the state of the world, their strategies are state-independent. Allowing for the possibility of mixed strategies, a strategy for an incompetent candidate $i \in \{A, B\}$ is represented by a probability mass function $g^i : X \rightarrow [0, 1]$.¹¹ In contrast, since competent candidates observe the state, their strategies can be state-dependent. A strategy for a competent candidate $i \in \{A, B\}$ when the state of the world is θ is represented by a probability mass function $g_\theta^i : X \rightarrow [0, 1]$.

Since the voter does not observe the state, she has to decide which candidate to vote for based only on the candidates' platforms. Her voting strategy is described by a voting function $v : X \times X \rightarrow [0, 1]$, which measures the probability of voting for Candidate A.

Given Candidate B's strategy and the voter's voting strategy, Candidate A's expected payoff from choosing platform x^A when he is competent, observing the state of θ is written as

$$\sum_{x^B \in X} \{v(x^A, x^B)(k + u(x^A, \theta)) + (1 - v(x^A, x^B))u(x^B, \theta)\}(cg_\theta^B(x^B) + (1 - c)g^B(x^B)), \quad (1)$$

which we denote by $U^A(x^A, \theta)$. Candidate A's expected payoff when he is incompetent is

$$\mathbb{E}_\theta[U^A(x^A, \theta)] = \sum_{\theta \in \Theta} U^A(x^A, \theta) f(\theta). \quad (2)$$

Candidate B's expected payoff can be described in an analogous manner.

As this is a signaling game, the voter's beliefs about the state of the world are critical. Let $\varphi(\cdot | x^A, x^B) : \Theta \rightarrow [0, 1]$ be the posterior probability mass over the states given Candidate A's and Candidate B's platforms x^A and x^B , respectively. Given the posterior belief over the states, φ , the voter votes for Candidate A if

$$\left| x^A - \sum_{\theta \in \Theta} \theta \varphi(\theta | x^A, x^B) \right| < \left| x^B - \sum_{\theta \in \Theta} \theta \varphi(\theta | x^A, x^B) \right|, \quad (3)$$

and votes for Candidate B if the opposite inequality holds.¹² Any voting rule is optimal when those two terms in (3) are equal.

⁹It can be shown that when $k \leq 2$ is very low, we can support an equilibrium such that all types of candidates simply choose the best platform given their information, and the voter simply votes for a candidate proposing an extreme platform because she knows that the candidate is proposing the most appropriate policy.

¹⁰In benchmark Hotelling-Downs models of electoral competition, candidates are purely office motivated, i.e., $k = \infty$.

¹¹As in Kartik and McAfee (2007), we take the interpretation of mixed strategies according to the Bayesian view of opponents' conjectures, originating in Harsanyi (1973). That is, a candidate's mixed strategy need not represent him literally randomizing over platforms; instead, it represents the uncertainty that the other candidate and the electorate have about his pure strategy choice.

¹²This follows from the form of the voter's utility function.

Our solution concept is that of perfect Bayesian equilibrium (Fudenberg and Tirole 1991). This requires that the platform distributions, g^A , g^B , g_θ^A and g_θ^B , maximize the expected payoff of each candidate given voter beliefs φ . They also need to be consistent with Bayes' rule. Therefore, competent candidates maximize (1), whereas incompetent candidates maximize (2).

In order to simplify the analysis, we focus on the equilibrium that satisfies the following two conditions: 1. Anonymity $g^A = g^B$, $g_\theta^A = g_\theta^B$ for all θ , and $v(x, y) = 1 - v(y, x)$ for all $x, y \in X$. 2. Symmetry: $g(-1) = g(1)$ and $v(-1, 1) = \frac{1}{2}$. Anonymity implies that two candidates choose the same strategy and the voting rule treats the two candidates equally. Symmetry implies that the two extreme platforms are treated in the same way. From anonymity, we drop the superscript on candidate strategies.

In an equilibrium, the voter's belief when she observes (x^A, x^B) takes the form

$$\varphi(\theta | x^A, x^B) = \frac{f(\theta) \Delta(x^A, x^B, \theta)}{\sum_{\theta} f(\theta) \Delta(x^A, x^B, \theta)}, \quad (4)$$

where

$$\begin{aligned} \Delta(x^A, x^B, \theta) &= c^2 g_\theta(x^A) g_\theta(x^B) \\ &\quad + c(1-c) [g_\theta(x^A) g(x^B) + g(x^A) g_\theta(x^B)] + (1-c)^2 g(x^A) g(x^B), \end{aligned}$$

which is the probability that a particular pair of platforms (x^A, x^B) is chosen, conditional on the realization of the state. We impose no restrictions on out-of-equilibrium beliefs.

Given an equilibrium $(g, g_\theta, v, \varphi)$, the (ex-ante) probability of winning associated with platform x before observing the state, which is denoted by $W(x)$, is

$$W(x) = c \cdot \mathbb{E}_\theta \left[\sum_{y \in X} g_\theta(y) v(x, y) \right] + (1-c) \sum_{y \in X} g(y) v(x, y).$$

This follows because with probability c , the opponent is competent and uses the state-contingent strategy $g_\theta(y)$, while with probability $1-c$ the opponent is incompetent and uses the non-state-contingent strategy $g(y)$.

In what follows, we let $T = \{I, (C, -1), (C, 0), (C, 1)\}$ be the type space of candidates, where $I \in T$ corresponds to an incompetent type, and $(C, \theta) \in T$ corresponds to the type of a competent candidate observing the state θ .

3 Signaling Competence

In this section, we characterize the set of equilibria that satisfy certain conditions. Those equilibria share some important properties, and it is useful to first summarize these in a proposition.

Proposition 1 *In an equilibrium that satisfies anonymity and symmetricity and passes the D1 criterion (defined in Appendix B), the following hold:*

Fact 1: *The probability of winning associated with proposing an extreme platform is strictly higher than that from proposing the median platform. That is,*

$$W(-1) = W(1) > W(0).$$

Fact 2: *Competent candidates who observe an extreme state choose the corresponding platform. That is*

$$g_{-1}(-1) = g_1(1) = 1.$$

It might be useful to explicitly define the usage of the term polarization in our analysis. Fact 2 says that competent candidates observing an extreme state simply choose the (ex-post) optimal platform that matches the true state. Because such a platform actually maximizes the voter’s utility given the state, we do not call such behavior of candidates “polarization”. We say that a candidate is polarizing when he is choosing a platform -1 or 1 , although the median platform is most preferred according to his available information. This implies that the word is used only for the behavior of incompetent candidates and competent candidates observing that the true state is the median. Furthermore, we say that a candidate is completely polarizing if he never chooses the median and mildly polarizing when he does not put the whole mass on choosing the median.

Fact 2 implies that the probability that a candidate proposing an extreme platform is competent is higher than the probability that he is incompetent. This makes the voter willing to vote for an extreme, which thereby makes it possible to support Fact 1, which in turn supports the behavior of competent candidates as described in Fact 2.

3.1 Polarizing Competence

We first characterize an equilibrium in which competent candidates polarize more than do incompetent candidates. Although the equilibrium varies continuously with respect to parameter changes, for expositional purposes we divide cases into two; one in which the median state is likely to be realized and one in which it is not likely to be realized. In the former case, the equilibrium involves mild polarization by competent candidates and moderation by incompetent ones.

Theorem 1 *There is a perfect Bayesian equilibrium that satisfies the following three conditions:*

1-*Competent candidates mildly polarize. That is,*

$$g_0(0) \in (0, 1), \quad g_0(-1) = g_0(1) > 0, \quad \text{and} \quad g_{-1}(-1) = g_1(1) = 1.$$

2-*Incompetent candidates choose the median platform. That is,*

$$g(0) = 1.$$

3-*The voting strategy satisfies*

$$v(-1, 0) = v(1, 0) > \frac{1}{2} \quad \text{and} \quad v(-1, 1) = \frac{1}{2},$$

if and only if either $m > 1/2$ or $c > \rho(m)$, where ρ is a function of m such that $\rho(m) > 1/2$ for all m .

The proof is in the Appendix. In this equilibrium, competent candidates observing the median state ($(C, 0)$ -candidates) mix between the three platforms and thereby “mildly” polarize. Incompetent candidates choose the moderate policy for certain. On the other hand,

the voter is indifferent between the two candidates, the one who is proposing an extreme and the other who is proposing the median and she mixes between voting for the two.¹³ These require that the way the voter mixes makes $(C, 0)$ -candidates indifferent between proposing the two platforms, and hence the voter attaches a higher winning probability to the extremes, and that the way $(C, 0)$ -candidates mix strategies on policy announcements makes the voter indifferent between candidates.

To see this point, keeping the other types' strategies fixed as stated in the theorem, suppose that $(C, 0)$ -candidates' strategy puts a very small probability on choosing an extreme platform, say platform 1. Then, choosing the platform works as a very strong signal that the candidate is of type $(C, 1)$ and that the state is 1. Hence, it makes the voter vote for him. On the other hand, if $(C, 0)$ -candidates choose an extreme platform with high probability, that platform loses signaling power and hence the voter prefers to vote for the median if another candidate chooses it. Only with an adequate degree of mixing can the expected value of the state after observing a platform pair of 0 and 1 become exactly $1/2$ ($\sum_{\theta \in \Theta} \theta \varphi(\theta|1, 0) = 1/2$), and this makes the voter remains indifferent between voting for the candidate proposing the extreme platform and the candidate proposing the median platform. On the other hand, to make $(C, 0)$ -candidates indifferent between the median and the extremes, the probability of winning associated with extreme platforms must be larger than the probability of winning associated with the median platform, otherwise extreme platforms attain lower policy utility than the median platform for $(C, 0)$ -candidates.

This equilibrium is supported only when the median state is likely or when the probability of competence is sufficiently high. The equilibrium requires that the voter is indifferent between the median platform and an extreme platform, and hence the expected value of the state is exactly in the middle. When m is high, or the state is likely to be the median, mixing by type $(C, 0)$, is enough to generate indifference and thus we can support the equilibrium. If m is small, the expected value of the state remains too far from the median, even if type $(C, 0)$ candidates completely polarize the equilibrium cannot be supported. On the other hand, even if m is small, if most candidates are competent, so that c is close to one, given a pair of a moderate and an extreme platform, the voter attaches high probability to the event that both are competent. Consequently, it is possible to generate the indifference only by $(C, 0)$ -candidates' mixing.

Roughly speaking, when the median state is likely, *ceteris paribus*, proposing an extreme platform is not very credible. By making only competent candidates polarize can we sustain an adequate degree of credibility of the extreme platform so that the voter is indifferent between the median platform and the extreme platform. Also, even if the median state is unlikely, if c is very high, incompetent candidates' strategies do not matter much for the voter's belief formation, and hence type $(C, 0)$ candidates' strategies can fully control the degree of credibility of extreme platforms.

¹³Kartik and McAfee (2007)'s model also shares this property. In their model, this property and the assumption that voters flip a fair coin when there is a tie induces an ex-post equilibrium, in which the same behavior of candidates remains an equilibrium, regardless of whether one candidate announces first or second, or both announce simultaneously. The equilibrium in our model, however, is not an ex-post equilibrium, since the voter is more likely to vote for an extreme platform. This implies that an extreme platform is more preferred when the opponent chooses an extreme and vice versa.

We now turn to the case in which extreme states are likely to occur. The following theorem shows that if the probability of competence is not too high, there is an equilibrium such that competent candidates completely polarize while incompetent candidates mix between all platforms.

Theorem 2 *There is a perfect Bayesian equilibrium that satisfies the following three conditions:*

1-Competent candidates completely polarize. That is,

$$g_0(0) = 0 \text{ and } g_{-1}(-1) = g_1(1) = 1.$$

2-Incompetent candidates mildly polarize. That is,

$$g(0) \in (0, 1) \text{ and } g(-1) = g(1).$$

3-The voting strategy satisfies

$$v(-1, 0) = v(1, 0) > \frac{1}{2} \text{ and } v(-1, 1) = \frac{1}{2},$$

if and only if $m < 1/2$ and $c < \frac{1}{2(1-m)}$.

Now the way the voter mixes strategies makes incompetent candidates indifferent between all platforms, while in the equilibrium of Theorem 1 it makes $(C, 0)$ -candidates indifferent. The way incompetent candidates mix makes the voter indifferent between candidates for all combination of platform choices. As in Theorem 1 the fact that a type of candidate whose optimal policy is the median platform (incompetent candidates) is indifferent between the median platform and extreme platforms means that extreme platforms have higher probability of winning than the median.

The equilibria characterized in Theorem 2, however, cannot be supported when the competence probability is very high. This is because in such a case, even when incompetent candidates put a high probability on choosing extremes, the voter still strictly prefers to vote for an extreme platform because the median state is sufficiently unlikely. Roughly speaking, when the median state is unlikely and candidates are unlikely to be competence, *ceteris paribus*, proposing an extreme platform is too credible, which makes the voter always vote for an extreme. Hence the equilibria cannot be supported when the competence probability is very high, i.e., $c \geq \frac{1}{2(1-m)}$.

The next question is what type of ‘‘Polarizing Competence’’ equilibrium we have when m is small and c is higher than $\frac{1}{2(1-m)}$. We see that the equilibrium characterized in the next theorem is compelling, although it contains an off-equilibrium platform choice, and it can be supported as an equilibrium for all parameter values. Actually, if we do comparative statics on the equilibrium of Theorem 2, we have $\lim_{c \uparrow \frac{1}{2(1-m)}} g(0) = 0$ and hence it converges to the equilibrium characterized in the next theorem.

Theorem 3 *For all parameter values, there is a perfect Bayesian equilibrium such that*

1-Competent candidates completely polarize. That is,

$$g_{-1}(-1) = g_1(1) = 1, \text{ and } g_0(0) = 0.$$

2-Incompetent candidates completely polarize. That is,

$$g(-1) = g(1) = \frac{1}{2}.$$

3-The voting strategy satisfies $v(1, -1) = \frac{1}{2}$.

In this equilibrium, the probability of winning associated with the median platform is sufficiently low so that no candidate chooses it. This is possible by making the voter who faces one candidate who chooses the median and another who chooses an extreme prefer the extreme candidate by making her off-equilibrium beliefs extreme.¹⁴

Although the existence of the equilibrium in Theorem 3 is ensured for all parameter specifications, a simple intuition, however, tells us that it is plausible only when the median state is unlikely and candidates are very likely to be competent. To see this, suppose that the voter uses the simple (out-of-equilibrium) updating rule where after a candidate deviates to the median, she updates her belief about the state based only on the platform choice made by the non-deviating candidate. Then it can be shown that we need $c \geq 1/2(1 - m)$ to make the voter want to vote for a non-deviating candidate.¹⁵ In this sense, we may say that the equilibrium is plausible only when candidates are likely to be competent (high c) and the median state is less likely to be realized (low m). We provide a more formal argument on this point in Appendix B.

3.2 Polarizing Incompetence

In the equilibria presented so far, competent candidates observing the median state had stronger incentives to polarize than incompetent candidates. This was because candidates have risk-averse preference over policies, and hence, for incompetent candidates, choosing an extreme platform is very risky, which prevented incompetent candidates from polarizing relative to competent candidates. The next theorem, however, demonstrates that there is an equilibrium in which only incompetent candidates polarize.

Theorem 4 *For all parameter values, there is a perfect Bayesian equilibrium that satisfies the three conditions:*

1-Competent candidates choose the (ex-post) best platform. That is,

$$g_0(0) = 1 \text{ and } g_{-1}(-1) = g_1(1) = 1.$$

2-Incompetent candidates mildly polarize. That is,

$$g(0) \in (0, 1) \text{ and } g(-1) = g(1) > 0.$$

3-The voting strategy satisfies

$$v(-1, 0) = v(1, 0) > \frac{1}{2} \text{ and } v(-1, 1) = \frac{1}{2}.$$

¹⁴We are free to attach any belief, from the definition of perfect Bayesian equilibrium.

¹⁵To see this, observe that if the non-deviating candidate is choosing 1 and the voter updates her belief based only on it, the voter attaches probability $(1 - m)(c + \frac{1-c}{2})$, m , $(1 - m)(\frac{1-c}{2})$ to the state being 1, 0, and -1, respectively. This implies that the expected value of the state is $(1 - m)c$.

The key to understanding how this equilibrium is supported is the fact that different types of candidates face different lotteries over the opponent's choice. In the equilibrium, a type- $(C, 0)$ candidate thinks that the opponent is very likely to choose the median, because c is high. Hence, for him, the median platform is not too disadvantageous in terms of winning the election. On the other hand, an incompetent candidate knows that the opponent is likely to choose an extreme, because of the high c and the small m . Given that extreme platforms have higher winning probabilities, this means that the (interim) probability of winning that incompetent candidates can expect from choosing the median platform is small. This induces them to polarize, while keeping type- $(C, 0)$ candidates choosing the median. Intuitively, a high probability of competence enables type- $(C, 0)$ candidates to implicitly coordinate their strategy to choose the ex-post best policy.

Note that in this equilibrium it can happen that proposing an extreme platform hurts the voter's belief about the candidate's competence. Because the voter is sufficiently confident that an extreme state occurs and candidates are competent, she has a very strong incentive to vote for an extreme. In order to make her indifferent between candidates choosing an extreme platform and the median platform, incompetent candidates must have a high probability of choosing the extreme platform, thereby reducing its competence signaling effect.

Although the model has multiple equilibria, there are some relationships between strategies and parameter values that all the equilibria, except for the one in Theorem 3 in which candidates' strategies are invariant with parameters, characterized in this section share and it is worth mentioning them briefly. First, in each equilibrium, there is a positive relationship between the probability of winning associated with the median platform and the probability of competence. This is because, for a candidate, it implies that the opponent is more likely to choose an extreme platform. This effect decreases the probability of winning associated with the median platform, which must be offset by increasing the odds that the median platform wins over extremes. Also, there are positive relationships between the degree of office motivation and the probability of winning associated with the median platform. The intuition is that when a policy becomes less important relative to the importance of winning the election, candidates are not willing to choose the median platform unless it has good odds of winning over extremes.

Our model, which is essentially a signaling game, allows multiple equilibria. A source of multiplicity comes from its large strategy space for players, and another source of multiplicity comes from the freedom of off-equilibrium specification of beliefs, which is common in signaling games. As long as we restrict attention to a particular class of equilibria, however, we have full characterization of equilibria in the class. In Appendix B, we provide an equilibrium refinement criterion D1, appropriately modified to fit to our model. Then we can actually eliminate the equilibrium such that one of the extreme platforms is never chosen on-equilibrium. The intuition is simple: the type of candidate who has the strongest incentive to deviate to an extreme platform is the competent type who observes the corresponding extreme state, which induces the voter to vote for the deviating candidate. This enables us to have the following result:

Theorem 5 *Any anonymous and symmetric equilibrium that passes the D1 criterion is one of those characterized in Theorems 1 to 4.*

4 Conclusion

This paper examined a signaling game in which a fraction of the candidates are competent, but competence is unobservable by voters. The general insight is that being extreme is advantageous for winning the election, because it makes the candidate who is so appear competent.

An important assumption of our model is that candidates are assumed to commit to policies during the election, and the commitment is assumed to be credible. Some justification for this assumption can be made. For example, real-life implementation of a policy requires preparations and there is a lag between proposing a policy and actually implementing it. This makes it impossible to change policies flexibly. In our model, if the commitment is not credible,¹⁶ the representative voter knows that, after the election, competent candidates will implement the optimal policy contingent on the revealed state of the world and incompetent candidates will implement the median policy. Then all the platforms that are announced in equilibrium have the same winning probability.

In this paper, in order to focus on the role of vertical differences between candidates (competence), we assumed away the possibility that candidates have policy biases. It may be interesting to relax this assumption. In such a case, there may be an effect in which proposing a platform that is opposite to a candidate's policy bias serves as a stronger signal about his competence. This type of extension needs to enlarge the policy space to the standard continuum policy space. In such an extension, however, the number of possible combinations of policy announcements becomes large and it is necessary to construct a large number of equilibrium beliefs, contingent on policy choices. This is a very difficult task that requires some simplifying assumptions about the way a voter's beliefs are formed. For the same reason, extending our model to a continuum of state space is also left to future research.

5 Appendix A: Proofs

5.1 Proof of Theorem 1

Suppose that the players' strategies are as follows: $v(0, -1) = v(0, 1) = \beta$, $v(1, -1) = 1/2$, $g(0) = 1$, $g_{-1}(-1) = g_1(1) = 1$, $g_0(0) = d$, and $g_0(-1) = g_0(1) = (1 - d)/2$. We will show that there are $\beta \in [0, 1/2)$ and $d \in (0, 1)$ such that these strategies constitute an equilibrium, if and only if $m > 1/2$ or $c > \rho(m)$ for a function ρ that satisfies $\rho(m) > 1/2$. Note that $W(-1) = W(1) > W(0)$ immediately follows from $\beta < 1/2$.

A strategy of type $(C, 0)$ candidates, who are mixing, is optimal when $U(1, 0) = U(0, 0)$ and $U(-1, 0) = U(0, 0)$. These can be rewritten as

$$G(\beta, d, k) = U(1, 0) - U(0, 0) = \left(\frac{1}{2} - \beta\right)k + c - cd + \beta - 2c\beta + 2cd\beta - 1 = 0. \quad (5)$$

On the other hand, the voting strategy is optimal if and only if $\sum_{\theta \in \Theta} \theta \varphi(\theta|1, 0) = 1/2$ and $\sum_{\theta \in \Theta} \theta \varphi(\theta|-1, 0) = -1/2$, since in such a case, candidates proposing 0 and -1 (and 0 and

¹⁶Osborne and Slivinski (1996) and Besley and Coate (1997) consider models in which candidates cannot make commitments at all.

1) are equally preferred. Thus we have

$$\frac{\frac{1-m}{2}c(1-c)}{\frac{1-m}{2}c(1-c) + mc(1-c)\frac{1-d}{2} + mc^2d\left(\frac{1-d}{2}\right)} = \frac{1}{2},$$

which can be rewritten as

$$F(d) = mcd^2 + m(1-2c)d + (1-2m)(1-c) = 0. \quad (6)$$

Because $F(1) = \frac{1}{2} - \frac{1}{2}c > 0$, there is $\hat{d} \in (0, 1)$ such that $F(\hat{d}) = 0$ if $\min_{d \in [0,1]} F(d) < 0$. A sufficient condition is $F(0) < 0$, which is ensured when $m > 1/2$. On the other hand, if $m < 1/2$, $F(0) > 0$ and $\arg \min_{d \in \mathbb{R}} F(d) = \frac{2c-1}{2c}$. Hence if $c \leq 1/2$, we have $F(d) \geq 0$ for all $d \in [0, 1]$ and we cannot find such $\hat{d} \in (0, 1)$. Think of the case $c > 1/2$. Let $\tilde{d} = \arg \min_{d \in \mathbb{R}} F(d) = \arg \min_{d \in (0,1)} F(d)$. If $F(\tilde{d}) < 0$, there are two such $\hat{d} \in (0, 1)$. By substituting $\tilde{d} = \frac{2c-1}{2c}$, it is verified that $F(\tilde{d}) < 0$ as long as c is higher than a threshold value determined by m . Take $\rho(m)$ as the maximum of such a threshold and $1/2$.

Let this value of \hat{d} be $d(c, m)$ (if there are multiple such m , take the smaller one). In order to complete the proof, it is enough to show that given k , there is $\beta \geq 0$ such that $G(\beta, d(c, m), k) = 0$. Since it is a strictly decreasing function of β and $G(\beta, d, k) < 0$ for all $\beta \geq 1/2$, this is possible if $G(0, d(c, m), k) > 0$, which holds if and only if $\frac{k+2c-2}{2c} > d(c, m)$. Hence it is an equilibrium if $k > 2cd(c, m) + 2 - 2c$. Since we assumed that $k > 2$, this completes the proof. *Q.E.D.*

5.2 Proof of Theorem 2

Suppose that $m < 1/2$ and the players' strategies are as follows: $v(0, -1) = v(0, 1) = \beta$, $v(1, -1) = 1/2$, $g(0) = 1$, $g_{-1}(-1) = g_1(1) = 1$, $g_0(-1) = g_0(1) = 1/2$, and $g(-1) = g(1) = (1-d)/2$, and $g(0) = d$. We will show that we can find $\beta \in (0, 1/2)$ and $d \in (0, 1)$ such that those strategies constitute an equilibrium if and only if $m < 1/2$ and $c < \frac{1}{2(1-m)}$. Note that $W(-1) = W(1) > W(0)$ immediately follows from $\beta < 1/2$.

In order to be an equilibrium, $\mathbb{E}[U(-1, \theta)] = \mathbb{E}[U(0, \theta)]$ and $\mathbb{E}[U(1, \theta)] = \mathbb{E}[U(0, \theta)]$, because all the platforms should be equally preferred. Hence we have a condition

$$G(\beta, d, k) = \mathbb{E}_\theta[U(1, \theta)] - \mathbb{E}_\theta[U(0, \theta)] = 0. \quad (7)$$

Also, it must hold that $\sum_{\theta \in \Theta} \theta \varphi(\theta|1, 0) = 1/2$ and $\sum_{\theta \in \Theta} \theta \varphi(\theta|-1, 0) = -1/2$. Those can be rewritten as

$$\frac{\frac{1-m}{2}[c(1-c) + (1-c)^2\frac{1-d}{2}] - \frac{1-m}{2}(1-c)^2\left(\frac{1-d}{2}\right)}{\frac{1-m}{2}[c(1-c) + (1-c)^2\frac{1-d}{2}] + m[c(1-c)\frac{1}{2} + (1-c)^2\left(\frac{1-d}{2}\right)] + \frac{1-m}{2}(1-c)^2\left(\frac{1-d}{2}\right)} = \frac{1}{2},$$

which can be simplified as

$$F(d) = 2c + d - 2cm - cd - 1 = 0. \quad (8)$$

The solution \hat{d} is given by $\hat{d} = \frac{1-2c+2cm}{1-c}$ and hence $\hat{d} \in (0, 1)$ if $m < \frac{1}{2}$ and $c < \frac{1}{2(1-m)}$. Therefore, in order to prove the theorem, it is enough to show that given k , equation (7), as

a function of β , has a solution in $(0, 1/2)$ when $d = \widehat{d}$. It can be seen that $G(\beta, d, k) < 0$ for all $\beta \geq 1/2$, d , and k . Let κ be the value of k that satisfies $G(0, \widehat{d}, \kappa) = 0$. Since $G(0, d, k)$ is a strictly increasing function of k , we can find such κ . Then it follows that when $k > \kappa$, we can find β such that $G(\beta, \widehat{d}, k) = 0$ from $(0, 1/2)$. Since

$$\begin{aligned} G(0, d, k) &= \mathbb{E}_\theta[U(1, \theta)] - \mathbb{E}_\theta[U(0, \theta)] \\ &= \frac{1}{2}k + m[cd - d] + \frac{(1-m)}{2} \left[\frac{3}{2}cd - 2d - \frac{3}{2}c - \frac{1}{2}c^2 + \frac{1}{2}c^2d \right] = 0, \end{aligned} \quad (9)$$

it is easy to see that $\kappa < 2 - 2cm$. Because we assumed that $k > 2$, this completes the proof. *Q.E.D.*

5.3 Proof of Theorem 3

Let $\varphi(-1|-1, 0) = 1$ and $\varphi(1|1, 0) = 1$. Then $v(-1, 0) = 1$ and $v(1, 0) = 1$ follow, which support the candidates' strategy. *Q.E.D.*

5.4 Proof of Theorem 4

Suppose that the players' strategies are as follows: $v(0, -1) = v(0, 1) = \beta$, $v(1, -1) = 1/2$, $g_0(0) = 1$, $g_{-1}(-1) = g_1(1) = 1$, and $g(-1) = g(1) = (1-d)/2$.

For the voter's strategy to be optimal, it has to be satisfied that $\sum_{\theta \in \Theta} \theta \varphi(\theta|1, 0) = 1/2$ and $\sum_{\theta \in \Theta} \theta \varphi(\theta|-1, 0) = -1/2$, and these are written as

$$\frac{-\frac{1-m}{2}(1-c)^2 d^{\frac{1-d}{2}} + \frac{1-m}{2}[c(1-c)d + (1-c)^2 d^{\frac{1-d}{2}}]}{\frac{1-m}{2}(1-c)^2 d^{\frac{1-d}{2}} + m[c(1-c)\frac{1-d}{2} + (1-c)^2 d^{\frac{1-d}{2}}] + \frac{1-m}{2}[c(1-c)d + (1-c)^2 d^{\frac{1-d}{2}}]} = \frac{1}{2}. \quad (10)$$

Because the right hand side is strictly increasing with d , and it is 0 when $d = 0$ and 1 when $d = 1$, there is a unique $\widetilde{d} \in (0, 1)$ that satisfies the above equation.

On the other hand, it can be computed that

$$\sum_{x \in \{-1, 1\}} \sum_{\theta \in \{-1, 1\}} [U(x, \theta) - U(0, \theta)] = 2 \sum_{x \in \{-1, 1\}} [U(x, 0) - U(0, 0)] \quad (11)$$

, and also there is a unique $\widetilde{\beta} \in (0, 1/2)$ such that $U(1, 0) - U(0, 0) = U(-1, 0) - U(0, 0) = 0$ when $d = \widetilde{d}$. For such combination of $(\widetilde{\beta}, \widetilde{d})$, from (11), $\mathbb{E}_\theta[U(x, \theta)] = \mathbb{E}_\theta[U(0, \theta)] = 0$ and $U(x, 0) = U(0, 0)$ for all $x \in X$. Hence candidates' strategies are also optimal. This completes the proof. *Q.E.D.*

5.5 Proof of Theorem 5

In the following, denote by $\Upsilon(g, g_\theta)$ the set of platforms that are chosen with strictly positive probabilities, i.e., $\Upsilon(g, g_\theta) = \{x | \max\{g(x), \sum_\theta g_\theta(x)\} > 0\}$. Note that $\Upsilon(g, g_\theta) = X$ does not necessarily imply that there is no off-equilibrium platform choices, because candidates' choices are correlated with each other. Furthermore, denote by $P(x)$ the set of types of

candidates who choose a particular platform with a strictly positive probability, that is, $(C, \theta) \in P(x)$ if $g_\theta(x) > 0$ and $I \in P(x)$ if $g(x) > 0$.

Note that for $x \in \{-1, 1\}$, $U(x, \theta) - U(0, \theta) =$

$$\begin{aligned} & \Pr[y \neq -x|\theta]\{v(x, 0) - 1/2\}k + \Pr[y = -x]\{v(x, y) - v(0, y)\}k \\ & + \sum_{s \in X} \Pr[y = s|\theta]\{v(x, s)u(x, \theta) + v(s, x)u(s, \theta)\} \\ & - \sum_{s \in X} \Pr[y = s|\theta]\{v(0, s)u(0, \theta) + v(s, 0)u(s, \theta)\}, \end{aligned} \quad (12)$$

and $\mathbb{E}_\theta[U(-1, \theta)] - \mathbb{E}_\theta[U(0, \theta)] =$

$$\begin{aligned} & \Pr[y \neq -x|\theta]\{v(x, 0) - 1/2\}k + \Pr[y = -x]\{v(x, y) - v(0, y)\}k \\ & + \mathbb{E}_\theta \left[\sum_{s \in X} \Pr[y = s|\theta]\{v(x, s)u(x, \theta) + v(s, x)u(s, \theta)\} \right] \\ & - \mathbb{E}_\theta \left[\sum_{s \in X} \Pr[y = s|\theta]\{v(0, s)u(0, \theta) + v(s, 0)u(s, \theta)\} \right], \end{aligned} \quad (13)$$

where $\Pr[y = s|\theta] = cg_\theta(s) + (1 - c)g(s)$, which is the probability that a candidate chooses platform s when the state is θ .

Then we have the following three lemmata.

Lemma 1 *In an anonymous and symmetric equilibrium such that $\Upsilon(g, g_\theta) = X$, $g_{-1}(-1) = 1$ and $g_1(1) = 1$.*

Proof. Take an equilibrium such that $\Upsilon(g, g_\theta) = X$. We first demonstrate that $g_{-1}(0) = 0$ holds ($g_1(0) = 0$ is also proven in an analogous way). In order to accomplish this, suppose that $g_{-1}(0) > 0$, which implies $U(-1, -1) \leq U(0, -1)$. In order for this to hold, however, from (12), we must have $v(-1, 0) < 1$. Then from (12), $U(-1, 0) < U(0, 0)$ and $U(-1, 1) < U(0, 1)$ follow. These also imply $\mathbb{E}_\theta[U(-1, \theta)] < \mathbb{E}_\theta[U(0, \theta)]$. Therefore, we have $\{(C, -1)\} = P(-1)$ and hence $\sum_{\theta \in \Theta} \varphi(\theta | -1, 0) = 1$. Then, however, $v(-1, 0) = 1$ follows, which is a contradiction. Similarly, we can also prove that $g_1(0) = 0$ holds.

Next, suppose that $g_{-1}(1) > 0$ holds. Because in a symmetric equilibrium, for all g and g_θ , it can be computed that $U(1, 1) - U(-1, 1) > U(1, 0) - U(-1, 0) > U(1, -1) - U(-1, -1)$ follows, and hence we have $\mathbb{E}_\theta[U(1, \theta)] > \mathbb{E}_\theta[U(-1, \theta)]$. Then we have $\{(C, -1)\} = P(-1)$ and hence $\sum_{\theta \in \Theta} \varphi(\theta | -1, 0) = 1$. From these, however, $v(-1, 0) = 1$ follows, which is a contradiction. ■

Lemma 2 *In an anonymous and symmetric equilibrium such that $\Upsilon(g, g_\theta) = X$, if $g_0(0) \in (0, 1)$ then $g(0) = 1$. Also, if $g_0(0) = 1$ then $g(0) \in (0, 1)$.*

Proof. Define the following functions:

$$\begin{aligned} \alpha(v, g, g_\theta) &= (1 - 2v(0, -1))k - 2v(0, -1), \\ \beta(v, g, g_\theta) &= (1 - v(0, 1) - v(0, -1))k + v(0, -1) + v(0, 1) - 2, \\ \gamma(v, g, g_\theta) &= (1 - 2v(0, 1))k - 2v(0, 1). \end{aligned}$$

In a symmetric equilibrium, we have $g_{-1}(-1) = g_1(1)$, and by using this it can be computed that

$$\begin{aligned}\phi(v, g, g_\theta) &= \frac{1}{2} \sum_{\theta \in \{-1, 1\}} \sum_{x \in \{-1, 1\}} \{U(x, \theta) - U(0, \theta)\} \\ &= (1 - c) \{g(-1) \alpha(v, g, g_\theta) + g(0) \beta(v, g, g_\theta) + g(1) \gamma(v, g, g_\theta)\} + c\beta(v, g, g_\theta)\end{aligned}$$

and

$$\begin{aligned}\Phi(v, g, g_\theta) &= \sum_{x \in \{-1, 1\}} \{U(x, 0) - U(0, 0)\} \\ &= (1 - c) \{g(-1) \alpha(v, g, g_\theta) + g(0) \beta(v, g, g_\theta) + g(1) \gamma(v, g, g_\theta)\} \\ &\quad + c\{g_0(-1) \alpha(v, g, g_\theta) + g_0(0) \beta(v, g, g_\theta) + g_0(1) \gamma(v, g, g_\theta)\}.\end{aligned}$$

In order to obtain a contradiction, suppose that there is an equilibrium such that $g_0(0) \in (0, 1)$ and $g(0) \in (0, 1)$. Observe that $g_0(0) > 0$ and $g(0) > 0$ imply $\phi(v, g, g_\theta) \leq 0$ and $\Phi(v, g, g_\theta) \leq 0$. If $\min\{\alpha(v, g, g_\theta), \gamma(v, g, g_\theta)\} > 0$, then from $\Phi(v, g, g_\theta) \leq 0$ we must have $\beta(v, g, g_\theta) < 0$. However, then $\phi(v, g, g_\theta) < \Phi(v, g, g_\theta)$ follows and hence we have $\mathbb{E}_\theta \left[\sum_{x \in \{-1, 1\}} \{U(x, \theta) - U(0, \theta)\} \right] > \sum_{x \in \{-1, 1\}} \{U(x, 0) - U(0, 0)\}$, which induces $g(0) = 0$, a contradiction. Also, if $\max\{\alpha(v, g, g_\theta), \gamma(v, g, g_\theta)\} < 0$, we must have $\beta(v, g, g_\theta) > 0$. Then it follows that $\phi(v, g, g_\theta) > \Phi(v, g, g_\theta)$ and hence we must have either $\mathbb{E}_\theta[U(-1, \theta)] > \mathbb{E}_\theta[U(0, \theta)]$ or $\mathbb{E}_\theta[U(1, \theta)] > \mathbb{E}_\theta[U(0, \theta)]$, and hence $g(0) = 0$, a contradiction. If $\alpha(v, g, g_\theta) = \gamma(v, g, g_\theta) = 0$, we must have $\beta(v, g, g_\theta) = 0$, but it can be checked that there is no v such that these are simultaneously satisfied. Finally, suppose that $\alpha(v, g, g_\theta) > 0$ and $\gamma(v, g, g_\theta) < 0$. Then, we must have $g_0(1) \alpha(v, g, g_\theta) + g_0(-1) \gamma(v, g, g_\theta) = 0$, since otherwise, we have either $\phi(v, g, g_\theta) > \Phi(v, g, g_\theta)$ or $\phi(v, g, g_\theta) < \Phi(v, g, g_\theta)$ each leads a contradiction. This also implies $\beta(v, g, g_\theta) = 0$, since otherwise we have $g(0) = 1$ or $g(1) = 0$. Then, it must also hold that $g(1) \alpha(v, g, g_\theta) + g(-1) \gamma(v, g, g_\theta) = 0$, and hence $g_0(1)/g_0(-1) = g(1)/g(-1) = 1$. Now we have $\alpha(v, g, g_\theta) + \gamma(v, g, g_\theta) = 0$, from which $(1 - v(0, -1) - v(0, 1))k - v(0, -1) - v(0, 1) = 0$ follows. This, however, contradicts $\beta(v, g, g_\theta) = 0$. Hence there is no equilibrium such that $g_0(0) \in (0, 1)$ and $g(0) \in (0, 1)$.

Next, suppose $g_0(0) = g(0) = 1$. Then, it follows that $\sum_{\theta \in \Theta} \theta \varphi(\theta | -1, 0) = -1$, and hence $v(-1, 0) = 1$. Then $U(-1, 0) \leq U(0, 0)$ implies $k \leq 2$, which is a contradiction.

Finally, suppose that $1 > g_0(0) > 0$. In a symmetric equilibrium, it must hold that $g_0(1) = g_0(-1)$, because otherwise $|\sum_{\theta \in \Theta} \theta \varphi(\theta | -1, 0)| \neq \sum_{\theta \in \Theta} \theta \varphi(\theta | 1, 0)$ follows, which leads a contradiction. Then we must have $\Phi(v, g, g_\theta) = 0$ and $v(0, -1) < 1/2$ from $k > 2$. Those imply $\alpha(v, g, g_\theta) > \beta(v, g, g_\theta)$ and hence $\beta(v, g, g_\theta) < 0$. This implies $\Phi(v, g, g_\theta) < 0$, and thus $g(0) = 1$ follows. ■

Lemma 3 *In an anonymous and symmetric equilibrium such that $\Upsilon(g, g_\theta) = X$, $g_0(-1) = g_0(1)$.*

Proof. Suppose that $g_0(-1) \neq g_0(1)$. Then, from Lemma 2, $g(0) = 1$ follows. Now because it holds that $|\sum_{\theta \in \Theta} \theta \varphi(\theta | -1, 0)| \neq \sum_{\theta \in \Theta} \theta \varphi(\theta | 1, 0)$, we have $v(-1, 0) \neq v(1, 0)$. Suppose $v(-1, 0) > v(1, 0)$, without loss of generality. Then, $U(-1, 0) > U(1, 0)$ follows and hence we have $\{(C, 1)\} = P(1)$. This implies $\sum_{\theta \in \Theta} \theta \varphi(\theta | 1, 0) = 1$ and thus $v(1, 0) = 0$, which is a contradiction. ■

Those lemmata show that in any anonymous and symmetric equilibrium such that every platform may be chosen, candidates' strategies are one of those characterized in Section 3. Then, Theorem 5 follows from this fact and the discussion in Appendix B. *Q.E.D.*

5.6 Appendix B: Equilibrium Refinement

In this appendix, we discuss refinement issues. Towards this end, we apply the D1 refinement, proposed by Cho and Kreps (1987). In our context, it requires that the voter does not attribute a deviation to a particular type of candidate if there is some other type who is willing to make the deviation for a strictly larger set of possible voting strategies.

In our model, however, a simple application of the D1 criterion is not appropriate, because there are first movers (candidates) whose strategies are strongly correlated with the state. This makes it impossible to make an inference about a deviating player's type (which must have some implication about the state) independently from the action of a non-deviating player. Therefore, we modify the usual D1 criterion by applying the idea used in Bagwell and Ramey (1991). It uses the fact that the types of the two candidates are strongly correlated and that the types of the candidates are also correlated with the state. For example, it is not possible that the voter perceives a candidate's type to be $(C, 1)$ at the same time as she perceives another candidate's type to be type $(C, 0)$. Once we require that after one candidate deviates the voter should still believe that another candidate follows the equilibrium strategy, this property of correlated types should impose some conditions on off-equilibrium beliefs.¹⁷

As in before, given an equilibrium $(g, g_\theta, v, \varphi)$, let $\Upsilon(g, g_\theta)$ be the set of platforms that are chosen with strictly positive probabilities. Also, denote by $P(x)$ the set of types of candidates that choose a particular platform with a strictly positive probability. Finally, let $V(t)$ be the equilibrium payoff for type t candidates in equilibrium $(g, g_\theta, v, \varphi)$, which is defined by (1) for competent candidates and (2) for incompetent candidates.

Next, given an equilibrium $(g, g_\theta, v, \varphi)$, for each pair of type $t \in T$ and an off-equilibrium platform choice $p \notin \Upsilon(g, g_\theta)$ let $D_t(p)$ and $D_t^+(p)$ be sets of functions from $X \times X$ to $[0, 1]$ that are defined, respectively, as follows:

$$\begin{aligned} D_t(p) &= \{v^0 : U(p, \theta) \geq V(t)\} \text{ if } t = (C, \theta) \\ &= \{v^0 : \mathbb{E}_\theta[U(p, \theta)] \geq V(t)\} \text{ if } t = I. \\ D_t^+(p) &= \{v^0 : U(p, \theta) > V(t)\} \text{ if } t = (C, \theta) \\ &= \{v^0 : \mathbb{E}_\theta[U(p, \theta)] > V(t)\} \text{ if } t = I, \end{aligned}$$

where $U(p, \theta)$ is calculated by (1) with voting rule v^0 and candidates' strategies g and g_θ of the equilibrium. Note that these sets can be empty.

The above definitions say that, $D_t(p)$ is the set of voting rules that make type t candidates weakly prefer to deviate to the off-equilibrium platform p , when the voter's belief and voting strategy satisfy the particular restriction and the opponent is supposed to follow the equilibrium strategy (g, g_θ) . Similarly, $D_t^+(p)$ is defined with strict preferences. An equilibrium that

¹⁷Bagwell and Ramey (1991) construct a two-period oligopoly model in which the incumbents' pricing choices, which may signal their production costs, are followed by the entrant's entering decision. In their refinement of what they call unprejudiced beliefs, the entrant assumes a single deviation happens after the off-equilibrium choices of incumbents.

satisfies the D1 criterion is the equilibrium satisfying the following condition, in addition to the conditions for a Bayes-Nash equilibrium:

Definition 1 *An equilibrium $(g, g_\theta, v, \varphi)$ satisfies D1 if for all pairs of x and y such that $x \in \Upsilon(g, g_\theta)$, $y \notin \Upsilon(g, g_\theta)$, the following are satisfied:*

Condition 1: If $I \in P(x)$ and there is (C, θ) such that $D_{t'}(y) \subset D_{(C, \theta)}^+(y)$ for all $t' \neq (C, \theta)$, then $\varphi(y|x, y) = 1$.

Condition 2: If $I \notin P(x)$ and there is $(C, \theta) \in P(x)$ such that $D_I(y) \subset D_{(C, \theta)}^+(y)$ and $D_{(C, \theta')}(y) \subset D_{(C, \theta)}^+(y)$ for all $(C, \theta') \in P(x)$, then $\varphi(y|x, y) = 1$. If there is no such (C, θ) , then

$$\sum_{\theta \in \Theta} \theta \varphi(\theta|x, y) \in \text{int } \text{co}\{\theta | D_{(C, \theta)}(y) \not\subset D_t^+(y) \text{ for all } t \in P(x)\}.$$

In above definition, $\text{int } \text{co}A$ means the interior of the convex hull of set A . Condition 1 describes the case in which the voter knows that the non-deviator may be incompetent and hence cannot elicit any information from him. In this case, the voter attributes a deviation to a particular type, if that type is the most likely to deviate among all types. Condition 2 describes the case in which the voter knows that the non-deviator is competent. In this case, the voter can ignore the possibility that the deviating candidate is a competent type who does not choose the platform that the non-deviator is choosing, because two candidates cannot be different competent types. Then, she attributes a deviation to the type who has the strongest incentive among all possible types of deviator. Also, even if she cannot attribute a deviation to a single competent type, she still uses information obtained from the non-deviating candidate to update her belief. We have no restriction on off-equilibrium beliefs when incompetent candidates are not excluded from the possibility of being the deviator, because the voter is free to think that the deviator should be incompetent, in which case his deviation reveals nothing about the state.

Now we have the following lemma, which completes the proof of Theorem 5.

Lemma 4 *Any equilibrium such that $\{-1, 1\} \subsetneq \Upsilon(g, g_\theta)$ fails D1. Also, the equilibria characterized in Theorem 1 through Theorem 4 satisfy D1.*

Proof. We first show that there is no equilibrium that satisfies D1 and $\Upsilon(g, g_\theta)$ is singleton. Towards this end, suppose that such an equilibrium exists. First think of the case in which $\Upsilon(g, g_\theta) = \{1\}$. Because whether a particular v is included in $D_t(-1)$ or $D_t^+(-1)$ depends only on $v(-1, 1)$, we can easily see that

$$D_t(-1) \subset D_{(C, -1)}^+(-1) \text{ for all } t \in \{I, (C, 1), (C, 0)\}.$$

Then, D1 implies that if the voter observes platform pair $(-1, 1)$, her off-equilibrium belief puts the whole mass on the event that the deviating candidate is type $(C, -1)$. Then, however, it must hold that $v(-1, 1) = 1$. This implies that type $(C, -1)$ candidates have an incentive to deviate to 0, which is a contradiction. The same proof applies to the cases in which $\Upsilon(g, g_\theta) = \{-1\}$ and $\Upsilon(g, g_\theta) = \{0\}$.

Next, we show that there is no equilibrium that satisfies D1 such that $\Upsilon(g, g_\theta) = \{0, 1\}$ or $\Upsilon(g, g_\theta) = \{0, -1\}$. Towards this end, take an equilibrium such that $\Upsilon(g, g_\theta) = \{0, 1\}$. By using $g_\theta(-1) = g(-1) = 0$ for all θ and (1) we can show that $g_{-1}(0) = 1$. To see this, if $g_{-1}(0) < 1$, it must hold that $U(1, -1) - U(0, -1) \geq 0$, which implies $(v(0, 1) - \frac{1}{2})k + 3v(0, 1) \leq 0$. This in turn implies that both $U(1, 0) - U(0, 0) > 0$ and $U(1, 1) - U(0, 1) > 0$ hold, and thus $\mathbb{E}_\theta[U(0, \theta)] > \mathbb{E}_\theta[U(1, \theta)]$. Then only type $(C, -1)$ candidates choose 0, and hence $v(0, 1) = 1$, which contradicts $U(1, -1) - U(0, -1) \geq 0$. Hence, $g_{-1}(0) = 1$. Similarly, we can show that $g_1(1) = 1$. Moreover, we have $g_0(0) > 0$, since otherwise $\sum_{\theta \in \Theta} \varphi(\theta, 0, 1) < 1/2$ and $v(0, 1) = 1$, which is a contradiction. Also, it can be shown that $v(1, 0) > 1/2$, since otherwise, we must have $U(0, 0) - U(1, 0) > 0$ and $\mathbb{E}_\theta[U(0, \theta)] - \mathbb{E}_\theta[U(1, \theta)] \geq 0$, which imply $g_0(0) = 1$ and $g(0) = 1$, respectively. This leads, however, to $\sum_{\theta \in \Theta} \varphi(\theta, 0, 1) = 1$ and thereby $v(0, 1) = 1$, which is a contradiction. Finally, $g_0(0) > 0$ and $\mathbb{E}_\theta[U(0, \theta)] - \mathbb{E}_\theta[U(1, \theta)] > U(0, 0) - U(1, 0)$ imply $g(0) = 1$.

We will show that

$$D_t(-1) \subset D_{(C, -1)}^+(-1) \text{ for all } t \in \{I, (C, 0), (C, 1)\}. \quad (14)$$

Note that from $(C, -1) \notin P(1)$, $I \notin P(1)$, and D1, it must hold that $v'(-1, 1) = 0$. Then whether $v' \in D_{(C, -1)}^+(-1)$ or not depends only on whether $v'(-1, 0)$ is strictly larger than some threshold value or not. More precisely, $v' \in D_{(C, -1)}^+(-1)$ if and only if

$$v'(-1, 0)k - (1 - v'(-1, 0)) > \frac{1}{2}k - 1. \quad (15)$$

It is easy to see that (15) is a necessary condition for $v' \in D_t(t)$, from which (14) follows.

It is straightforward to prove the second statement and hence the proof is omitted. ■

The above result, however, is not powerful enough to enable us to discuss under what parameter values we can justify the equilibrium of Theorem 3. One possible idea is to simply assume that the voter updates her belief about the state of the world based only on the choice made by the other candidate, when D1 is of no bite. This argument seems compelling, given that candidates' types are independent from other.

Definition 2 Take a pair of platforms $x \in \Upsilon(g, g_\theta)$, $y \notin \Upsilon(g, g_\theta)$ such that for all $t \neq I$, $D_I(y) \subsetneq D_t(y)$. Then the voter uses simple off-equilibrium updating if

$$\varphi(\theta|x, y) = \frac{f(\theta)(cg_\theta(x) + (1-c)g(x))}{\sum_{\theta} f(\theta)(cg_\theta(x) + (1-c)g(x))}.$$

Then, we can see easily the following result, which provides some argument for supporting the equilibrium of Theorem 3 only for the parameter range of $c \geq 1/2(1-m)$.

Proposition 2 The equilibrium of Theorem 3 is supported by simple off-equilibrium updating if and only if $c \geq 1/2(1-m)$.

Proof. Given the candidates' strategies in Theorem 3, there is v such that $v(-1, 0) = v(1, 0) < 1/2$ and $v(-1, 1) = 1/2$ and that satisfies $v \in D_I(y)$ and $v \notin D_t(y)$ for all $t \neq I$. The formula of simple off-equilibrium updating implies $\varphi(\theta|-1, 0) < -1/2$ and $\varphi(\theta|1, 0) > 1/2$, if and only if $c \geq 1/2(1-m)$. ■

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