Strategic Communication Networks *

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October 5, 2009

Abstract

We consider situations in which every agent would like to take an action that is coordinated with those of others, as well as close to a common state of nature, with the ideal proximity to that state varying across agents. Before this coordination game is played, agents decide to whom they reveal their private information about the state. The information transmission occurring in the cheap-talk communication stage is characterized by a *strategic communication network* whose links represent truthful information transmission. In equilibrium, whether communication takes place between two agents depends not only on the conflict of interest between these agents, but also on the number and preferences of the other agents with whom they communicate. In particular, communication to a large group of recipients may be feasible even though communication to a small subset of that group may not be. We show that agents who are more central in terms of preference tend to communicate more and to have a greater impact on decisions.

Keywords: Cheap talk, coordination, incomplete information, networks.
JEL Classification: C72, D82, D83, D85.

*We thank Bruno Biais, Antoni Calvó-Armengol, Ying Chen, Andrew Clark, Olivier Compte, Wouter Dessein, Françoise Forges, Matthew Jackson, Philippe Jehiel, Navin Kartik, David Martimort, Eduardo Perez, Ludovic Renou, Aggey Semenov, Francesco Squintani, Joel Sobel, Jean-Marc Tallon, Jean-Christophe Vergnaud, and seminar participants at Bristol, Cergy-Pontoise, École Polytechnique, Leicester, Lille, Mannheim, Marseille, Montpellier, Paris 1, PSE, Strasbourg, Toulouse, University of California at Santa Barbara (Southwest Economic Theory), the 2007 workshop on strategic communication and networks in Valencia, the 2008 world congress of the Game Theory Society in Northwestern, the 2009 CESifo area conference on applied microeconomics in Munich, and the 9th SAET conference in Ischia for useful comments and discussions. This research was partly carried out while the first author was at University Paris 1.

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1 Introduction

Many economic situations involve agents who share an interest in coordinating their actions as well as in adapting them to an unknown state of the world. The analysis presented here considers this type of context but departs from the typical assumption that agents agree on the state-contingent profile of decisions. Because their tastes may differ, we let the interacting agents vary in their ideal proximity to the underlying fundamentals. For example, it is widely admitted that the different divisions of an organization have to coordinate their actions to maximize the firm's profit, with such actions also corresponding to the uncertain environment of the firm. For a number of reasons, ranging from local adaptation costs to career concerns, it is likely that each of those divisions will also attempt to adapt its choice to some local particularities.[1] Similarly, when advocating policies, members of a political party will wish to best suit the situation, but also to be in line with the announcements made by other members to ensure the cohesion of the party. At the same time, activists may have heterogenous preferences regarding the right policy to implement.[2] We here consider this type of coordination game of incomplete information in which every player incurs losses from any mismatch between his action and both others' actions and his own "ideal action". Every ideal action depends on the state and on a systematic positive or negative bias, as in the cheap-talk or delegation models of Crawford and Sobel (1982) and Dessein (2002). These biases vary across agents, and the profile of biases in the population is a measure of the conflict of interest that they face.

The aim of this paper is to analyze how agents strategically transmit to each other the signals they privately hold about the fundamentals in these types of situation. Indeed, *before* taking their payoff-relevant actions, we offer players the opportunity to send costless, non-verifiable, and private messages about their information. Within this stylized framework, the communication stage consists of a cheap-talk game in which every player is, at the same time, a sender and a receiver, and we address the question of *who truthfully speaks with whom*. Our precise focus is on how agents' heterogeneity in ideal actions affects decentralized and strategic communication between them. We propose to characterize the transmission of private information by what we call a *communication network*, described by a set of receivers for every player. A player is said to be a receiver of another player if the latter truthfully reveals his private information to the former. A complete characterization of the information transmission occurring in equilibrium is provided, which roughly boils down to the intuitive statement that agents are more prone to communicate when their ideal actions are more similar, and as the need for coordination becomes larger.

Informational incentive constraints require that no player have an interest in lying about his type to his (endogenous) set of receivers. As in standard cheap-talk games (e.g., Crawford and Sobel, 1982), this condition can be formulated as a condition on the proximity between the sender's and the receivers' biases. In existing models extending communication to multiple but strategically-independent decision-makers (see, e.g., Farrell and Gibbons, 1989, Goltsman and Pavlov, 2009 and Galeotti, Ghiglino, and Squintani, 2009), we have only to check that the sender has no incentive to lie to any *single* receiver. In our model, the informational incentive constraints are more sophis-

[1] A multi-divisional organization in which decisions must be adapted to local conditions and information but also coordinated between divisions is considered in Alonso, Dessein, and Matouschek (2008) and Rantakari (2008).

[2] See, for example, Dewan and Myatt (2008).

ticated than in these games since all of the agents want to coordinate their actions. Due to the strategic interaction between receivers, how each receiver reacts to a sender's signal depends not only on this signal but also on (his expectation over) the total number of receivers of this signal. At the same time, since the sender also wants to coordinate his action with the receivers, any deviation by the sender in the communication stage induces coordination costs that depend on both the total number of his receivers and the number of receivers he lies to. Combined with the assumption that loss functions are quadratic, our informational incentive constraints require that the sender's bias be close enough to the average bias of *every subset* of receivers. Exactly how close biases should be is determined by some threshold that depends on both the total number of receivers and the respective subsets of receivers the sender could lie to.

This feature reveals a key insight of our paper: communication between two agents depends on not only the conflict of interest between them, but also the preferences and the size of all the agents with whom they communicate. In particular, one main result is that communication to a large group of recipients may occur in equilibrium even though communication only to a strict subset of that group may not. To understand the intuition, consider a simplified 3-player situation in which there is a unique informed agent (the sender): the sender and one uninformed agent both want to choose an action exactly adapted to the true state of the nature, and another uninformed agent is positively biased, i.e. wants to choose an action higher than the true state. Assume that every player also wants to coordinate his action with that of the two others. When the sender communicates only to the biased agent, he has a strong incentive to under-report his type in order to decrease this agent's expectation about the state so that his action gets closer to the sender's ideal action. On the contrary, when the sender communicates to both the biased and unbiased agents, he may have no incentive to jointly lie to both of them because their average bias is small. He may also have no incentive to misrepresent his information only to the biased agent. Indeed, both agents are now more responsive to the sender's message than when he communicates only to one agent, so this deviation would increase the dispersion of players' actions and thus induce large coordination losses. It is worth noting that the disciplinary effect that the coordination of multiple audiences has on communication is different from the disciplinary effect of public communication identified by Farrell and Gibbons (1989). These latter show that communication to two independent decision-makers may occur in equilibrium when communication is public, whereas information is revealed to neither decision-maker when communication is private. Our *disciplinary effect of coordination* appears even though communication is *not* public and relies on the fact that the receivers we consider are not independent decision-makers.

We provide sharp predictions regarding equilibrium communication networks for several configurations of preferences. First, when players' biases are uniformly distributed we show that a player's tendency to communicate increases with the proximity of his bias to the average bias in the population. Communication is therefore typically not symmetric: centrists tend to influence the decisions of the other players more because they transmit their information truthfully to more distant players than do extremists, with this effect becoming stronger as the need for coordination increases. When the coordination motive is very strong, middle-biased players may communicate to all of the other players even with a wide dispersion of preferences, while other players may never truthfully report their private information. Second, when players are arranged in groups with the same preferences, we again show that information transmission across groups is typically asymmetric: members from the larger group tend to communicate more easily to members of other groups than do members of a smaller group. That is, large groups of agents tend to influence the decisions of small groups by credibly reporting information, while there is less truthful communication from small to large groups.

The paper is organized as follows. The model is presented in Section 2, which also shows that strategic communication networks cannot be completely Pareto-ranked. Equilibrium communication networks are analyzed and illustrated in Section 3, and Section 4 concludes. Most of the proofs are relegated to the Appendix.

2 Model

2.1 A Class of Coordination Games with Incomplete Information

Let $N = \{1, ..., n\}$ be a finite set of agents, with $n \geq 2$. Each agent chooses an action $a_i \in A_i = \mathbb{R}$. The action profile is denoted $a = (a_1, \ldots, a_n)$. Each agent's payoff depends on the action profile and a state of nature θ. Before the game starts, nobody knows the state of nature, but each agent $i \in N$ receives a private signal $s_i \in S_i = \{\underline{s}_i, \overline{s}_i\}$ about θ, where $\underline{s}_i < \overline{s}_i$. We assume that agents' types are independent and denote by $q_i \in \Delta(S_i)$ the prior probability distribution over agent i's set of types, for every $i \in N$. When the type profile is $s = (s_1, ..., s_n)$, the underlying state of nature is $\theta(s) \in \mathbb{R}$.

Agent i's payoff function is given by

$$u_i(a_1, \ldots, a_n; \theta(s)) = -(1-\alpha)(a_i - \theta(s) - b_i)^2 - \frac{\alpha}{n-1} \sum_{j \neq i} (a_i - a_j)^2. \tag{1}$$

The first component of agent i's utility function is a quadratic loss in the distance between his action a_i and his ideal action $\theta(s) + b_i$, where $b_i \in \mathbb{R}$. We allow the bias parameter b_i to vary across individuals to reflect agents' conflicts of interest with respect to their ideal actions. The second component is a miscoordination quadratic loss which increases in the average distance between i's action and other agents' actions. The constant $\alpha \in (0, 1)$ weights both sources of quadratic loss, i.e. it parameterizes agents' coordination motives arising from the strategic complementarity in their actions. Without loss of generality, players are indexed in increasing order of their biases: $b_1 \leq \cdots \leq b_n$.

We assume that the state of nature is the aggregated term $\theta(s) = \sum_{i \in N} s_i$.[3] The sum of players' private signals is indeed a good approximation to the payoff-relevant state in many situations. In an organizational setting for instance, a signal s_i for division i may represent division i's time, budget or expected benefit from a joint project (which is private information), and the state that matters for the whole organization may be the total time, budget or expected benefit of the project. More broadly, considering a state of nature which is additive in types is a simplifying standard assumption in common-value environments, especially in auction theory (see, amongst many others, Bulow and Klemperer, 2002, Mares and Harstad, 2003 and Levin, 2004), in some models of lobbying

[3] Note that the state can be any *weighted* sum of players' types (since we do not make any assumptions about the two possible values of each signal), and players' signals are not assumed to be i.i.d. (they are only assumed to be independent).

with multiple experts (e.g., Wolinsky, 2002) or in organization theory (e.g., Jehiel, 1999). From a theoretical point of view, assuming an additive state and independent types implies that the impact of player i's signal on the fundamentals, $\overline{s}_i - \underline{s}_i$, which can be interpreted as the value of player i's private information, is independent of others' signals. It follows that in our analysis we abstract from any effects that the correlation and the degree of complementarity between players' signals may have on informational incentive constraints. We focus instead on the effect of players' coordination motives and preference heterogeneity. The robustness of our results to this independence property will be discussed and related to the literature at the end of Subsection 3.1.

2.2 Communication Game

Before the coordination game described above is played, but after each player has learnt his type, a communication stage is introduced in which players can send costless and private messages to each other. More precisely, every player i can send a different message $m_i^j \in M_i$ to every other player $j \neq i$, with M_i denoting the (non-empty) set of messages available to player i. Let $m_i = (m_i^j)_{j \neq i} \in (M_i)^{n-1}$ be the vector of messages sent by player i, and $m^i = (m_j^i)_{j \neq i} \in \prod_{j \neq i} M_j \equiv M_{-i}$ the vector of messages received by player i.

The information transmission occurring during the cheap-talk extension of the game will be characterized by a *communication network*, whose directed links represent revelation of private information from one player to another. In order to focus on the presence or absence of such information-transmission links between the agents, we restrict the analysis to pure communication strategies and abstract from the partial transmission of information generated by random strategies.[4] For the same reason, we only consider two possible types for each player. It follows that any message from player i to player j will either be fully revealing or non-informative, and we consider that a communication link is formed from i to j when i's message to j is fully revealing. Without loss of generality, we assume that message spaces are binary: $M_i = \{\underline{m}, \overline{m}\}$.

Player i's communication strategy is a profile $\sigma_i = (\sigma_i^j)_{j \neq i}$ with

$$\sigma_i^j : S_i \to M_i.$$

Let $\sigma_i^j(m_i^j \mid s_i)$ be the probability (0 or 1) that player i send the message m_i^j to player j according to his strategy σ_i when his type is s_i.

Since each player i's utility function is strictly concave with respect to a_i, his best response is necessarily unique, so we can consider without loss of generality pure second-stage strategies. Player i's second-stage strategy is a mapping

$$\tau_i : S_i \times (M_i)^{n-1} \times M_{-i} \to A_i,$$

where $\tau_i(s_i, m_i, m^i)$ is the action chosen by player i when his type is $s_i \in S_i$, the vector of private messages $m_i = (m_i^j)_{j \neq i} \in (M_i)^{n-1}$ was sent, and the vector of private messages $m^i = (m_j^i)_{j \neq i} \in M_{-i}$ was received. Let $\tau(s, (m_i)_{i \in N}) = (\tau_i(s_i, m_i, m^i))_{i \in N}$ be the corresponding action profile.

[4] We do not exclude the existence of non-trivial mixed equilibria, as in the discrete quadratic version of Crawford and Sobel (1982), but the full characterization of such equilibria is quite difficult since we have to consider the possibility that any combination of players randomize over their messages, for any possible combination of receivers.

As is usual in cheap-talk games, the set of Nash equilibrium outcomes in our model coincides with the set of sequential equilibrium outcomes because messages off the equilibrium path can simply be treated as synonyms of equilibrium messages. Hence we do not have to specify a complete belief system: an equilibrium of the communication game is simply a strategy profile $(\sigma, \tau) = ((\sigma_i)_{i \in N}, (\tau_i)_{i \in N})$ satisfying the following properties:

(i) For all $i \in N$, and $s_i \in S_i$,

$$(\sigma_i^j(s_i))_{j \neq i} \in \arg \max_{m_i \in M_i^{n-1}} \sum_{s_{-i} \in S_{-i}} q_{-i}(s_{-i})\, u_i\left(\tau(s, (\sigma_{-i}(s_{-i}), m_i)); \theta(s)\right),$$

where $q_{-i}(s_{-i}) = \prod_{j \neq i} q_j(s_j)$.

(ii) For all $i \in N$, $m_i \in (M_i)^{n-1}$ and $m^i \in \text{supp}[\sigma_{-i}^i]$,

$$\tau_i(s_i, m_i, m^i) \in \arg \max_{a_i \in A_i} \sum_{s_{-i} \in S_{-i}} \mu_i(s_{-i} \mid m^i)\, u_i\left((\tau_j(s_j, (\sigma_j^{-i}(s_j), m_j^i), (\sigma_{-i}^j(s_{-i}), m_i^j)))_{j \neq i}, a_i; \theta(s)\right),$$

where $\mu_i(s_{-i} \mid m^i) = \prod_{j \neq i} \frac{\sigma_j^i(m_j^i | s_j) q_j(s_j)}{\sum_{t_j \in S_j} \sigma_j^i(m_j^i | t_j) q_j(t_j)}$.

A *communication network*, that characterizes a communication strategy profile $(\sigma_i)_{i \in N}$, is denoted by $(R_i)_{i \in N}$, where, for every player i, the *set of receivers*

$$R_i \equiv \{j \in N \backslash \{i\} : \sigma_i^j(\underline{s}_i) \neq \sigma_i^j(\overline{s}_i)\},$$

is the set of individuals to whom player i truthfully reveals his type. Let $|R_i|$ be the number of individuals who learn player i's type in the communication stage. Using the terminology of network theory, R_i corresponds to player i's (out)neighborhood and $|R_i|$ to player i's (out)degree.

2.3 Second-Stage Equilibrium Characterization

The quadratic formulation of players' utility functions, together with the independence of players' types, enable us to obtain a unique and tractable second-stage equilibrium characterization whatever the information structure generated by the communication stage. Indeed, as in Calvó-Armengol and Martí (2009) who consider the same utility functions but without heterogeneity in biases, it can be shown that our payoffs admit a potential that represents common interests for all players. The corresponding common-interest game satisfies the sufficient conditions in Marschak and Radner's (1972) team theory for the equilibrium to be unique and linear. More precisely, we show in Appendix A.1 that, given a profile of types $(s_i)_{i \in N}$ and a communication strategy profile characterized by $(R_i)_{i \in N}$, the second-stage equilibrium strategy of each player $i \in N$ is uniquely given by:

$$a_i = \sum_{j \in I_i} \frac{\alpha(n-1-|R_j|)E(s_j) + (1-\alpha)(n-1)s_j}{n-1-\alpha|R_j|} + \sum_{j \in \overline{I}_i} E(s_j) + B_i, \tag{2}$$

where $I_i = \{k : i \in R_k\} \cup \{i\}$ is the set of signals which are known by player i after the communication stage, $\overline{I}_i = \{k : i \notin R_k\} \backslash \{i\}$ is the set of signals which are not known by player i after the communication stage, and

$$B_i = \frac{[(n-1)-(n-2)\alpha]b_i + \alpha \sum_{j\neq i} b_j}{n+\alpha-1}. \tag{3}$$

Player i's second-stage equilibrium action has three components. The first component is a weighted sum of j's actual type, s_j, and the expected value of j's type, $E(s_j)$, for each player j whose type is known by player i (including himself). The weight put on the actual type of player j increases with the number of players who know j's type, $|R_j|$. This is because a player who wants to be coordinated with the others has a greater incentive to act according to a signal when many other players act according to the same signal. In other words, the larger the set of receivers, the more the sender and those receivers choose an action which is responsive to the sender's private information. This is one of the key effects that will drive our results regarding efficient and equilibrium communication networks. The second component of player i's equilibrium action corresponds to the sum of the expected values of j's type for each player j whose type is not known by player i. The last component adjusts the action of player i with respect to the bias profile. This increases in all players' biases, with more weight being put on player i's own bias, b_i, as the coordination motive decreases.

2.4 Efficient Communication Networks

Before characterizing the networks that arise as equilibria of the communication game, we consider the efficiency of communication networks. The following proposition compares players' ex ante expected payoffs as the communication network expands, assuming that equilibrium actions are played in the second-stage game.[5] While an increase in the size of player i's set of receivers is always strictly beneficial for player i and for these receivers, this increase always makes the players who do not learn player i's type strictly worse off.

Proposition 1 *Consider two communication networks $R = (R_i, R_{-i})$ and $R' = (R'_i, R_{-i})$ such that $|R_i| < |R'_i|$.*

i) Every player $j \in R'_i \cup \{i\}$ is strictly better off*, ex-ante, with the communication network R' than with the communication network R;*

ii) Every player $j \in N \backslash (\{i\} \cup R'_i)$ is strictly worse off*, ex-ante, with the communication network R' than with the communication network R.*

Proof. See Appendix A.2. ■

The intuition of this result is the following. Consider one sender's private signal. As we have observed in the previous subsection (see Equation (2)), when the number of players informed about this signal increases, they become more responsive to it. While this increase benefits informed players whose actions are now better coordinated and adapted to the state, uninformed players suffer larger miscoordination losses, and are therefore worse off.

This result implies that, in general, communication networks cannot be ranked in the sense of Pareto. In particular, defining a communication network $R' = (R'_i)_{i\in N}$ as *more informative* than

[5] As in Crawford and Sobel (1982), it is not possible to compare players' expected payoffs at the interim stage.

$R = (R_i)_{i \in N}$ if $R_i \subseteq R_i'$ for every $i \in N$ (with at least one strict inequality), a more-informative communication network does not Pareto dominate, in general, a less-informative communication network. However, using Proposition 1 iteratively, we obtain that the complete communication network ($R_i = N \backslash \{i\}$ for all $i \in N$) Pareto dominates every other communication network.

3 Equilibrium Communication Networks

In this section we provide a full characterization and the general qualitative features of equilibrium communication networks, and derive a number of comparative-static results. We examine how a large number of receivers with strong coordination motives can discipline communication, which is one main novelty of the paper. In particular, we show that there may exist an equilibrium in which a sender reveals his information to a large group of recipients, whereas there is no equilibrium in which he does so to a strict subset of that group only. We also show that players who are more central in terms of preferences communicate more and have a greater impact on the decisions taken. These features are illustrated in two major configurations of preferences. When players' biases are uniformly distributed we show that an individual communicates more and to more distant individuals as the proximity of his preference to the average preference of the population increases. When preferences are the same within groups, but differ across groups, the impact of group size on communication again produces an interesting result: information transmission across groups is typically asymmetric, since players from the larger group communicate more easily to members of the smaller group than the reverse.

3.1 Full Characterization

Our main theorem provides a full characterization of the communication networks that arise as equilibrium outcomes of the cheap-talk stage of the game. In short, the theorem states that a player truthfully reveals his information to a group of players if his taste is not too different from the average taste of every subset of that group. More precisely, there exists an equilibrium network in which player i's set of receivers is $R_i \subseteq N \backslash \{i\}$ if and only if, for every subset of players in R_i, the average bias of the players in the subset is close enough to player i's bias.

Theorem 1 *There exists an equilibrium network in which player i's set of receivers is $R_i \subseteq N \backslash \{i\}$ iff for all $R_i' \subseteq R_i$ we have*

$$\left| b_i - \frac{\sum_{k \in R_i'} b_k}{|R_i'|} \right| \leq \frac{(n-1+\alpha)(n-1-\alpha|R_i'|)}{2(n-1)(n-1-\alpha|R_i|)} (\overline{s}_i - \underline{s}_i). \tag{4}$$

Proof. See Appendix A.3. ■

To understand the intuition of this characterization, observe that when a sender's bias is close to the average bias of the receivers, a lie about his type may move these receivers' actions too far from the sender's ideal point. On the contrary, if the distance between the sender's bias and the average bias of the receivers is substantial, then the sender has an incentive to over-report or under-report his type so that the receivers' actions become closer to his ideal action. Since private communication allows the sender to lie to any subset R_i' of the set R_i of receivers, informational incentive constraints require that the sender's bias be close enough to the average bias of every subset $R_i' \subseteq R_i$.

The exact proximity between the sender's and the receivers' biases required for truthful communication depends on the threshold given by the RHS of Equation (4). It is worth noting that this threshold depends on the numbers of receivers $|R_i|$ and $|R_i'|$. In existing models of cheap talk to multiple audiences who do not interact strategically in the decision stage (e.g., Farrell and Gibbons, 1989; Goltsman and Pavlov, 2009; Galeotti et al., 2009), the proximity of players' biases required for truthful communication depends only on the information structure and on some game parameters (here, $\overline{s}_i - \underline{s}_i$, α and n). Hence, with independent decision-makers, a *necessary* condition for truthful information transmission from a sender to a set of receivers is that, for every member of this set, there exists an equilibrium in which the sender transmits his information truthfully to this member only. In contrast, in our model, whether communication from a sender to a given receiver can be sustained in equilibrium depends on the *whole* set of players to whom the sender truthfully reveals his information. In particular, the incentive to communicate to a receiver not only depends on the conflict of interest between the sender and this receiver, but also on the number and the preferences of all the receivers to whom the sender sends a truthful message.

To understand why the RHS of Equation (4) depends on $|R_i|$ and $|R_i'|$, observe two differences between a cheap-talk model with independent decision-makers and ours. First, since the equilibrium actions of players in R_i depend on i's signal and others' actions, they react differently to i's signal depending on the total number of receivers $|R_i|$. Second, the sender also wants to coordinate his action with his receivers' actions. Hence, when he deviates and sends the wrong signal to a subset of receivers $R_i' \subseteq R_i$ (this deviation cannot be observed by any player different from player i himself), the coordination costs induced by that lie depend on the size of R_i'. Precisely, the threshold of the RHS of Equation (4) is decreasing in $|R_i'|$, meaning that it is less costly for the sender to lie to large subsets R_i' of R_i than to small ones.

As an illustration, and for future reference, consider a game with $n = 4$ players, $\alpha = 1/2$, and assume that every player has the same value of private information, given by $\overline{s}_i - \underline{s}_i = \frac{12\times 3}{7}$. The RHS of Equation (4) in Theorem 1 simplifies to $3\frac{6-|R_i'|}{6-|R_i|}$. It follows that player i reveals his type to all of the other players if for all $k, l \in N\backslash\{i\}$,

$$\left| b_i - \frac{\sum_{j\neq i} b_j}{3} \right| \leq 3, \quad \left| b_i - \frac{b_k + b_l}{2} \right| \leq 4, \quad \text{and} \quad |b_i - b_k| \leq 5. \tag{5}$$

Similarly, player i reveals his type only to players in $\{j, k\} \subsetneq N\backslash\{i\}$ if

$$\left| b_i - \frac{b_j + b_k}{2} \right| \leq 3, \quad \text{and} \quad |b_i - b_j|, |b_i - b_k| \leq 3.75. \tag{6}$$

Finally, player i reveals his type only to player $j \neq i$ if $|b_i - b_j| \leq 3$.[6] It appears clearly that the conditions ensuring that player i truthfully communicates with j depend on the whole set of receivers to which j may belong. Given a set of receivers, we can also see that the thresholds on

[6]For example, with the bias profile $b = (b_1, b_2, b_3, b_4) = (0, 3.8, 4.8, 9)$, $(R_i)_{i\in N} = (\emptyset, \{3\}, \{1, 2, 4\}, \emptyset)$ is the most informative equilibrium communication network.

the RHS decrease with the size of the subset considered.

The origin of one main insight of our work is given by the following observation: given $|R'_i|$, the RHS of Equation (4) is increasing in $|R_i|$. That is, the conditions given by Theorem 1 on the proximity between i's bias and the average bias of the strict subsets $R'_i \subsetneq R_i$ of receivers become weaker as the set of all receivers, R_i, is larger. The intuition is that, as we had already seen in the second-stage equilibrium characterization (Equation (2)), as $|R_i|$ increases, receivers are more responsive to whatever the sender is revealing to them. But the more responsive receivers are to a message by the sender, the less the latter has an incentive to over-report or under-report his information as it may affect the actions of the fixed set of receivers R'_i too much.

This feature implies that when the informational incentive constraints are satisfied for information transmission to a set of receivers, these constraints are not necessarily satisfied for information transmission to a strict subset of these receivers only. In particular, one key effect revealed by our model is that there may exist an equilibrium in which an individual reveals his true type to a group of players whereas there is no equilibrium in which he reveals it only to strict subsets of this group. As an example, consider the bias profile $b = (-4.1, 0, 3.8, 4.1)$ in the previous four-player game. There is then an equilibrium communication network in which player 2's set of receivers is $R_2 = \{1, 3, 4\}$, but there is no equilibrium communication network in which player 2's set of receivers is a strict subset of $\{1, 3, 4\}$.

The fact that communication to a large group of recipients may occur in equilibrium even though communication to a small subset of that group may not relies on the receivers' need for coordination. This feature, therefore called the *disciplinary effect of coordination*, will be illustrated for particular configurations of biases in Subsection 3.4. Note that this disciplinary effect of coordination differs from the disciplinary effect of public communication identified by Farrell and Gibbons (1989) and further analyzed recently by Goltsman and Pavlov (2009). Indeed, our effect does not rely on the public nature of a sender's message and appears even when communication is private.

To see how coordination motives make large equilibrium communication networks feasible when intermediate communication networks are not, it is instructive to look at extreme situations. When there is almost no coordination motive ($\alpha \to 0$), incentive constraints are as in a model without strategic interactions in the decision stage: the condition for R_i to be an equilibrium set of receivers for player i reduces to

$$|b_i - b_j| \leq \frac{\overline{s}_i - \underline{s}_i}{2}, \text{ for all } j \in R_i.$$

That is, there is an equilibrium in which player i truthfully reveals his information to the players in R_i if and only if, for every agent in R_i, there is an equilibrium in which player i truthfully communicates with that agent only. This is because when the need for coordination vanishes, the responsiveness of the receivers' actions to i's signal and the sender's coordination costs mentioned above no longer depend on the number of receivers.

Consider now the opposite situation in which coordination costs are extremely high ($\alpha \to 1$) and let player i transmit his information to all the other players ($R_i = N \backslash \{i\}$) so that the responsiveness of players' actions to player i's signal is maximal. In that case, the incentive compatibility of

Theorem 1 for player i reduces to

$$\left| b_i - \frac{\sum_{j \neq i} b_j}{n-1} \right| \leq \frac{n}{2(n-1)} (\overline{s}_i - \underline{s}_i).$$

That is, the incentive compatibility conditions ensuring that player i does not misrepresent his information to strict subsets of receivers are irrelevant. In particular, full revelation of information from player i to all the other players is possible whenever i's bias b_i is close enough to the average bias of the other players, $\frac{\sum_{j \neq i} b_j}{n-1}$, whatever the distribution of players' biases. The intuition is that, with extremely high coordination costs, player i never wants to lie about his type to only a subset of the other players, as, if he does so, his action cannot be perfectly coordinated with both the players to whom he lies and the players to whom he reveals his true type. This also means that as the weight of the coordination motive tends to one the conditions for full information revelation from any player are equivalent to the conditions for full information revelation were communication to be *public*. Indeed, if communication were public, informational incentive constraints would by definition be weaker than under private communication: the only possible deviation from a message sent publicly would be to jointly lie to the whole audience of players, while private communication enables the sender to lie to any subset of these players.

As shown in Theorem 1, each player's equilibrium communication strategy does *not* depend on other players' communication strategies. To obtain the intuition for this independence property, consider a strategy profile in which some player (say, player 1) reveals his type to players in R_1. This strategy is optimal if, whatever his type, player 1 has no incentive to lie to some or all players in R_1. Now consider the (unobservable) deviation that consists in player 1 lying to all players in R_1 (the intuition is exactly the same when he lies only to a subset of the players in R_1). Player 1's deviation affects player 1's expected utility by changing (i) the second-stage actions of players in R_1, who now act in believing player 1's wrong type instead of the true one and (ii) player 1's best reply to the latter actions. As can be seen from the (linear and additive) form of second-stage equilibrium actions given by Equation (2), the change in the actions of the players in R_1 is *independent* of what they learn about the types of players other than 1. This relies on our specific utility functions, on the additivity of the state and also on the independence of players' types.[7] Consequently, player 1's best reply to other players' actions and player 1's expected utility are affected by the change in the actions of players in R_1 independently of the information transmitted by other senders. In contrast, Austen-Smith (1993, Proposition 1) shows that the incentive for an expert to reveal truthfully his type to a decision-maker *depends* on the communication strategy of the other expert. While we assume that types are independent and the state is additive in types, in Austen-Smith (1993) players' signals are independent *conditional* on the state. Hence, in his model, the effect of the message of an expert on the beliefs and action of the decision-maker depends on how well informed the decision-maker is. More precisely, if the decision-maker is well informed, the expert's message affects his action only slightly, whereas if the decision-maker is poorly informed, the message will affect his action significantly. This implies that communication from a given expert

[7] As an extreme example consider the situation in which some player $j \neq 1$ is almost perfectly informed about player 1's type, i.e. s_1 and s_j are strongly correlated. Then, when players in R_1 know player j's signal, i.e. players in R_1 also belong to R_j, player 1's message only has a limited impact on players' actions (and thus, on player 1's expected payoff), whereas if player j does not reveal his type to players in R_1, then player 1's signal has a stronger impact on the actions of players in R_1.

to the decision-maker is more difficult when the other expert communicates with him.[8]

3.2 Comparative Statics

The next corollary, easily deduced from Theorem 1, details the effect of the disparity of players' preferences, the coordination motive, and the information structure on equilibrium sets of receivers.

Corollary 1 (Comparative Statics) *The equilibrium conditions for information transmission from any player i to any set of receivers become weaker as:*

(i) All biases are reduced by the same factor: $(b_1, \ldots, b_n) \mapsto r(b_1, \ldots, b_n)$, where $r \in [0, 1)$;

(ii) The weight on coordination motives, α, increases;

(iii) The value of private information, $\bar{s}_i - \underline{s}_i$, increases.

Proof. Reducing the absolute values of the biases as in (i) clearly decreases the LHS of Inequality (4). The RHS of Inequality (4) is also clearly increasing in $\bar{s}_i - \underline{s}_i$. Finally, it is increasing in α because $\frac{\partial}{\partial \alpha} \frac{(n-1-\alpha|R_i'|)}{(n-1-\alpha|R_i|)} = \frac{(n-1)(|R_i|-|R_i'|)}{(n-1-\alpha|R_i|)^2} \geq 0$. ■

The intuition for (i) is quite clear. As all the biases become more similar to each other, the conflict of interest between all players falls, so informational incentive constraints become weaker as in existing cheap-talk models.

Point (ii) is not as direct as (i) since, in our framework, it is the need for coordination that itself results in incentive conflicts between players. Indeed, were there no need for coordination ($\alpha = 0$), an equilibrium with perfect communication would always exist because a sender would be indifferent between revealing truthfully his information or not. When α is positive his message has an impact on his payoff through the modification of others' actions, and misrepresenting his type may be beneficial when his bias is significantly different from the receivers' biases. However, the higher is α, the more costly it is for the sender to coordinate his action to the actions of the receivers he lies to. The effect of the need for coordination on strategic information transmission is also analyzed in Alonso et al. (2008) who consider a two-division organization in which the decisions of the divisions are both responsive to local conditions and coordinated with each other.[9] Decision-makers' payoffs are similar to those we consider, but the conflict of interest regarding decisions is modeled differently. In Alonso et al. (2008), each division manager has an ideal action that depends on an idiosyncratic state, and maximizes a weighted sum of his own division's profit and that of the other division. Under decentralization, they also show that an increase in the need for coordination facilitates communication between the two divisions. On the contrary, under centralization, when a benevolent principal makes all decisions by relying on cheap-talk statements from the divisions, an increase in the need for coordination worsens communication.

Finally, the intuition of (iii) is standard. As we already observed, the value of player i's information, $\bar{s}_i - \underline{s}_i$, measures the impact of his message on the receivers' belief about the state. So, when

[8] A similar statistical structure is used in, e.g., Morgan and Stocken (2008) and Galeotti et al. (2009), where the willingness of a sender to communicate with a player also declines in the number of senders communicating with that player.

[9] See also Rantakari (2008).

$\bar{s}_i - \underline{s}_i$ is small, his influence on the receivers' actions is also small and his incentive to lie about his type is greater. In the extreme case in which $\bar{s}_i - \underline{s}_i$ tends to zero, the incentive constraints of player i would be similar to the condition for full revelation of information in a model with a continuum of types, as in Crawford and Sobel (1982), which is never satisfied except when players' preferences exactly coincide.

We can also note that, for a given sender and a given set of receivers, increasing the total number of players, n, strengthens the conditions on the proximity between the sender's and receivers' biases, since the RHS of Equation (4) is decreasing in n.[10] The intuition of this effect is similar to the intuition of (iii) in the previous corollary: as the total number of players rises, the influence of the actions of a fixed set of receivers is smaller, so the sender's incentive to misrepresent his type is greater. To account for variations in the size of the population, we could describe the state as the *average* of players' signals, instead of the sum (this is irrelevant when n is fixed, since we impose no restrictions on $\bar{s}_i$ and $\underline{s}_i$). Equivalently, we can replace each signal s_i of every player i by $\frac{s_i}{n}$. In this case, the RHS of Inequality (4) always tends to zero as n increases, whatever the set of receivers, so that information transmission becomes impossible between any pair of players who do not have the same preferences. This effect is similar to that observed by Morgan and Stocken (2008, Proposition 1) who show that truthful reporting is never an equilibrium for a sufficiently large sample of constituents.

3.3 General Properties

By taking a closer look at the way in which the informational incentive constraints given in Theorem 1 intersect, some general properties of the equilibrium sets of receivers can be established. First, we can always construct larger equilibrium sets of receivers by adding agents whose biases are closer to the sender's bias than to those of any of his receivers.

Corollary 2 *If there exists an equilibrium network in which player i's set of receivers is R_i, then there also exists an equilibrium network in which player i's set of receivers is $R_i \cup \{j\}$ for every player j whose bias is closer to i's bias than to those of any player in R_i, i.e.,*

$$|b_i - b_j| \leq |b_i - b_k|, \quad \forall\, k \in R_i. \tag{7}$$

Proof. See Appendix A.4. ■

In particular, the above corollary implies that there always exists an equilibrium communication network in which, for every player i, his set of receivers R_i includes all the players with the same bias b_i. Note that if condition (7) holds for only *some* players in R_i, but not for all of them, then the result may not hold. To see this, consider again the four-player example introduced in Subsection 3.1 with the bias profile $b = (0, 2.2, 3.2, 3.7)$. In this case, there is an equilibrium in which player 1's set of receivers is $R_1 = \{2, 4\}$ but no equilibrium in which it is $\{2, 3, 4\}$.

Second, it is obvious from Theorem 1 that the existence of an equilibrium network in which player i's set of receivers is $\{j\}$ implies the existence of an equilibrium in which his set of receivers

[10] The sign of the derivative with respect to n is $2\alpha(n-1)|R_i'| - \alpha^2|R_i||R_i'| - (n-1)^2(|R_i| + 1 - |R_i'|)$, which is always negative.

is $\{k\}$, for every k whose bias is between b_i and b_j. Applying the previous corollary, this yields:

Corollary 3 *If there exists an equilibrium network in which player i's set of receivers is $\{j\}$, $j > i$, then there also exists an equilibrium network in which player i's set of receivers is R_i for every $R_i \subseteq \{i+1, \dots, j\}$.*

Proof. Directly from Theorem 1 and Corollary 2. ■

Note that if players i and j have the same value of private information, i.e. $\bar{s}_i - \underline{s}_i = \bar{s}_j - \underline{s}_j$, and if there is an equilibrium network in which player i's set of receivers is $\{j\}$, then there is also an equilibrium network in which player j's set of receivers is $\{i\}$. Combined with the previous corollary, this reciprocity property implies that if all players in $\{i, \dots, j\}$ have the same value of private information, then any communication network among these players is an equilibrium network. In particular, full revelation of information between all of the players in $\{i, \dots, j\}$ is an equilibrium.

Under some conditions, larger communication networks can also be constructed by forming the union of existing equilibrium networks. As stated in the following corollary, combining two equilibrium sets of receivers R_i and $\tilde{R}_i$ for player i yields an equilibrium set of receivers $R_i \cup \tilde{R}_i$ for player i if R_i and $\tilde{R}_i$ do not overlap, i.e. $R_i \cap \tilde{R}_i = \emptyset$.

Corollary 4 *If there is an equilibrium network in which player i's set of receivers is R_i, and an equilibrium network in which player i's set of receivers is $\tilde{R}_i$, and if R_i and $\tilde{R}_i$ do not overlap, then there is also an equilibrium network in which player i's set of receivers is $R_i \cup \tilde{R}_i$.*

Proof. See Appendix A.4. ■

More generally, the proof of the previous corollary reveals that a sufficient condition for the result to hold is that the distance between i's bias and the average bias in $R_i \cup \tilde{R}_i$ is smaller than the distance between i's bias and the average bias in R_i or $\tilde{R}_i$. When this condition does not hold, the union of the two equilibrium receiver sets does not necessarily yield another equilibrium receiver set. To see this, consider once more our four-player example with the bias profile $b = (0, 2.2, 3.2, 3.7)$. Here, there is an equilibrium network in which player 1's set of receivers is $R_1 = \{2, 3\}$ and an equilibrium network in which it is $\tilde{R}_1 = \{2, 4\}$, but no equilibrium network in which player 1's set of receivers is $R_1 \cup \tilde{R}_1 = \{2, 3, 4\}$. This implies that, in general, there may not exist a "maximal" equilibrium communication network which is more informative than all the other communication networks.

Necessary and sufficient conditions for the complete, Pareto dominant, communication network to be an equilibrium are easily deduced from Theorem 1. More precisely:

Corollary 5 (Complete Network) *The complete communication network is an equilibrium network if and only if for all $i \in N$ and $R_i \subseteq N \backslash \{i\}$,*

$$\left| b_i - \frac{\sum_{j \in R_i} b_j}{|R_i|} \right| \leq \frac{(n-1+\alpha)(n-1-\alpha|R_i|)}{2(n-1)^2(1-\alpha)} (\bar{s}_i - \underline{s}_i). \tag{8}$$

Proof. Directly from Theorem 1. ■

Note that when all players have the same value of private information, the set of these conditions is reduced, since we only have to check that the incentive constraints are satisfied for the two extreme players (player 1 and player n). In that case, a simple sufficient condition for the complete network to be an equilibrium is that there exist an equilibrium network in which player 1's set of receivers is $R_1 = \{n\}$ and player n's set of receivers is $R_n = \{1\}$.

3.4 Illustrations

In this subsection, we analyze two particular configurations of biases and obtain further results regarding the structure of equilibrium communication networks. Our aim is also to illustrate the disciplinary effect of coordination identified after Theorem 1. First, we consider *uniformly-distributed biases*, assuming that players' biases are equidistant: $b_{i+1} - b_i = \beta \geq 0$ for every $i \in N$. Second, we consider *two-spike biases* situations in which the players are partitioned into two groups, $L = \{1, \ldots, l\}$ and $M = \{l+1, \ldots, n\}$. Players in the first group have a bias b_L and players in the second group have a bias b_M, with $b_M - b_L = \beta > 0$. For both configurations of biases we know from Corollary 1 (i) that in equilibrium the maximal number of receivers of every player falls with the distance β.

To focus on the impact of players' positions on their communication behavior, we assume from now on that they all have the same value of private information: $\bar{s}_i - \underline{s}_i = \Delta$ for all i. In addition, we restrict our attention to equilibrium communication networks R which are maximal (i.e., such that there exists no equilibrium communication network more informative than R) and such that, for every i, players in $\{i\} \cup R_i$ are consecutive.[11]

3.4.1 Uniformly-Distributed Biases

When biases are uniformly distributed, observe that, for any size $|R_i|$, the distance between i's bias and the average bias of the $|R_i|$ players who are the closest to i in the population falls with the proximity of i's bias to the average bias in the whole population. Hence, from the equilibrium characterization of Theorem 1 we have:

Corollary 6 *If players' biases are uniformly distributed, then the maximal number of equilibrium receivers increases with the proximity of a sender's bias to the average bias in the population.*

After stating Corollary 5, we noted that full revelation of information between all players is possible whenever the two extreme players reveal their information to all the other players. With uniformly-distributed biases, the previous corollary further asserts that middle-biased players (i.e. players whose biases are close to the average bias in the population) communicate more than extremists (i.e. players whose biases are far from the average bias in the population).

The impact of players' position on their communication behavior is even stronger than that stated in Corollary 6. If every player i, whatever his position, communicates to all players whose

[11]By consecutive, we mean that there is no player in $N \backslash (\{i\} \cup R_i)$ whose bias lies between the biases of any two players in $\{i\} \cup R_i$. From Theorem 1 it is easy to show that such an equilibrium always exists.

biases are less distant than some threshold $d > 0$, i.e., $R_i = \{k \neq i : |b_i - b_k| \leq d\}$, then it is clear that central players communicate more than extremists since $|\{k \neq j : |b_j - b_k| \leq d\}| \geq |\{k \neq i : |b_i - b_k| \leq d\}|$ whenever j is more central than i. The next corollary shows that not only the *number* of receivers increases with the sender's centrality, but so does the distance between the sender's bias and his receivers' biases. In other words, central players can truthfully communicate with agents with whom they have higher conflicts of interest than less central players can do.

Corollary 7 *If players' biases are uniformly distributed and $R_i = \{k \neq i : |b_i - b_k| \leq d\}$ is an equilibrium set of receivers for some player i and some distance $d \geq 0$, then $R_j = \{k \neq j : |b_j - b_k| \leq d\}$ is also an equilibrium set of receivers for every player j who is more central than i. In general, the reverse is not true.*

These results are illustrated by Figure 1 which plots the number of receivers $|R_i|$ as a function of the coordination motive α and player i's position, when $n = 7$, $\Delta = 2$ and $\beta = 0.6$. The figure shows how the number of receivers increases with players' centrality whatever the value of α, and with α whatever the players' position. When α is high enough (in this figure, when $\alpha \geq 0.3$) we also see that more central senders communicate to more distant receivers. For example, when $\alpha = 0.5$, the most central player (player 4) communicates to all the other players, while players 1 and 7 only communicate to a single player.

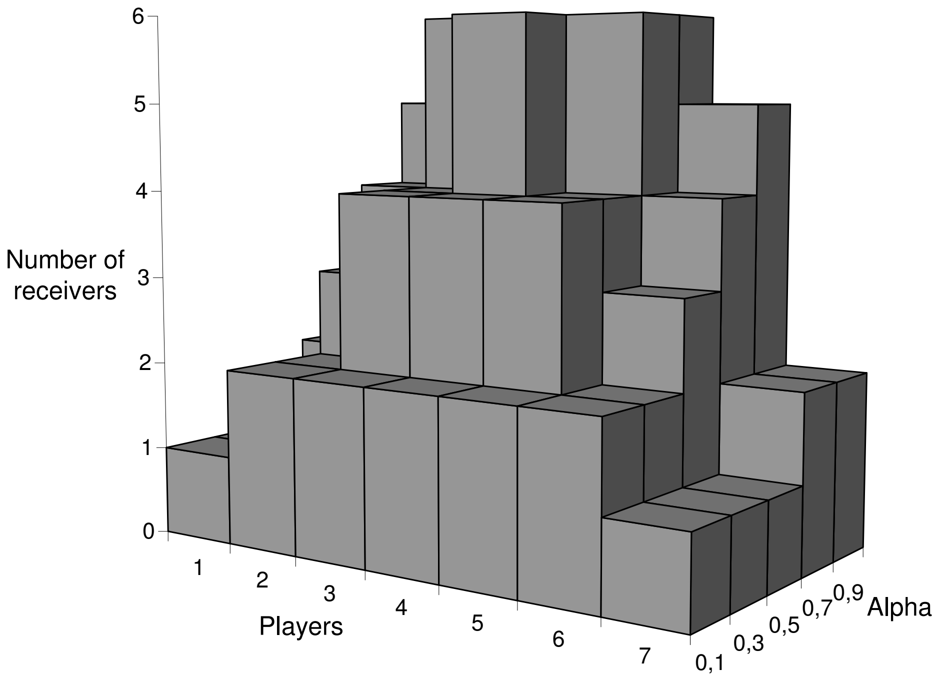

Figure 1: Number of equilibrium receivers with $n = 7$ players, uniformly-distributed biases, $\beta = 0.6$ and $\Delta = 2$.

To understand better the role of the coordination motive on the impact of players' position on their incentive to communicate, consider again extreme situations. When $\alpha \to 0$ we have already observed that the equilibrium condition for R_i to be a equilibrium set of receivers for player i

reduces to $\max_{j \in R_i} |b_i - b_j| \leq \frac{\Delta}{2}$ whatever player i's position. So, in that case, only the distance between a sender and one receiver matters for incentive compatibility, and the reverse of Corollary 7 is true. More generally, when $\alpha > 0$, the equilibrium condition for a single receiver, namely

$$\beta \leq \frac{n-1+\alpha}{2(n-1)}\Delta,$$

is the same for every sender, but will depend on the sender's position for more than two receivers. To see this, notice that incentive compatibility for the less central players (player 1 and player n) to communicate to $r_i \in \{1, \ldots, n-1\}$ receivers can be written as:

$$\beta \leq \frac{(n-1+\alpha)(n-1-\alpha r_i')}{(n-1)(2r_i - r_i' + 1)(n-1-\alpha r_i)}\Delta, \quad \forall\, r_i' \in \{1, \ldots, r_i\}.$$

It can be shown[12] that for every $\alpha \in (0,1)$ this incentive compatibility condition becomes strictly stronger as r_i increases. That is, it is always more difficult for extremists to communicate to a larger group than to a smaller group, even when the coordination motive is very strong. On the contrary, for strong enough coordination motives, incentive compatibility conditions for more central players are not necessarily monotonic in the number of receivers. That is, large receiver sets may be equilibrium outcomes for more central players, while small receiver sets are not. For example, when n is odd, the incentive compatibility condition for the central player $i = \frac{n+1}{2}$ is always strictly weaker for $r_i + 1$ receivers than for r_i receivers when $r_i \leq n-1$ is odd. As α tends to one the condition always holds for $r_i = n-1$ receivers, but continues to represent a constraint for $r_i < n-1$ receivers. In particular, a central player may reveal his information to all players, while an extreme player may transmit his information to none.

3.4.2 Two-Spike Biases

When players are partitioned into two groups, full revelation of information amongst all players in the same group is always an equilibrium. In addition, Corollary 2 implies that if a player $i \in L$ ($i \in M$, resp.) transmits his information to players in R_i in equilibrium, then there is also an equilibrium in which i reveals his information to players in $R_i \cup (L \backslash \{i\})$ ($R_i \cup (M \backslash \{i\})$, resp.). Hence, there is a unique maximal equilibrium network, such that the set of receivers of each player $i \in L$ includes all players in $L \backslash \{i\}$ and the set of receivers of each player $i \in M$ includes all players in $M \backslash \{i\}$.

Since it is always possible for players to communicate to players with the same bias, and since the RHS of the informational incentive constraint (4) is increasing in the total number of receivers, a player's informational incentive constraints are relaxed as the relative number of players with the same bias increases. More precisely:

Corollary 8 (Two-Spike Biases) *In the maximal equilibrium network with two-spike biases, a player's set of receivers increases, and includes more players from the other group, as the relative number of players in his own group increases.*

[12] A formal proof is available from the authors upon request.

Proof. Directly from Theorem 1 and the observation above. ■

In particular, this corollary implies that intergroup information transmission is higher for players in the larger group than for players in the smaller group. As a simple example, consider the situation in which $\alpha \to 1$. Then, there is complete communication from players in group L if and only if $\beta \leq \frac{n}{2(n-l)}\Delta$, which becomes easier to satisfy as the size of this group, l, increases.

This property does not extend to more than two groups of players. For example, in the four-player example, when $\alpha = 0.9$ and $\bar{s}_i - \underline{s}_i = 1$ for all i, the condition for player i to reveal his information to all players is

$$\left| b_i - \frac{\sum_{j \neq i} b_j}{3} \right| \leq 0.65, \quad \left| b_i - \frac{b_k + b_l}{2} \right| \leq 2.6, \quad \text{and} \quad |b_i - b_k| \leq 4.55. \tag{9}$$

Hence, when $b = (-3, 0, 0, 3)$ there is an equilibrium in which players with zero bias transmit their information to all of the other players, but the first inequality does not hold when $b = (0, 0, 0, 3)$. Actually, when there are more than two groups of players with the same bias, the corollary above only applies, in general, for players in the two extreme groups, i.e., the group with bias b_1 and that with bias b_n.

4 Conclusion

In this paper, we consider a class of economically-relevant coordination games in which information about a common state of nature is distributed among the players. Each of these players chooses an action by trading off the benefit of it being close to his own "ideal action", which depends both on the state and on an idiosyncratic bias, with that of being close to the other players' decisions. Before taking such actions, the players are offered the opportunity to communicate with each other in a decentralized and strategic manner. In this setting, our focus is on the way heterogeneity in preferences shapes strategic information transmission. We provide explicit conditions on the proximity of players' biases for information to be revealed by any sender to any group of receivers. Precisely, we show that an agent reveals his information to a group as long as this group is large enough and his ideal action is close enough to the average ideal action of every subset of agents in this group.

Similar coordination games with incomplete information have already been analyzed in the literature, but under the assumption that there is no conflict of interest between agents regarding the ideal state-contingent action profile (see, for example, Morris and Shin, 2002 and Angeletos and Pavan, 2007). When agents' goals are aligned, but there are physical or cost constraints on the number of communication links between agents, another literature has identified the most efficient communication structures; see, amongst others, Marschak and Radner (1972), Radner (1993), Jehiel (1999), Chwe (2000), Calvó-Armengol and Martí (2007, 2009), and Morris and Shin (2007). In these papers, efficient networks are characterized under *physical* communication constraints. On the contrary, our approach studies the equilibrium communication networks that arise under *strategic* communication constraints. To that extent, our work mainly borrows from the literature on strategic information transmission based on Crawford and Sobel (1982) but then proposes a framework in which every player is at the same time a sender and a receiver.

One key insight that stems from our characterization of equilibrium communication networks is that large networks may be easier to sustain in equilibrium that smaller ones. In other words, we show that the need for coordination of multiple interacting audiences can discipline communication, in the sense that truthful communication may be feasible in a large group but not in strict subsets of this group. A similar phenomenon is obtained in a team-theoretic framework in Dessein and Santos (2006), which also considers a coordination-adaptation situation with quadratic costs, but where communication is non-strategic.[13] Another main result is that agents who are more central in terms of preferences communicate more and have a greater impact on the decisions taken. Note that such a prediction contrasts with those of models of *costly* communication (e.g., Banerjee and Somanathan, 2001) where there is a tendency for extremists to express more voice.[14]

The way in which communication links have been constructed in the current analysis completely departs from usual non-cooperative network-formation games in a number of ways.[15] In typical games of this type, players' strategies mainly consist in listing desired contacts, given the exogenous costs and benefits of direct and indirect connections. In addition, since it is commonly admitted that much of the information required for economic decision-making is exchanged via networks of relationships, the value of these connections is often interpreted as being informational. However, whether agents have an effective interest in transmitting information once a link exists has not yet been investigated to the best of our knowledge. By way of contrast, we explicitly model agents' informational frameworks and derive the equilibrium links directly from the informativeness of agents' communication strategies. Given that the connection conveys truthful information, the benefits from linking are then endogenously determined by the way in which the information transmitted is used in the decision stage.

A Appendix

A.1 Second-Stage Equilibrium Characterization

We first characterize the unique equilibrium action profile under complete information. The utility function of player i (see Equation (1)) can be rewritten as (minus a constant):

$$a_i \left[2(1-\alpha)(\theta + b_i) + \frac{2\alpha}{n-1} \sum_{j \neq i} a_j - a_i \right] - \frac{\alpha}{n-1} \sum_{j \neq i} (a_j)^2. \tag{10}$$

The best response of each player i to a_{-i} is given by:

$$a_i(a_{-i}; \theta) = (1-\alpha)(\theta + b_i) + \frac{\alpha}{n-1} \sum_{j \neq i} a_j. \tag{11}$$

[13] In that model, a communication link requires "bundling of tasks", which is assumed to be costly. Bundling a few tasks together may reduce profits relative to stand-alone tasks (no communication links at all), while bundling a lot of them together may actually improve profitability.

[14] In their model, individuals only differ in terms of beliefs about a binary state of nature. Centrists are those who put relatively equal weight on the two states of the world, while extremists firmly believe in one or the other of the states.

[15] See Jackson (2008) for an extensive survey of such models.

If a_i is a best response to a_{-i}, then it follows from Equations (10) and (11) that player i's utility takes the following simple form (minus a constant):

$$u_i(a_i(a_{-i};\theta), a_{-i};\theta) = (a_i(a_{-i};\theta))^2 - \frac{\alpha}{n-1}\sum_{j\neq i}(a_j)^2. \tag{12}$$

The system of equations formed by Equation (11) can be written as:

$$\begin{pmatrix} a_1 \\ \vdots \\ \vdots \\ a_n \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & -\frac{\alpha}{(n-1)} & \cdots & -\frac{\alpha}{(n-1)} \\ -\frac{\alpha}{(n-1)} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -\frac{\alpha}{(n-1)} \\ -\frac{\alpha}{(n-1)} & \cdots & -\frac{\alpha}{(n-1)} & 1 \end{pmatrix}}_{I}^{-1} \begin{pmatrix} (1-\alpha)(\theta + b_1) \\ \vdots \\ \vdots \\ (1-\alpha)(\theta + b_n) \end{pmatrix}.$$

Simple algebra yields:

$$I^{-1} = \frac{1}{(n-1)-(n-2)\alpha-\alpha^2}\begin{pmatrix} (n-1)-(n-2)\alpha & \alpha & \cdots & \alpha \\ \alpha & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \alpha \\ \alpha & \cdots & \alpha & (n-1)-(n-2)\alpha \end{pmatrix}.$$

Therefore, when every player knows the state of nature, equilibrium actions are given by:

$$a_i(\theta) = \theta + \frac{[(n-1)-(n-2)\alpha]b_i + \alpha\sum_{j\neq i} b_j}{n+\alpha-1} \equiv \theta + B_i, \text{ for every } i \in N. \tag{13}$$

Since players' best responses are linear, exactly the same algebra shows that, under incomplete information, and whatever the information structure generated by the communication strategy profile, expected equilibrium actions are uniquely characterized by

$$E(a_i) = E(\theta) + B_i, \text{ for every } i \in N. \tag{14}$$

The uniqueness of the linear equilibrium identified in (2) is proved as in Calvó-Armengol and Martí (2009, Theorem 1). We define the following payoff function:

$$v(a_1, ..., a_n; s) = -(1-\alpha)\sum_{i\in N}(a_i - \theta(s) - b_i)^2 - \frac{\alpha}{2(n-1)}\sum_{i\in N}\sum_{j\neq i}(a_i - a_j)^2. \tag{15}$$

The set of equilibria of our second-stage coordination game is the same as that in the corresponding Bayesian game with identical agent preferences in which every player's payoff function is given by (15), as the best responses are identical in both games.

Theorem 4 of Marschak and Radner (1972)[167–168] provides a sufficient condition for the equilibrium of a Bayesian game with identical agent preferences to be determined uniquely by a system of linear equations when the set of states of the world is finite and payoff functions are given by:

$$\lambda(s) + 2\sum_{i\in N} \mu_i(s)a_i - \sum_{i,j\in N} v_{ij}(s)a_i a_j, \tag{16}$$

where the λ, μ_i and v_{ij} are all real-valued functions of the state of the world, $s \in S$. It is easily checked that the payoff function (15) can be written as (16), with

$$\begin{aligned}
\lambda(s) &= -(1-\alpha)\sum_{i\in N}(\theta(s)+b_i)^2 \ , \\
\mu_i(s) &= (1-\alpha)(\theta(s)+b_i) \ , \\
v_{ii}(s) &= v_{ii} = 1 \ , \\
v_{ij}(s) &= v_{ij} = -\frac{\alpha}{n-1} \ .
\end{aligned}$$

The sufficient condition in Theorem 4 of Marschak and Radner (1972) then boils down to the n-square matrix $[v_{ij}]_{i,j\in N}$ being positive definite. The determinant of $[v_{ij}]_{i,j\in N}$ is:

$$\begin{vmatrix} 1 & -\frac{\alpha}{n-1} & \cdots & -\frac{\alpha}{n-1} \\ -\frac{\alpha}{n-1} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -\frac{\alpha}{n-1} \\ -\frac{\alpha}{n-1} & \cdots & -\frac{\alpha}{n-1} & 1 \end{vmatrix} = (1-\alpha)\begin{vmatrix} 1 & -\frac{\alpha}{n-1} & \cdots & -\frac{\alpha}{n-1} \\ 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -\frac{\alpha}{n-1} \\ 1 & \cdots & -\frac{\alpha}{n-1} & 1 \end{vmatrix}$$

$$= (1-\alpha)\begin{vmatrix} 1 & -\frac{\alpha}{n-1} & \cdots & -\frac{\alpha}{n-1} \\ 0 & 1+\frac{\alpha}{n-1} & 0 & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1+\frac{\alpha}{n-1} \end{vmatrix} = (1-\alpha)(1+\frac{\alpha}{n-1})^{n-1}.$$

The first equality results from the replacement of the elements in the first column by the row sum, and then taking out the common factor $(1-\alpha)$. The second equality is obtained by subtracting the first row from every other row. We are left with an upper triangular matrix whose determinant is just the product of the diagonal term, which is positive. Similarly, we deduce that the leading principal minors of $[v_{ij}]_{i,j\in N}$ are positive. The matrix $[v_{ij}]_{i,j\in N}$ is thus positive definite.

Next, by explicitly solving some particular incomplete-information situations as above, it is possible to guess the general form of the unique second-stage equilibrium actions. To check that the solution given by Equation (2) is indeed the equilibrium when the communication strategy profile is characterized by $(R_i)_{i\in N}$, fix some player $l \in N$ and suppose that the second-stage equilibrium action of every player $i \neq l$ is given by Equation (2). We then show that player l's best response to this profile of second-stage actions $(a_i)_{i\neq l}$ is also of the form of Equation (2).

After the communication stage, for all $i \in N$, recall that $I_i = \{k : i \in R_k\} \cup \{i\}$ is the set of players whose signals are known by player i, $\overline{I}_i = \{k : i \notin R_k\}\backslash\{i\}$ the set of players whose signals are not known by player i, and let $E_i(\cdot) = E(\cdot \mid \{s_k : \ k \in I_i\})$ be player i's expectation operator conditional on the set of signals that he knows.

The expected payoff of player l after the communication stage is as follows:

$$-(1-\alpha)E_l\big[(a_l-\sum_{j\in N}s_j-b_l)^2\big]-\frac{\alpha}{n-1}\sum_{i\neq l}E_l\big[(a_l-a_i)^2\big], \tag{17}$$

so his best response is given by:

$$a_l=(1-\alpha)\left(\sum_{j\in I_l}s_j\ +\sum_{j\in\overline{I}_l}E(s_j)\ +b_l\right)+\frac{\alpha}{n-1}\sum_{i\neq l}E_l(a_i). \tag{18}$$

From now on and for every $i\in N$, we use the notation r_i for $|R_i|$. Using Equation (2) for $i\neq l$, player l's conditional expectation of player i's action is given by:

$$\begin{aligned}E_l(a_i) &= \sum_{j\in I_i}\frac{\alpha(n-1-r_j)E(s_j)}{n-1-\alpha r_j}+\sum_{j\in I_i\cap I_l}\frac{(1-\alpha)(n-1)s_j}{n-1-\alpha r_j}\\ &\quad+\sum_{j\in I_i\cap\overline{I}_l}\frac{(1-\alpha)(n-1)E(s_j)}{n-1-\alpha r_j}+\sum_{j\in\overline{I}_i}E(s_j)+B_i.\end{aligned}$$

Summing over all agents other than l, we can write:

$$\begin{aligned}&\sum_{i\neq l}E_l(a_i)=\\ &=\sum_{i\neq l}\sum_{j\in I_i\cap I_l}\frac{\alpha(n-1-r_j)E(s_j)}{n-1-\alpha r_j}+\sum_{i\neq l}\sum_{j\in I_i\cap\overline{I}_l}\frac{\alpha(n-1-r_j)E(s_j)}{n-1-\alpha r_j}+\sum_{i\neq l}\sum_{j\in I_i\cap I_l}\frac{(1-\alpha)(n-1)s_j}{n-1-\alpha r_j}\\ &+\sum_{i\neq l}\sum_{j\in I_i\cap\overline{I}_l}\frac{(1-\alpha)(n-1)E(s_j)}{n-1-\alpha r_j}+\sum_{i\neq l}\sum_{j\in\overline{I}_i\cap I_l}E(s_j)+\sum_{i\neq l}\sum_{j\in\overline{I}_i\cap\overline{I}_l}E(s_j)+\sum_{i\neq l}B_i.\end{aligned} \tag{19}$$

Every signal s_j known by player l is known by r_j players other than l and not known by $n-1-r_j$ players different from l; every signal s_j not known by player l is known by r_j+1 players other than l and not known by $n-2-r_j$ players other than l. This enables us to deduce that:

$$\begin{aligned}\sum_{i\neq l}E_l(a_i) &= \sum_{j\in I_l}r_j\,\frac{\alpha(n-1-r_j)E(s_j)}{n-1-\alpha r_j}+\sum_{j\in\overline{I}_l}(r_j+1)\,\frac{\alpha(n-1-r_j)E(s_j)}{n-1-\alpha r_j}\\ &\quad+\sum_{j\in I_l}r_j\,\frac{(1-\alpha)(n-1)s_j}{n-1-\alpha r_j}+\sum_{j\in\overline{I}_l}(r_j+1)\,\frac{(1-\alpha)(n-1)E(s_j)}{n-1-\alpha r_j}\\ &\quad+\sum_{j\in I_l}(n-1-r_j)\,E(s_j)+\sum_{j\in\overline{I}_l}(n-2-r_j)\,E(s_j)+\sum_{i\neq l}B_i.\\ &= \sum_{j\in I_l}\frac{r_j(1-\alpha)(n-1)s_j+(n-1)(n-1-r_j)E(s_j)}{n-1-\alpha r_j}\\ &\quad+\sum_{j\in\overline{I}_l}(n-1)\,E(s_j)+\sum_{i\neq l}B_i.\end{aligned} \tag{20}$$

In addition, we have:

$$\sum_{i\neq l} B_i = \frac{\alpha(n-1)b_l + (n-1)\sum_{i\neq l} b_i}{n+\alpha-1}. \tag{21}$$

Plugging (21) and (20) into (18) and simplifying, we obtain player l's optimal action, which takes exactly the same form as that in Equation (2).

A.2 Proof of Proposition 1

The ex ante equilibrium payoff of player $j \in N$ is given by:

$$\begin{aligned} U_j &= -(1-\alpha)Var(a_j - \sum_{i\in N} s_i - b_j) - (1-\alpha)[E(a_j - \sum_{i\in N} s_i - b_j)]^2 \\ &\quad -\frac{\alpha}{n-1}\sum_{m\neq j} Var(a_j - a_m) - \frac{\alpha}{n-1}\sum_{m\neq j}[E(a_j - a_m)]^2. \end{aligned}$$

It follows from (14) that $E(a_j) = \sum_{i\in N} E(s_i) + B_j$, so we have:

$$U_j = -(1-\alpha)Var(a_j - \sum_{i\in N} s_i) - \frac{\alpha}{n-1}\sum_{m\neq j} Var(a_j - a_m) - (1-\alpha)[B_j - b_j]^2 - \frac{\alpha}{n-1}\sum_{m\neq j}[B_j - B_m]^2.$$

We consider two communication networks $R = (R_k)_{k\in N}$ and $R' = (R'_k)_{k\in N}$ such that $R_i \neq R'_i$ and $R_k = R'_k$ for all $k \in N\backslash\{i\}$. That is, R and R' are identical except that player i has a different set of receivers in R'. Player i is fixed throughout the analysis. The ex ante equilibrium payoff of every player $j \in N$ with the communication network R (R', resp.) is denoted by U_j (U'_j, resp.). Given the communication network R (R', resp.), the second-stage equilibrium action of every player $j \in N$ is denoted by a_j (a'_j, resp.). For all $j \in N$, given a strategic communication network R (R', resp.), let $I_j = \{k : j \in R_k\} \cup \{j\}$ ($I'_j = \{k : j \in R'_k\} \cup \{j\}$, resp.) denote the set of players whose signals are known by player j, and $\overline{I}_j = \{k : j \notin R_k\}\backslash\{j\}$ ($\overline{I'}_j = \{k : j \notin R'_k\}\backslash\{j\}$, resp.) the set of players whose signals are not known by player j.

For every player $j \in N$, we have:

$$\begin{aligned} U_j - U'_j &= (1-\alpha)\left(Var(a'_j - \sum_{i\in N} s_i) - Var(a_j - \sum_{i\in N} s_i)\right) \\ &\quad + \frac{\alpha}{n-1}\left(\sum_{m\neq j} Var(a'_j - a'_m) - \sum_{m\neq j} Var(a_j - a_m)\right). \end{aligned} \tag{22}$$

The second-stage equilibrium action a_j given by (2) enables us to write:

$$Var(a_j - \sum_{i\in N} s_i) = Var\left(\sum_{l\in I_j}\frac{\alpha(n-1-r_l)[E(s_l) - s_l]}{n-1-\alpha r_l} + \sum_{l\in\overline{I}_j}[E(s_l) - s_l] + B_j\right).$$

The independence of signals yields:

$$
\begin{aligned}
&Var(a_j - \sum_{i \in N} s_i) \\
&= \sum_{l \in I_j} Var\left(\frac{\alpha(n-1-r_l)s_l}{n-1-\alpha r_l}\right) + \sum_{l \in \overline{I}_j} Var(s_l) = \sum_{l \in I_j} \left(\frac{\alpha(n-1-r_l)}{n-1-\alpha r_l}\right)^2 Var(s_l) + \sum_{l \in \overline{I}_j} Var(s_l) \\
&= \sum_{l \in I_j \backslash \{i\}} \left(\frac{\alpha(n-1-r_l)}{n-1-\alpha r_l}\right)^2 Var(s_l) + \sum_{l \in \overline{I}_j \backslash \{i\}} Var(s_l) + \ \mathbf{1}[i \in I_j] \ \left(\frac{\alpha(n-1-r_i)}{n-1-\alpha r_i}\right)^2 \ Var(s_i) \\
&\quad + \ \mathbf{1}[i \in \overline{I}_j] \ Var(s_i),
\end{aligned}
$$

where $\mathbf{1}[i \in I_j]$ is an indicator function that equals 1 when player j knows the signal s_i, and $\mathbf{1}[i \in \overline{I}_j]$ is an indicator function that equals 1 when player j does not know the signal s_i. A similar equation holds for $Var(a'_j - \sum_{i \in N} s_i)$, when the communication network is R'.

The two communication networks R and R' that we consider are such that $I_j \backslash \{i\} = I'_j \backslash \{i\}$ and $\overline{I}_j \backslash \{i\} = \overline{I'}_j \backslash \{i\}$, so that for all $j \in N$ we have:

$$
\begin{aligned}
&Var(a'_j - \sum_{i \in N} s_i) - Var(a_j - \sum_{i \in N} s_i) = Var(s_i) \\
&\left[\mathbf{1}[i \in I'_j] \ \left(\frac{\alpha(n-1-r'_i)}{n-1-\alpha r'_i}\right)^2 + \ \mathbf{1}[i \in \overline{I'}_j] - \mathbf{1}[i \in I_j] \ \left(\frac{\alpha(n-1-r_i)}{n-1-\alpha r_i}\right)^2 - \mathbf{1}[i \in \overline{I}_j]\right].
\end{aligned}
\tag{23}
$$

When the communication network is R, for all $j \in N$ and $m \neq j$, we have, from (2):

$$
\begin{aligned}
Var(a_j - a_m) &= \sum_{l \in I_j \cap \overline{I}_m} \left(\frac{(1-\alpha)(n-1)}{n-1-\alpha r_l}\right)^2 Var(s_l) + \sum_{l \in \overline{I}_j \cap I_m} \left(\frac{(1-\alpha)(n-1)}{n-1-\alpha r_l}\right)^2 Var(s_l) \\
&= \sum_{l \in (I_j \cap \overline{I}_m) \backslash \{i\}} \left(\frac{(1-\alpha)(n-1)}{n-1-\alpha r_l}\right)^2 Var(s_l) + \sum_{l \in (\overline{I}_j \cap I_m) \backslash \{i\}} \left(\frac{(1-\alpha)(n-1)}{n-1-\alpha r_l}\right)^2 Var(s_l) \\
&\quad + \left(\frac{(1-\alpha)(n-1)}{n-1-\alpha r_i}\right)^2 \ \left[\mathbf{1}[i \in I_j \cap \overline{I}_m] + \mathbf{1}[i \in \overline{I}_j \cap I_m]\right] Var(s_i).
\end{aligned}
$$

A similar equation holds for $Var(a'_j - a'_m)$ when the communication network is R'.

The two communication networks R and R' are such that $(I_j \cap \overline{I}_m) \backslash \{i\} = (I'_j \cap \overline{I'}_m) \backslash \{i\}$ and $(\overline{I}_j \cap I_m) \backslash \{i\} = (\overline{I'}_j \cap I'_m) \backslash \{i\}$, so for all $j \in N$ and $m \neq j$ we have:

$$
\begin{aligned}
&Var(a'_j - a'_m) - Var(a_j - a_m) \\
&= ((1-\alpha)(n-1))^2 \left[\frac{\mathbf{1}[i \in I'_j \cap \overline{I'}_m] + \mathbf{1}[i \in \overline{I'}_j \cap I'_m]}{(n-1-\alpha r'_i)^2} - \frac{\mathbf{1}[i \in I_j \cap \overline{I}_m] + \mathbf{1}[i \in \overline{I}_j \cap I_m]}{(n-1-\alpha r_i)^2}\right] Var(s_i).
\end{aligned}
\tag{24}
$$

Plugging (23) and (24) into (22), we obtain: $U_j - U'_j =$

$$(1-\alpha)\left[\mathbf{1}[i \in I'_j]\ \left(\frac{\alpha(n-1-r'_i)}{n-1-\alpha r'_i}\right)^2 + \ \mathbf{1}[i \in \overline{I'}_j] - \mathbf{1}[i \in I_j]\ \left(\frac{\alpha(n-1-r_i)}{n-1-\alpha r_i}\right)^2 - \mathbf{1}[i \in \overline{I}_j]\right.$$
$$\left. + \ \alpha(1-\alpha)(n-1)\sum_{m \neq j}\left(\frac{\mathbf{1}[i \in I'_j \cap \overline{I'}_m] + \mathbf{1}[i \in \overline{I'}_j \cap I'_m]}{(n-1-\alpha r'_i)^2} - \frac{\mathbf{1}[i \in I_j \cap \overline{I}_m] + \mathbf{1}[i \in \overline{I}_j \cap I_m]}{(n-1-\alpha r_i)^2}\right)\right] Var(s_i). \tag{25}$$

We next focus on the particular case in which $|R_i| < |R'_i|$ and let $L = R_i \cap R'_i$ be the set of agents who belong both to R_i and R'_i. The set L is fixed throughout the analysis, and $|L| = l$. Let $|R_i| = r_i$, $|R'_i| = r'_i$, $|R_i \backslash l| = r_i - l$ and $|R'_i \backslash l| = r'_i - l$. To evaluate the sign of $U_j - U'_j$, in order to establish who is better off and who is worse off under the networks R and R', the set N of players is divided into the following five types:

- *(i)* Players who belong to R'_i and also to R_i. For every such player $j \in L$, we have $i \in I_j$ and $i \in I'_j$.
- *(ii)* Players who belong to R'_i but not to R_i. For every such player $j \in R'_i \backslash L$, we have $i \in \overline{I}_j$ and $i \in I'_j$.
- *(iii)* Players other than player i who belong neither to R'_i nor to R_i. For every such player $j \in N \backslash (R_i \cup R'_i \cup \{i\})$, we have $i \in \overline{I}_j$ and $i \in \overline{I'}_j$.
- *(iv)* Players who do not belong to R'_i but belong to R_i. For every such player $j \in R_i \backslash L$, we have $i \in I_j$ and $i \in \overline{I'}_j$.
- *(v)* Player i, for whom we have $i \in I_i$ and $i \in I'_i$.

(i) For every player $j \in L$, the set of players other than j can be divided into four disjoint sets of players: $\{i\} \cup L \backslash \{j\}$, $N \backslash (R_i \cup R'_i \cup \{i\})$, $R'_i \backslash L$ and $R_i \backslash L$. We have:

- for every player $m \in \{i\} \cup L \backslash \{j\}$, $i \in I_m$ and $i \in I'_m$;
- for every player $m \in N \backslash (R_i \cup R'_i \cup \{i\})$, $i \in \overline{I}_m$ and $i \in \overline{I'}_m$, and we have $|N \backslash (R_i \cup R'_i \cup \{i\})| = n - 1 - r_i - r'_i + l$;
- for every player $m \in R'_i \backslash L$, $i \in \overline{I}_m$ but $i \in I'_m$;
- for every player $m \in R_i \backslash L$, $i \in I_m$ but $i \in \overline{I'}_m$.

Since $i \in I_j$ and $i \in I'_j$, Equation (25) simplifies to:

$$U_j - U'_j \quad = \quad \alpha(1-\alpha)\left(\frac{n-1-r'_i}{n-1-\alpha r'_i} - \frac{n-1-r_i}{n-1-\alpha r_i}\right) Var(s_i). \tag{26}$$

Using $r'_i > r_i$, we obtain $U_j - U'_j = -\left(\frac{\alpha(1-\alpha)^2(n-1)(r'_i - r_i)}{(n-1-\alpha r'_i)(n-1-\alpha r_i)}\right) Var(s_i) < 0$. Hence, for all $j \in L$, we have $U_j < U'_j$.

(ii) For every player $j \in R'_i \backslash L$, the set of players other than j can be divided into four disjoint sets of players: $\{i\} \cup L$, $N \backslash (R_i \cup R'_i \cup \{i\})$, $R'_i \backslash (L \cup \{j\})$ and $R_i \backslash L$. We have:

- for every player $m \in \{i\} \cup L$, $i \in I_m$ and $i \in I'_m$, and we have $|\{i\} \cup L| = l + 1$;
- for every player $m \in N \backslash (R_i \cup R'_i \cup \{i\})$, $i \in \overline{I}_m$ and $i \in \overline{I'}_m$, and we have $|N \backslash (R_i \cup R'_i \cup \{i\})| = n - 1 - r_i - r'_i + l$;
- for every player $m \in R'_i \backslash (L \cup \{j\})$, $i \in \overline{I}_m$ but $i \in I'_m$;
- for every player $m \in R_i \backslash L$, $i \in I_m$ but $i \in \overline{I'}_m$.

Since $i \in \overline{I}_j$ and $i \in I'_j$, Equation (25) simplifies to:

$$U_j - U'_j \quad = \quad -(1-\alpha)^2(n-1)\left(\frac{1}{n-1-\alpha r'_i} + \frac{\alpha\,(r_i+1)}{(n-1-\alpha r_i)^2}\right)Var(s_i) < 0. \tag{27}$$

Hence, for all players $j \in R'_i \backslash L$, we have $U_j < U'_j$.

(iii) For every player $j \in N \backslash (R_i \cup R'_i \cup \{i\})$, the set of players other than j can be divided into four disjoint sets of players: $\{i\} \cup L$, $N \backslash (R_i \cup R'_i \cup \{i, j\})$, $R'_i \backslash L$ and $R_i \backslash L$. We have:

- for every player $m \in \{i\} \cup L$, $i \in I_m$ and $i \in I'_m$, and we have $|\{i\} \cup L| = l + 1$;
- for every player $m \in N \backslash (R_i \cup R'_i \cup \{i, j\})$, $i \in \overline{I}_m$ and $i \in \overline{I'}_m$;
- for every player $m \in R'_i \backslash L$, $i \in \overline{I}_m$ but $i \in I'_m$;
- for every player $m \in R_i \backslash L$, $i \in I_m$ but $i \in \overline{I'}_m$.

Since $i \in \overline{I}_j$ and $i \in \overline{I'}_j$, Equation (25) simplifies to:

$$U_j - U'_j \quad = \quad \alpha(1-\alpha)^2(n-1)\left(\frac{r'_i+1}{(n-1-\alpha r'_i)^2} - \frac{r_i+1}{(n-1-\alpha r_i)^2}\right)Var(s_i). \tag{28}$$

Using $r'_i > r_i$, we have $\left[\frac{r'_i+1}{(n-1-\alpha r'_i)^2} - \frac{r_i+1}{(n-1-\alpha r_i)^2}\right] > 0$. Hence, for all $N \backslash (R_i \cup R'_i \cup \{i\})$, we have $U_j > U'_j$.

(iv) For every player $j \in R_i \backslash L$, the set of players other than j can be divided into four disjoint sets of players: $\{i\} \cup L$, $N \backslash (R_i \cup R'_i \cup \{i\})$, $R'_i \backslash L$ and $R_i \backslash (L \cup \{j\})$. We have:

- for every player $m \in \{i\} \cup L$, $i \in I_m$ and $i \in I'_m$, and we have $|\{i\} \cup L| = l + 1$;
- for every player $m \in N \backslash (R_i \cup R'_i \cup \{i\})$, $i \in \overline{I}_m$ and $i \in \overline{I'}_m$, and we have $|N \backslash (R_i \cup R'_i \cup \{i\})| = n - 1 - r_i - r'_i + l$;
- for every player $m \in R'_i \backslash L$, $i \in \overline{I}_m$ but $i \in I'_m$;
- for every player $m \in R_i \backslash (L \cup \{j\})$, $i \in I_m$ but $i \in \overline{I'}_m$.

Since $i \in I_j$ and $i \in \overline{I'}_j$, Equation (25) simplifies to:

$$U_j - U'_j \quad = \quad (1-\alpha)^2(n-1)\left(\frac{1}{n-1-\alpha r_i} + \frac{\alpha\,(r'_i+1)}{(n-1-\alpha r'_i)^2}\right) Var(s_i) > 0. \tag{29}$$

Hence, for every player $j \in R_i \backslash L$, we have $U_j > U'_j$.

(v) The set of players other than i can be divided into four disjoint sets of players: L, $N\backslash(R_i \cup R'_i \cup \{i\})$, $R'_i\backslash L$ and $R_i\backslash L$. We have:

- for every player $m \in L$, $i \in I_m$ and $i \in I'_m$;
- for every player $m \in N\backslash(R_i \cup R'_i \cup \{i\})$, $i \in \overline{I}_m$ and $i \in \overline{I'}_m$;
- for every player $m \in R'_i\backslash L$, $i \in \overline{I}_m$ but $i \in I'_m$
- for every player $m \in R_i\backslash L$, $i \in I_m$ but $i \in \overline{I'}_m$.

Since $i \in I_i$ and $i \in I'_i$, Equation (25) yields exactly the same difference as Equation (26). Hence, for player i such that $r_i < r'_i$, we have $U_i < U'_i$. This completes the proof of Proposition 1.

A.3 Proof of Theorem 1

Consider an equilibrium in which each player i reveals his type to players in $R_i \subseteq N\backslash\{i\}$. Without loss of generality, assume that each player i sends to every player $j \in R_i$ the message $m_i^j = \overline{m}$ when his type is $\overline{s}_i$ and the message $m_i^j = \underline{m}$ when his type is $\underline{s}_i$, and sends the same message whatever his type to players outside R_i. Given $(R_i)_{i\in N}$, the second-stage equilibrium actions are given by (2).

Without loss of generality, we look for the conditions under which player 1 does not deviate from his equilibrium communication strategy described above. First, assume that player 1's true type is $s_1 = \overline{s}_1$. In equilibrium, using Equation (2), the second-stage action of every player $i \in R_1 \cup \{1\}$ is given by

$$\begin{aligned}\overline{a}_i \quad = \quad & \sum_{j\in I_i\backslash\{1\}} \frac{\alpha(n-1-r_j)E(s_j) + (1-\alpha)(n-1)s_j}{n-1-\alpha r_j} + \sum_{j\in\overline{I}_i} E(s_j) + B_i \\ & + \frac{\alpha(n-1-r_1)E(s_1) + (1-\alpha)(n-1)\overline{s}_1}{n-1-\alpha r_1},\end{aligned} \tag{30}$$

and the second-stage action of every player $i \notin R_1 \cup \{1\}$ is given by

$$a_i \quad = \quad \sum_{j\in I_i} \frac{\alpha(n-1-r_j)E(s_j) + (1-\alpha)(n-1)s_j}{n-1-\alpha r_j} + \sum_{j\in\overline{I}_i\backslash\{1\}} E(s_j) + B_i + E(s_1). \tag{31}$$

The relevant deviations for player 1 in the communication stage consist in lying to a subset of players $M \subseteq R_1$, i.e. sending message $\underline{m}$ instead of $\overline{m}$ to players in M (and not deviating towards the other players).[16] Let $m = |M|$, and denote by $(a'_i)_{i\in N}$ the profile of players' actions after this

[16]In equilibrium, any message off of the equilibrium path is interpreted as exactly $\underline{m}$ or $\overline{m}$.

deviation. Every player $i \in M$ chooses action $a_i' = \underline{a}_i$, which is given by replacing $\overline{s}_1$ by $\underline{s}_1$ in (30). The action a_i' of every player $i \in N \backslash (M \cup \{1\})$ is the same as that in the original equilibrium. Player 1's optimal action in the second stage is obtained from the best response of Equation (18) to $(a_i')_{i \neq 1}$, and takes the following form:

$$a_1' = (1-\alpha)\left(\sum_{j \in I_1 \backslash \{1\}} s_j \ + \overline{s}_1 \ + \sum_{j \in \overline{I}_1} E(s_j) \ + b_1 \right) + \frac{\alpha}{n-1} \sum_{i \neq 1} E_1(a_i'). \tag{32}$$

Using the same reasoning as that used to obtain expression (20), we have:

$$\begin{aligned} \sum_{i \neq 1} E_1(a_i') \quad = \quad & \sum_{j \in I_1} r_j \ \frac{\alpha(n-1-r_j)E(s_j)}{n-1-\alpha r_j} + \sum_{j \in \overline{I}_1} (r_j+1) \ \frac{\alpha(n-1-r_j)E(s_j)}{n-1-\alpha r_j} \\ & + \sum_{j \in I_1 \backslash \{1\}} r_j \ \frac{(1-\alpha)(n-1)s_j}{n-1-\alpha r_j} + \frac{m(1-\alpha)(n-1)\underline{s}_1}{n-1-\alpha r_1} \\ & + \frac{(r_1-m)(1-\alpha)(n-1)\overline{s}_1}{n-1-\alpha r_1} + \sum_{j \in \overline{I}_1} (r_j+1) \ \frac{(1-\alpha)(n-1)E(s_j)}{n-1-\alpha r_j} \\ & + \sum_{j \in I_1} (n-1-r_j)E(s_j) + \sum_{j \in \overline{I}_1} (n-2-r_j) \ E(s_j) + \sum_{i \neq 1} B_i. \end{aligned} \tag{33}$$

Plugging (33) into (32), using (21) and simplifying, we obtain:

$$\begin{aligned} a_1' \quad = \quad & \sum_{j \in I_1 \backslash \{1\}} \frac{\alpha(n-1-r_j)E(s_j) + (1-\alpha)(n-1)s_j}{n-1-\alpha r_j} + \sum_{j \in \overline{I}_1} E(s_j) \\ & + \frac{\alpha m(1-\alpha)\underline{s}_1 + (n-1-\alpha m)(1-\alpha)\overline{s}_1 + \alpha(n-1-r_1)E(s_1)}{n-1-\alpha r_1} + B_1. \end{aligned} \tag{34}$$

We denote by V_1 the expected payoff of player 1 conditional on signal s_1 under the original equilibrium, and by V_1' his expected payoff conditional on signal s_1 when he deviates by lying to players in M (and thus plays action a_1' in the second-stage game). Player 1 does not deviate by lying to players in M if $V_1' - V_1 \leq 0$. We have:

$$\begin{aligned} & V_1' - V_1 = (1-\alpha)E\big[(\overline{a}_1 - \sum_{i \in N} s_i - b_1)^2 - (a_1' - \sum_{i \in N} s_i - b_1)^2 \mid s_1\big] \\ & + \frac{\alpha}{n-1}\left(\sum_{i \in M} E\big[(\overline{a}_1 - \overline{a}_i)^2 - (a_1' - \underline{a}_i)^2 \mid s_1\big] \right. \\ & \left. + \sum_{i \in R_1 \backslash M} E\big[(\overline{a}_1 - \overline{a}_i)^2 - (a_1' - \overline{a}_i)^2 \mid s_1\big] + \sum_{i \in N \backslash (R_1 \cup \{1\})} E\big[(\overline{a}_1 - a_i)^2 - (a_1' - a_i)^2 \mid s_1\big] \right). \end{aligned}$$

For the sake of simplicity, we examine separately the elements of the difference $V_1' - V_1$ and use the following notation for $i \neq 1$:

$$z_i = \sum_{j \in (I_1 \cap \overline{I}_i) \setminus \{1\}} \frac{(1-\alpha)(n-1)(s_j - E(s_j))}{n-1-\alpha r_j} + \sum_{j \in (\overline{I}_1 \cap I_i) \setminus \{1\}} \frac{(1-\alpha)(n-1)(E(s_j) - s_j)}{n-1-\alpha r_j} + B_1 - B_i.$$

Using (30), (31) and (34) and the fact that $E[z_i \mid s_1] = B_1 - B_i$:

$$\sum_{i \in M} E\left[(\overline{a}_1 - \overline{a}_i)^2 - (a_1' - \underline{a}_i)^2 \mid s_1\right] = \sum_{i \in M} E\left[z_i^2 - \left(z_i + \frac{(1-\alpha)(n-1-\alpha m)(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1} \right)^2 \middle| s_1 \right]$$
$$= -2\left(\frac{(1-\alpha)(n-1-\alpha m)(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1}\right) \sum_{i \in M} (B_1 - B_i) - m \left(\frac{(1-\alpha)(n-1-\alpha m)(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1}\right)^2 . \quad (35)$$

$$\sum_{i \in R_1 \setminus M} E\left[(\overline{a}_1 - \overline{a}_i)^2 - (a_1' - \overline{a}_i)^2 \mid s_1\right] = \sum_{i \in R_1 \setminus M} E\left[z_i^2 - \left(z_i - \frac{(1-\alpha)\alpha m(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1} \right)^2 \middle| s_1 \right]$$
$$= 2\left(\frac{(1-\alpha)\alpha m(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1}\right) \sum_{i \in R_1 \setminus M} (B_1 - B_i) - (r_1 - m) \left(\frac{(1-\alpha)\alpha m(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1}\right)^2 . \quad (36)$$

$$\sum_{i \in N \setminus (R_1 \cup \{1\})} E\left[(\overline{a}_1 - a_i)^2 - (a_1' - a_i)^2 \mid s_1\right] = \sum_{i \in N \setminus (R_1 \cup \{1\})} E\left[\left(z_i + \frac{(1-\alpha)(n-1)(\overline{s}_1 - E(s_1))}{n-1-\alpha r_1} \right)^2 \right.$$
$$\left. - \left(z_i + \frac{(1-\alpha)\alpha m \underline{s}_1 + (1-\alpha)(n-1-\alpha m)\overline{s}_1 - (1-\alpha)(n-1)E(s_1)}{n-1-\alpha r_1} \right)^2 \middle| s_1 \right]$$
$$= 2\left(\frac{(1-\alpha)\alpha m(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1}\right) \sum_{i \in N \setminus (R_1 \cup \{1\})} (B_1 - B_i) + (n-1-r_1) \left(\frac{(1-\alpha)(n-1)(\overline{s}_1 - E(s_1))}{n-1-\alpha r_1}\right)^2$$
$$- (n-1-r_1)\left(\frac{(1-\alpha)\alpha m \underline{s}_1 + (1-\alpha)(n-1-\alpha m)\overline{s}_1 - (1-\alpha)(n-1)E(s_1)}{n-1-\alpha r_1}\right)^2 . \quad (37)$$

In addition, using

$$\bar{a}_1 - a_1' = \frac{(1-\alpha)\alpha m(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1},$$

and

$$\bar{a}_1^2 - {a_1'}^2 = \left(\frac{\alpha(n-r_1-1)E(s_1) + (1-\alpha)(n-1)\overline{s}_1}{n-1-\alpha r_l}\right)^2$$
$$- \left(\frac{\alpha m(1-\alpha)\underline{s}_1 + (n-1-\alpha m)(1-\alpha)\overline{s}_1 + \alpha(n-r_1-1)E(s_1)}{n-1-\alpha r_1}\right)^2$$
$$+ 2\left(\sum_{j \in I_1 \setminus \{1\}} \frac{\alpha(n-r_j-1)E(s_j) + (1-\alpha)(n-1)s_j}{n-1-\alpha r_j} + \sum_{j \in \overline{I}_1} E(s_j) + B_1\right)\left(\frac{(1-\alpha)\alpha m(\overline{s}_1 - \underline{s}_1)}{n-1-r_1}\right),$$

we obtain:

$$
\begin{aligned}
&E\left[(\bar{a}_1-\sum_{i\in N}s_i-b_1)^2-(a_1'-\sum_{i\in N}s_i-b_1)^2\middle|\, s_1\right]\\
&=E\left[\bar{a}_1^2-{a_1'}^2\middle|\, s_1\right]-2E\left[(\bar{a}_1-a_1')(\sum_{i\in N\setminus\{1\}}s_i+s_1+b_1)\middle|\, s_1\right]\\
&=\left(\frac{\alpha(n-r_1-1)E(s_1)+(1-\alpha)(n-1)\overline{s}_1}{n-1-\alpha r_1}\right)^2+2\left(B_1-b_1-\overline{s}_1\right)\left(\frac{(1-\alpha)\alpha m(\overline{s}_1-\underline{s}_1)}{n-1-\alpha r_1}\right)\\
&-\left(\frac{\alpha m(1-\alpha)\underline{s}_1+(n-1-\alpha m)(1-\alpha)\overline{s}_1+\alpha(n-r_1-1)E(s_1)}{n-1-\alpha r_1}\right)^2. \qquad (38)
\end{aligned}
$$

Next, we plug (35), (36), (37) and (38) into $V_1'-V_1$ and simplify. To simplify the part of the difference $V_1'-V_1$ that deals with biases, note that:

$$
B_1-B_i=\frac{(1-\alpha)(n-1)(b_1-b_i)}{n+\alpha-1}\quad\text{and}\quad B_1-b_1=\frac{-\alpha(n-1)b_1+\alpha\sum_{j\neq 1}b_j}{n+\alpha-1}.
$$

Finally, simple but tedious calculus yields:

$$
V_1'-V_1=\frac{2\alpha(1-\alpha)^2(n-1)(\overline{s}_1-\underline{s}_1)}{(n+\alpha-1)(n-1-\alpha r_1)}\left(\sum_{i\in M}b_i-mb_1\right)-\frac{\alpha(1-\alpha)^2m(n-1-\alpha m)(\overline{s}_1-\underline{s}_1)^2}{(n-1-\alpha r_1)^2}.
$$

Hence, player 1 of type $s_1=\overline{s}_1$ does not deviate by lying to players in $M\subseteq R_1$ if $V_1'-V_1\leq 0$, i.e.:

$$
-\left(b_1-\frac{\sum_{i\in M}b_i}{m}\right)\leq\frac{(n-1+\alpha)(n-1-\alpha m)}{2(n-1)(n-1-\alpha r_1)}(\overline{s}_1-\underline{s}_1). \qquad (39)
$$

Applying the same reasoning, player 1 of type $s_1=\underline{s}_1$ has no profitable deviation if, for all $M\subseteq R_1$, the following condition holds:

$$
b_1-\frac{\sum_{i\in M}b_i}{m}\leq\frac{(n-1+\alpha)(n-1-\alpha m)}{2(n-1)(n-1-\alpha r_1)}(\overline{s}_1-\underline{s}_1). \qquad (40)
$$

Condition (4) is obtained from (39) and (40).

A.4 Other Proofs

Proof of Corollary 2. We have to show that for every $R_i''\subseteq R_i\cup\{j\}$ we have:

$$
\left|b_i-\frac{\sum_{k\in R_i''}b_k}{r_i''}\right|\leq\frac{(n-1+\alpha)(n-1-\alpha r_i'')}{2(n-1)(n-1-\alpha(r_i+1))}(\overline{s}_i-\underline{s}_i). \qquad (41)
$$

If $j\notin R_i''$, then (4) clearly implies (41), because the LHS is the same in both inequalities, but the RHS is larger in (41). Now, let $R_i''=R_i'\cup\{j\}$ for some $R_i'\subseteq R_i$. By (7), the LHS of (41) is smaller than the LHS of (4). Since $r_i''=r_i'+1$, it remains for us to check that the RHS of (41) is larger than the RHS of (4), i.e.:

$$\frac{n-1-\alpha(r_i'+1)}{n-1-\alpha(r_i+1)} \geq \frac{n-1-\alpha r_i'}{n-1-\alpha r_i} \iff \alpha^2(r_i - r_i') \geq 0, \tag{42}$$

which is satisfied since $r_i \geq r_i'$. ■

Proof of Corollary 4. Let $T_i = R_i \cup \tilde{R}_i$. For $T_i' \subseteq T_i$, let $R_i' \subseteq R_i$ and $\tilde{R}_i' \subseteq \tilde{R}_i$ be such that $T_i' = R_i' \cup \tilde{R}_i'$. Since R_i and $\tilde{R}_i$ do not overlap, we have:

$$\left| b_i - \frac{\sum_{k\in T_i'} b_k}{t_i'} \right| \leq \max\left\{ \left| b_i - \frac{\sum_{k\in R_i'} b_k}{r_i'} \right|, \left| b_i - \frac{\sum_{k\in \tilde{R}_i'} b_k}{\tilde{r}_i'} \right| \right\}.$$

Since R_i and $\tilde{R}_i$ are equilibrium sets of receivers, Theorem 1 implies:

$$\begin{aligned}
\left| b_i - \frac{\sum_{k\in T_i'} b_k}{t_i'} \right| &\leq \max\left\{ \frac{(n-1+\alpha)(n-1-\alpha r_i')}{2(n-1)(n-1-\alpha r_i)}(\overline{s}_i - \underline{s}_i), \frac{(n-1+\alpha)(n-1-\alpha \tilde{r}_i')}{2(n-1)(n-1-\alpha \tilde{r}_i)}(\overline{s}_i - \underline{s}_i) \right\} \\
&\leq \max\left\{ \frac{(n-1+\alpha)(n-1-\alpha(r_i'+\tilde{r}_i))}{2(n-1)(n-1-\alpha(r_i+\tilde{r}_i))}(\overline{s}_i - \underline{s}_i), \frac{(n-1+\alpha)(n-1-\alpha(\tilde{r}_i'+r_i))}{2(n-1)(n-1-\alpha(r_i+\tilde{r}_i))}(\overline{s}_i - \underline{s}_i) \right\} \\
&\leq \frac{(n-1+\alpha)(n-1-\alpha t'i)}{2(n-1)(n-1-\alpha t_i)}(\overline{s}_i - \underline{s}_i),
\end{aligned}$$

where the last inequality comes from $r_i' + \tilde{r}_i \geq t_i'$ and $\tilde{r}_i' + r_i \geq t_i'$. Hence, by Theorem 1, $R_i \cup \tilde{R}_i$ is an equilibrium set of receivers for player i. ■

Proof of Corollary 7. The equilibrium conditions for R_i to be a set of receivers for player i can be written as:

$$\max\left\{ \left| b_i - \frac{\sum_{k\in R_i'} b_k}{x} \right| : |R_i'| = x, R_i' \subseteq R_i \right\} \leq \frac{(n-1+\alpha)(n-1-\alpha x)}{2(n-1)(n-1-\alpha r_i)}\Delta, \quad \forall\, x = 1, \dots r_i.$$

If j is more central than i, then for every $x = 1, \dots r_i$,

$$\max\left\{ \left| b_i - \frac{\sum_{k\in R_i'} b_k}{x} \right| : |R_i'| = x, R_i' \subseteq R_i \right\} = \max\left\{ \left| b_j - \frac{\sum_{k\in R_j'} b_k}{x} \right| : |R_j'| = x, R_j' \subseteq R_j \right\}.$$

From $r_j \geq r_i$ we also have $\frac{(n-1+\alpha)(n-1-\alpha x)}{2(n-1)(n-1-\alpha r_i)}\Delta \leq \frac{(n-1+\alpha)(n-1-\alpha x)}{2(n-1)(n-1-\alpha r_j)}\Delta$ for every $x = 1, \dots r_i$, so we obtain:

$$\max\left\{ \left| b_j - \frac{\sum_{k\in R_j'} b_k}{x} \right| : |R_j'| = x, R_j' \subseteq R_j \right\} \leq \frac{(n-1+\alpha)(n-1-\alpha x)}{2(n-1)(n-1-\alpha r_j)}\Delta, \quad \forall\, x = 1, \dots r_i. \tag{43}$$

Next, for every $x = r_i, \dots r_j$, we have:

$$\max\left\{ \left| b_j - \frac{\sum_{k\in R_j'} b_k}{x} \right| : |R_j'| = x, R_j' \subseteq R_j \right\} \leq \left| b_i - \frac{\sum_{k\in R_i} b_k}{r_i} \right| \leq \frac{(n-1+\alpha)}{2(n-1)}\Delta.$$

Using $\frac{(n-1+\alpha)}{2(n-1)}\Delta \leq \frac{(n-1+\alpha)(n-1-\alpha x)}{2(n-1)(n-1-\alpha r_j)}\Delta$ for every $x = r_i, \ldots r_j$ we obtain:

$$\max\left\{\left|b_j - \frac{\sum_{k\in R'_j} b_k}{x}\right| : |R'_j| = x, R'_j \subseteq R_j\right\} \leq \frac{(n-1+\alpha)(n-1-\alpha x)}{2(n-1)(n-1-\alpha r_j)}\Delta, \quad \forall\, x = r_i, \ldots r_j. \quad (44)$$

Finally, from Inequalities (43) and (44) we deduce that R_j is an equilibrium set of receivers for player j. ∎

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