Fee Setting Intermediaries: On Real Estate Agents, Stock Brokers, and Auction Houses[*]

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January 20, 2010

Abstract

   Mechanisms where intermediaries charge a commission fee and have the sellers set the price are widely used in practice e.g. by real estate agents, stock brokers, art galleries, or auction houses. In this paper we analyze such fee setting mechanisms, and we model competition between intermediaries in a dynamic random matching model, where in every period a buyer, a seller, and an intermediary are randomly matched. In any period, every intermediary has a temporary monopoly and designs an exchange mechanism that maximizes his own expected profits. Traders' valuations for the indivisible good depend on their option value of future trade. The following results obtain. First, we show that the intermediary can achieve the highest possible profit with a fee setting mechanism, and we characterize when these fees are linear. Second, fee setting is an equilibrium outcome in a dynamic market. Third, when the rematching probability increases or, equivalently, the period length decreases, the equilibrium fees become smaller. Fourth, our model can explain, or is consistent with, several of the stylized facts observed in real estate brokerage, such as the 6 percent fee, the invariance of the mechanism employed within the industry with respect to the number of active intermediaries, the relation between listing price and time on market, inefficient free entry, higher prices for houses owned by brokers, and home owners who bought during a boom asking higher prices.

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Keywords: brokers, applied mechanism design, linear commission fees, optimal indirect mechanisms, internet auctions, auction houses.

JEL-Classification: C72, C78, L13

1   Introduction

Many markets are organized by intermediaries, and many of these intermediaries neither buy nor sell the goods whose exchange they enable. Instead they set percentage fees to be levied on the price, which is subsequently set by the seller. Buyers then either accept or reject the price. If the mechanism involves an auction where the seller sets a reserve price, the buyers bid in the auction, and the fee is levied on the realized price. We call such mechanisms "fee setting mechanisms".

Real estate brokers, stock brokers, art galleries, and auction houses or auction sites are just a few examples of fee setting intermediaries. Real estate brokers in the U.S. typically charge 5 to 6 percent. Commission fees by art galleries are said to be in the range of 30 to 50 percent. The auction houses Sotheby's and Christie's use a regressive fee structure and so does eBay.[1] Other industries where fee setting mechanisms are frequently used include stock brokerage, share-cropping in agriculture , contracts between authors and publishing companies, and retailers that charge a percentage on the revenue a manufacturer generates with his product. Similarly, electronic payment systems and credit cards charge percentage fees. Percentage fees are also used in a slightly different environment in investment banking[2] and by labor market intermediaries, in particular by head hunters.

As a matter of fact, industries where fee setting mechanisms are predominantly used

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[1]The marginal rate at Sotheby's is 25 percent for items with prices up to USD 20,000, 25 percent between USD 20,000 and USD 500,000 and 12 percent beyond. At eBay (ebay.com, accessed on May 5, 2008) the marginal fee on the closing price is 8.75 percent below USD 25, 3.5 percent between USD 25 and USD 1000, and 1.5 percent above USD 1000. Sotheby's and Christie's used a linear fee of 20 percent prior to being investigated by U.S. Department of Justice, convicted for collusive behavior, and induced to change the fee structure.
Similarly, real estate brokerage has come under scrutiny by the U.S. Department of Justice (DOJ, 2007). There is a widespread, though rarely explicit, suspicion that in particular the almost complete invariance of broker commission fees reflects collusive behavior by intermediaries.

[2]Underwriters on initial public offerings in the U.S. charge in most cases exactly 7 percent, see Chen and Ritter (2000).

are quite sizeable. For example, the sales generated by Sotheby's in 2007 alone exceeded USD 4 billions.[3] The annual operating revenue of eBay was more than USD 7.5 billion in 2007, and Christie's annual sales in 2006 exceeded USD 4.5 billion.[4] The real estate brokerage industry in the U.S. generates annual sales beyond USD 1000 billion and commission fees of more than USD 60 billion per year.[5] Credit card companies are big business, too. For example, MasterCard's annual revenue in 2007 exceeded USD 4 billion.[6]

Despite their widespread use and economic significance, fee setting mechanisms have received very little attention in the theoretical economic literature. In particular, no prior analysis of the optimality of fee setting and the structure of fees from a mechanism design perspective exists.[7] The purpose of this paper is to start filling this gap and to improve economists' understanding what determines whether intermediaries set commission fees and, if they choose to do so, what determines the size and form of these fees.[8]

Our paper makes two main contributions. First, we set up a dynamic random matching model with a continuum of buyers, sellers and intermediaries, and we derive the exchange mechanism of every intermediary as the endogenous outcome of a mechanism design problem that depends on the distributions of the types of buyers and sellers which are, in turn, endogenous to the choice of mechanisms by *all* intermediaries. Our second

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[3]See Sotheby's Annual Report 2007, p.28, available at Sothebys.com. The fee structure was reported in the New York Times (2008).

[4]See www.marketwatch.com and www.sgallery.net, respectively.

[5]See Rutherford, Springer, and Yavas (2005).

[6]MasterCard Worldwide, Annual Report 2007.

[7]The fact that, to the best of our knowledge, no name for this type of mechanism exists only goes to show how little theoretical interest these mechanisms have received. Two papers that provide explanations of when intermediaries may use percentage fees and when they set prices are Hagiu (2006) and Yavas (1992). Hagiu's argument relies on the presence and nature of network externalities, while Yavas' explanation depends on the presence and working of search markets.

[8]That said, this means also that we do not aim to explain why intermediaries emerge in equilibrium (as do e.g. Gehrig (1993); Spulber (1999); Rust and Hall (2003)). Rather, we take the existence and importance of intermediated exchange as given and ask why (some) intermediaries use fee setting mechanisms. Empirically the predominance of intermediation we presume in this paper is on solid ground. For example, real estate brokers account for approximately 80 percent of all single-family dwellings in the U.S. (see Rutherford, Springer, and Yavas, 2005). A rather simple explanation for the exclusivity of trade through intermediaries is that there is an alternative search market where traders meet directly, but search costs are prohibitively high. Alternatively, it should be possible to extend the framework by a search market as an outside option. Traders' willingness to pay an intermediary is limited by the outside option of going to the search market.

contribution, which can be seen as a by-product to the first, is that we analyze and characterize fee setting mechanisms in the one period model. The following is a sketch of the dynamic model. Buyers and sellers have private information about their valuations for an indivisible homogeneous good. In every period one buyer, one seller, and one intermediary are randomly matched. Intermediaries are free to choose the trading mechanism anew in every period. The equilibrium mechanism used by the intermediaries in the market determines the option value of future trade and hence the endogenous reservation values of buyers and sellers. The distributions of these values in turn determine the best response mechanism of an intermediary. We focus on a steady state equilibrium where every intermediary uses the same stationary mechanism. We show that this model permits an analytical solution for certain cases. Interestingly, the equilibrium mechanism does not vary with the number of intermediaries under standard assumptions on the matching technology. The equilibrium fees become smaller as the matching frequency increases, or equivalently the period length between subsequent rematchings decreases. Moreover, we derive empirically testable predictions, such as the implied time goods stay on the market as a function of their prices, the distribution of prices on the market, and the probability that a good is ever sold.

The intuition for the intermediaries' equilibrium choice of mechanism stems from the one period model, where a monopolistic intermediary is matched to one buyer and one seller.[9] Assume for simplicity that the mechanism design problem is regular, i.e. the buyer's virtual valuation and the seller's virtual cost are increasing everywhere. Then the mechanism that is optimal for the intermediary induces trade between the seller and the buyer if and only if the seller's virtual cost is less than the buyer's virtual valuation.[10] A fee setting mechanism is thus intermediary optimal if it induces a seller of a given cost type to set a price that is accepted by the buyer if and only if the buyer's virtual valuation exceeds his virtual cost. Such a fee setting mechanism exists. Moreover, this fee setting mechanism has the desirable properties of having an essentially unique equilibrium and of making payments to the seller and requiring payments from the buyer only if they

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[10]If the problem is not regular, insert the word 'ironed' in front of 'virtual' in the last phrase. For more on the ironing procedure see e.g. Myerson (1981) or Appendix C below.

actually trade. In the dynamic model a fee setting mechanism is still optimal for every intermediary, but now the fee depends on the distributions of reservation values that arise endogenously from the choice of mechanisms by all intermediaries. With many buyers whose valuations are i.i.d. draws and one seller, a fee setting mechanism followed by an optimal auction (such as used by eBay, Sotheby's or Christie's) is intermediary optimal.

The dynamic model admits various extensions. We study inefficient free entry by intermediaries whose opportunity costs of entry and levels of ability may be heterogeneous. Under the assumption that power distributions are a good approximation, the model also allows for costs of intermediation that vary with transactions. We also show that in the dynamic model a vertically integrated intermediary (who is also a seller) may charge a higher price than an independent seller, thereby providing an explanation for empirical observations (see Levitt and Syverson (2008) and Rutherford, Springer, and Yavas (2005)). Furthermore, we show that price posting by the intermediary is not optimal for the intermediary in a static setup with one buyer and one seller, but becomes intermediary optimal in a dynamic setup if the good in question can be stored without cost. Last, we show that slotting allowances, i.e. the practice of retailers to first auction off scarce shelf space to a producer and then charging a percentage fee on the revenue generated by the producer, can be interpreted as an intermediary optimal mechanism.

Our paper contributes to the large and growing literature on intermediation such as Gehrig (1993), Yavas (1992, 1996), Hackett (1992), Spulber (1996, 1999, 2008), Wooders (1997), Rust and Hall (2003), Duffie, Garleau, and Peddersen (2005), Loertscher (2007) and Burani (2008) by adding a mechanism design perspective to the notion of (dynamic) random matching present in most of these papers. Apart from Myerson and Satterthwaite (1983) the only articles applying mechanism design to intermediation we are aware of are Spulber (1988) and Matros and Zapechelnyuk (2006). Our paper differs from these papers by having dynamic random matching, multiple competing sellers and intermediaries, and predictions on price dispersion, fee structures, and time on market. Our paper also relates to the literature on bilateral trade with private information initiated by Myerson and Satterthwaite (1983) and Chatterjee and Samuelson (1983). That

a fee setting mechanism is intermediary optimal, has an essentially unique equilibrium and induces payments only from and to agents who actually trade, is interesting on its own. It provides a practical counterpart to the direct, and therefore abstract, intermediary optimal mechanism derived by Myerson and Satterthwaite. We show also that an appropriately chosen fee mechanism is an ex ante efficient mechanism.[11]

As we add intermediaries to a dynamic random matching model with incomplete information similar to Satterthwaite and Shneyerov (2007, 2008) and Atakan (2006b) it also relates to this strand of literature.[12] Insofar as the intermediaries in our dynamic model are competing mechanism designers, the paper is related to the work of McAfee (1993), Peters and Severinov (1997, 2006), and Damianov (2005) who study mechanism design by sellers whereas in our model the mechanisms are chosen by intermediaries.[13]

In that respect, and because real estate brokerage is an industry to which our model applies, the paper also contributes to the literature on real estate economics. Most of the literature analyzing real estate brokerage remains in the principal-agent framework, where the seller, and occasionally the buyer, is the principal and the broker the agent; see e.g. Anglin and Arnott (1991), Bagnoli and Khanna (1991), Arnold (1992), Williams (1998), Lewis and Ottaviani (2008)[14] for theoretical and Rutherford, Springer, and Yavas (2005) and Levitt and Syverson (2008) for empirical work. The present paper offers a novel perspective by explaining fees as a means of rent extraction by brokers rather than as an incentive for brokers. We think there are good reasons to depart from the principal-agent framework. First, in the case where a broker represents a buyer, the

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[11]Recall that the double auction described by Chatterjee and Samuelson (1983) satisfies the social optimality condition stated in Myerson and Satterthwaite (1983, Theorem 2) for uniform distributions. We show that the fee setting mechanism described here satisfies the intermediary optimality conditions for general distributions. Moreover, for bilateral trade and for trade between one seller and many buyers an ex ante efficient fee setting mechanism exists under the same sufficiency conditions as Gresik and Satterthwaite (1989) use to prove the existence of an ex ante efficient mechanism.

[12]See also Wolinsky (1988), De Fraja and Sakovics (2001), Serrano (2002), Moreno and Wooders (2002), Lauermann (2007), Lauermann and Wolinsky (2008). For complete information models see e.g. Mortensen and Wright (2002), Gale (2000), and the references therein.

[13]McAfee (1993, p.1304) notes that [his] "paper falls far short of a real theory of equilibrium institutions partly because it places the design of institutions in the hands of the sellers. A more satisfactory approach requires explicit modelling of the role of intermediaries, or auctioneers, who compete among each other for both buyers and sellers."

[14]Lewis and Ottaviani have a general model of dynamic search agency with research and development as the main application. However, real estate brokerage is one of the many applications of their model.

broker typically charges 3 percent of the price paid by the buyer. This percentage fee cannot be explained in a principal-agent framework where the buyer incentivizes the broker to find an advantageous price for him, since their interests are diametrically opposed under such a contract. Second, even in the cases where the broker represents the seller, it is not clear why the broker typically gets 6 percent of the total price. If it were the seller who proposes the contract to the broker, incentive compatibility implies that he would give a much higher percentage to the broker for the *marginal* increase of price he achieves. Making the individual rationality constraint binding should lead to a lower fee on the *inframarginal* price.[15] Third, the following observations suggest that bargaining rests with the broker rather than the buyer or the seller: the almost complete invariance of commission fees, the concerns about collusion by real estate brokers, and the fact that brokers are long-term players with substantial benefits from reputation, whereas individual buyers and sellers trade with very low frequency with brokers. It is also hard to make a case that, as a consequence of competition between brokers, sellers, or buyers, should propose the contract since in most other industries competing firms like e.g. retailers, car manufacturers or gas stations make take-it-or-leave-it (price) offers to their clients. Competition merely constrains these firms in what they can optimally offer. Fourth, empirical observations of price dispersion of houses with the same quality and the relation of the price and the time on market are difficult to explain in a principal agent framework.

Our dynamic model also gives possible explanations for, or is at least consistent with, the following stylized facts observed in real estate markets. First, real estate brokers charge 6 percent of the transaction price, a commission rate that shows very little variance over time and across regions. Second, sellers with a higher loan-to-value ratio ask higher prices (Genesove and Mayer, 1997). Third, sellers who had bought their houses when average real estate prices were high, ask for higher prices than those who had bought when prices were low (Genesove and Mayer, 2001). Fourth, quality adjusted prices and time on market of houses are positively correlated in cross sectional and

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[15]As Hsieh and Moretti (2003) point out in their empirical analysis, a 6 percent fee seems to be far above the costs incurred by a broker for a house selling for say USD 500,000, especially so as 6 percent is sufficient to cover the broker's costs for a house selling for USD 100,000.

negatively correlated in longitudinal data. Fifth, broker fees are the same irrespective of the number of intermediaries and house prices. Sixth, while industry profits doubled, the number of brokers doubled as well (Hsieh and Moretti, 2003). Seventh, direct sellers (either brokers selling their own houses or houses for sale by owner (FSBO)) sell at higher prices than indirect sellers selling through a broker (see Rutherford, Springer, and Yavas, 2005; Levitt and Syverson, 2008; Hendel, Nevo, and Ortalo-Magné, forthcoming).

The remainder of this paper is structured as follows. Section 2 introduces the basic model. Sections 3 and 4 derive the equilibrium and comparative statics. Section 5 applies the model to markets of real estate agents, auction houses, and stock brokers. Section 6 concludes. All proofs are in the Appendix.

2   The Model

We study a fairly general model of intermediation. A discussion of our assumptions is best deferred to Section 5, where we present and discuss various applications. Fig. 1 illustrates and summarizes the infinite horizon model we study. Every period a mass 1 of buyers and a mass 1 of sellers consider entering the market. An entering buyer's valuation of the good $\tilde{v}$ is drawn from the distribution $\tilde{F}_0$ with strictly positive density $\tilde{f}_0$ on the support $[\underline{\tilde{v}}_0, \bar{\tilde{v}}_0]$. Each entering seller has one unit of an indivisible good. His cost of selling the good is $\tilde{c}$ is drawn from $\tilde{G}_0$ with $\tilde{g}_0 > 0$ on $[\underline{\tilde{c}}_0, \bar{\tilde{c}}_0]$.[16] Both valuation and cost are private information, whereas the distributions are common knowledge. We refer to $\tilde{v}$ and $\tilde{c}$ as a buyer's and seller's static type (i.e. static valuation and cost, respectively).

Buyers enter a pool with mass σ. The endogenous distribution of the valuations of buyers in this pool is $\tilde{F}(\tilde{v})$ with support $[\underline{\tilde{v}}, \bar{\tilde{v}}]$. Similarly, sellers enter a pool of mass σ, and the (endogenous) distribution of their costs is $\tilde{G}(\tilde{c})$ with support $[\underline{\tilde{c}}, \bar{\tilde{c}}]$. We assume that the market is in steady state, i.e. the traders entering the market (pool) have the same mass and distribution of valuations as those who leave. We also assume that buyers and sellers who cannot trade for sure do not enter the market. Hence, $\tilde{G}$ and $\tilde{F}$ are the

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[16]For brevity, we refer to a seller's valuation of the good, or his opportunity cost of selling the good, as his cost. This makes clear that the model also applies to settings where the good has to be produced by the seller at a cost.

steady state cumulative distribution functions. Their densities are denoted with $\tilde{f}$ and $\tilde{g}$. There is an unlimited supply of intermediaries standing ready to offer their services. In the main analysis we abstract from the entry decision of intermediaries, but we address this issue in Section 5. In each period, each buyer, each seller and an intermediary are uniform randomly matched[17] in a triple consisting of one member each of the three groups. All agents are risk neutral, and preferences are quasilinear, i.e. when trading at price p a buyer whose static valuation is $\tilde{v}$ gets the instantaneous net utility $\tilde{v} - p$ and a seller with static cost $\tilde{c}$ enjoys a net payoff of $p - \tilde{c}$. In accordance with the literature we assume that buyers and sellers who cannot trade do not enter the market in steady state.[18]

Buyers and sellers who do not trade stay in the market with the exogenous probability $e^{-\eta\tau}$ until the next period, where τ represents the length of a period and η is a parameter of the hazard rate. With probability $1 - e^{-\eta\tau}$ a trader drops out of the market and has utility 0. For simplicity, assume that intermediaries stay in the market forever.[19] Future utility is discounted with a factor $e^{-r\tau}$. As for most of the analysis only the product of these two factors matters, we define $\delta := e^{-(\eta+r)\tau}$ as the total discount factor. The period length τ can be interpreted as the extent of frictions in the market or, as we will show later, as a parameter of the degree of competition: the shorter the time of a new match after a failed trade, the more fiercely intermediaries compete.

The dynamics offer buyers and sellers positive probability of trading in the future if trade fails in the presence. That is, for given mechanisms employed by intermediaries a buyer whose static type is $\tilde{v}$ will have a maximal willingness to pay $v \leq \tilde{v}$ because of her outside option of waiting and trading in the future. We call this lowered willingness to pay her dynamic valuation or her dynamic type. Similarly, a seller with static type $\tilde{c}$ will have a dynamic cost $c \geq \tilde{c}$.[20] The crucial point of our analysis is that there are

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[17]This matching technology is essentially the same as in Atakan (2006a,b). It differs from Satterthwaite and Shneyerov (2007, 2008) who assume a seller who is matched with zero, one, or many buyers.

[18]This is equivalent to assuming that the inequalities in Assumption 1 below are binding in steady state.

[19]This assumption is not important. We could just as well have intermediaries arriving each period and others dropping out.

[20]Put differently, the fact that there is a future drives a positive (negative) wedge between a buyer's (seller's) static type and his dynamic type.

monotonic relations between static and dynamic types, so that $v = B(\tilde{v}) = \tilde{v} - \delta W_B(\tilde{v})$ and $c = S(\tilde{c}) = \tilde{c} + \delta W_S(\tilde{c})$, where both B and S are increasing functions and W_B and W_S are the option values of continuing for buyers and sellers. This allows us to use the dynamic valuation of the buyer $v = B(\tilde{v})$ and the dynamic cost of the seller $c = S(\tilde{c})$, which of course remain to be determined, to derive the endogenous distributions $F(v) = \tilde{F}(B^{-1}(v))$ and $G(c) = \tilde{G}(S^{-1}(c))$ with densities f and g.

We assume that intermediaries have all the instantaneous bargaining power. That is, matched to a buyer and seller in a given period an intermediary chooses a mechanism that maximizes his expected profit subject to the buyer's and seller's individual rationality and incentive constraints. The relevant distributions an intermediary has to take into account when designing his exchange mechanism are the distributions of buyers' and sellers' dynamic types F and G. Note that from the point of view of an intermediary this is equivalent to a one-shot game because the probability that he will meet the same buyer or the same seller in a subsequent period is zero and because he takes the mechanisms offered by other intermediaries in subsequent periods as given. Though the individual rationality and incentive constraints are endogenous to the game, they are exogenous for every intermediary. This motivates to study the one-shot game in some detail, which is what we do in Section 3.1.

There are three types of distributions in our model: static entrant $\tilde{F}_0$ and $\tilde{G}_0$, static steady-state $\tilde{F}$ and $\tilde{G}$, and dynamic steady-state F and G. The distributions $F(v)$ and $G(c)$ and the mechanisms employed by the intermediaries which in turn determine the probabilities $\rho_B(v)$ and $\rho_S(c)$ that a buyer v and a seller c trade in a given period. The exogenous per period exit probability is $1 - e^{-\eta\tau}$. Therefore, the probabilities of staying in the market are $(1 - \rho_B(v))e^{-\eta\tau}$ and $(1 - \rho_S(c))e^{-\eta\tau}$. The steady state mass of agents in the market being σ and the mass of entrants being 1, the densities $\tilde{f}_0(v)$ and $\tilde{g}_0(c)$ entrants have to integrate to 1 and satisfy

$$\tilde{f}_0(v) = \sigma[1 - (1 - \rho_B(B(\tilde{v})))e^{-\eta\tau}]\tilde{f}(\tilde{v}) \tag{1}$$

and

$$\tilde{g}_0(\tilde{v}) = \sigma[1 - (1 - \rho_S(S(\tilde{c})))e^{-\eta\tau}]\tilde{g}(\tilde{v}). \tag{2}$$
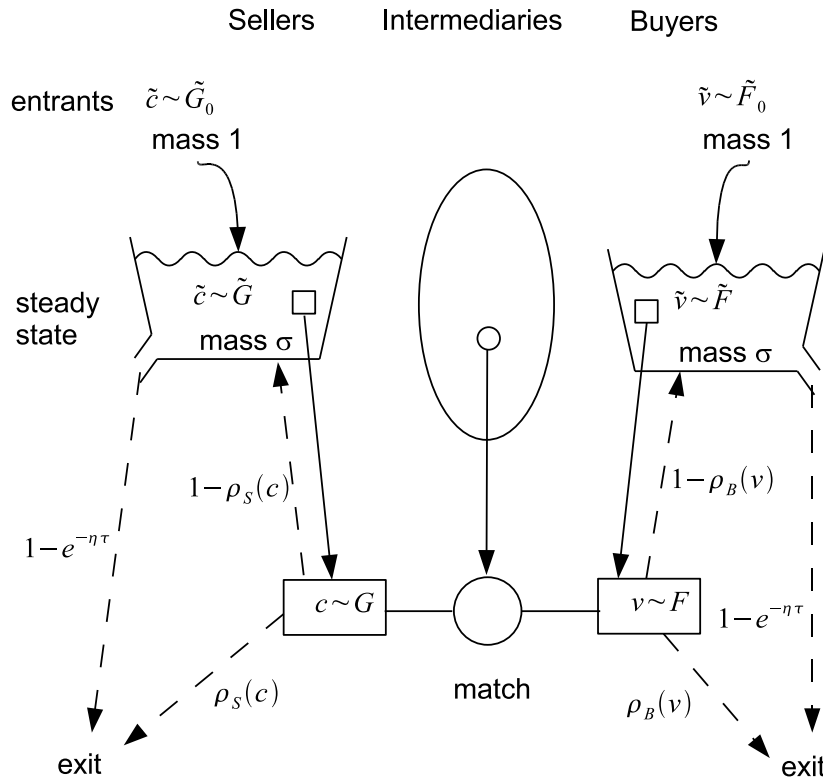
Figure 1: Market in steady state. Each period mass 1 of traders with distributions $\tilde{F}_0$ and $\tilde{G}_0$ enter the market and join pools with distributions $\tilde{F}$ and $\tilde{G}$. Traders have dynamic type distributions F and G. With probabilities $\rho_B(v)$ and $\rho_S(c)$ they leave the market because they trade, with probability $1 - e^{-\eta\tau}$ they leave the market for exogenous reasons.

3   Equilibrium

We now turn to the equilibrium analysis of the model. We begin with the stage game, which corresponds to a static, one-shot game.

3.1   The Stage Game

The stage game is of some interest of its own. In this game, distributions of costs and valuations are the exogenously given primitives of the model. The intermediary is a monopolist who faces a buyer and a seller whose valuation v and cost c for a homogenous good of known quality are private information and independently drawn from distribu-

tions F and G.[21] We assume that the intermediary has all the bargaining power, subject to the individual rationality and incentive constraints of the buyer and the seller. The distributions F and G have strictly positive density f and g on the support $[\underline{v}, \bar{v}]$ and $[\underline{c}, \bar{c}]$, respectively.

As all agents are risk neutral and preferences are quasilinear, the buyer's utility is $v - p$ and the seller's payoff if $p - c$ in case of trade at price p. The seller and the buyer can only trade through the monopolistic intermediary who has all the bargaining power and can hence choose the trade mechanism. The intermediary's expected profit under some mechanism is the equilibrium payment by the buyer minus the equilibrium payment to the seller in expectations. A mechanism[22] is intermediary optimal if there is no mechanism that gives strictly higher profits to the intermediary in expectations over the buyer's and the seller's valuation for the good, subject to the constraints that the buyer and seller want to participate and that buyer's and seller's strategies constituting a (Bayes) Nash equilibrium of the game.

Denote by $\Phi(v) := v - (1 - F(v))/f(v)$ the buyer's virtual valuation function. Analogously let the seller's *virtual cost function* be denoted as $\Gamma(c) := c + G(c)/g(c)$. The intermediary's mechanism design problem is said to be regular if both virtual type functions are increasing. To simplify notation, in particular dealing with the inverses Φ^{-1} and Γ^{-1}, we make the following two assumptions throughtout the paper. First,

Assumption 1. $\Phi(\bar{v}) \leq \Gamma(\bar{c})$ *and* $\Phi(\underline{v}) \leq \Gamma(\underline{c})$.[23]

Second, we assume Myerson's regularity condition, i.e. Φ and Γ are increasing.[24] The general case without these assumptions can be easily dealt with by standard techniques

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[21]As the model is static and has neither entry nor exit, trivially, no distinction between static and dynamic types has to be made nor do we need to distinguish between distribution of entrant types and types in the pool. This simplifies the notation.

[22]A mechanism means the following. First, the mechanism designer (here the intermediary) offers a menu of possible actions to the seller and the buyer, for each combination of actions he announces the payments a participant pays or receives and whether the good is exchanged, then both seller and buyer pick actions that are mutually best replies. For the example of fee setting, the seller's choice of actions is the choice which price to ask. The buyer chooses between the actions of accepting or rejecting the offer. A more detailed explanation of these concepts is provided e.g. in Krishna (2002, Ch.5).

[23]As it will become clear later on, this assumption means that an intermediary's profit maximizing mechanism excludes certain buyers and sellers from trade for sure.

[24]This implies a convex optimization problem for the intermediary.

such as ironing and a slight modification of the definition of the inverses (see Appendix C).

Fee Setting Mechanisms The main focus of this paper is on the following type of indirect mechanisms, which we call *fee setting* mechanisms.

First, the intermediary first announces a fee function $\omega(.)$ that determines the amount the intermediary gets upon successful sale at price p, leaving $p - \omega(p)$ to the seller. Then the seller sets the price p, knowing ω and his own cost c. Finally, observing p and her own valuation v, the buyer then accepts or rejects the offer p and the game ends. If the buyer accepts, the seller gets the net price $p - \omega(p)$ and the intermediary the fee $\omega(p)$.

Notice that by construction a fee setting mechanism has the property that payments occur only if there is trade. Thus, there is no regret. Obviously the buyer accepts if and only if $v \geq p$. The choice of the fee are analyzed next. Below we also study a slightly modified version of this game with multiple buyers whose valuations are independent draws from the distribution F while there is still one seller and one intermediary.

Our assumption that the seller pays the fee is of course without loss of generality, i.e. how the intermediary's fee $\omega(p)$ is allocated between the buyer and the seller does not matter. We use the convention of saying that an equilibrium is *essentially unique* if all seller and buyer types who trade with positive probability in some equilibrium take the same action in every equilibrium.[25] Analogously, a fee structure is said to be *essentially unique* if it is unique pinned down for all those prices that are accepted by some type of buyer with positive probability.

The Simple Economics of Optimal Intermediation The buyer's virtual valuation $\Phi(v)$ can be interpreted as the marginal revenue of increasing the probability of trade, the seller's virtual cost as marginal cost.[26] Therefore, the intermediary wants the seller and

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[25]Put differently, an equilibrium is essentially unique if equilibria only differ with respect to actions of types who never trade in any equilibrium.

[26]The reasoning is similar to Bulow and Roberts (1989)'s for optimal auctions: interpret the probability that $\tilde{V} \geq v$ and $\tilde{C} \leq c$ as quantity demanded and supplied, i.e. $q := 1 - F(v)$ and $q := G(c)$. Thus the inverse demand and supply function are $v = F^{-1}(1-q)$ and $c = G^{-1}(q)$, yielding $R(q) = qF^{-1}(1-q)$ and $C(q) = qG^{-1}(q)$ as revenue and cost functions. Taking derivative w.r.t. q and substituting back in yields $R'(q) = \Phi(v)$ and $C'(q) = \Gamma(c)$.

the buyer to trade if and only if marginal revenue exceeds marginal cost, i.e. whenever $\Phi(v) \geq \Gamma(c)$.

As shown by Myerson and Satterthwaite (1983), this is indeed the optimal allocation rule for the intermediary.[27] Due to payoff equivalence (see e.g. Krishna (2002)), once the allocation rule is determined the equilibrium expected payoffs for all types of all players are determined up to an additive constant. It is in the intermediary's interest to minimize this constant subject to the individual rationality constraint that all types of buyers and sellers are willing to participate in the mechanism. Therefore, under the intermediary optimal mechanism the worst off agents are just indifferent between participating and not, i.e. the lowest valuation buyer $\underline{v}$ and the least efficient seller $\bar{c}$ get expected payoffs of zero. See Lemma 2 in the Appendix for a summary and formalization of these results.

Intermediary Optimal Fee Setting Mechanism   We now show that an intermediary optimal fee setting mechanism exists. Let us denote $P(c) := \Phi^{-1}(\Gamma(c))$ for notational ease, which will turn out to be the price the seller sets in equilibrium, and denote its inverse as P^{-1}.

An intuitive derivation of the optimal fee setting mechanism can be obtained by taking a brief detour through a dominant strategy direct mechanism implementation.[28] The dominant strategy implementation is that the intermediary asks agents to report their types and allows trade iff $v \geq P(c)$ (or equivalently $c \leq P^{-1}(v)$) and in case of trade the buyer pays $P(c)$ and the seller gets $P^{-1}(v)$.[29] Conditional on trade, the seller thus gets $E_v[P^{-1}(v)|v \geq P(c)]$ in expectations over v. Since the seller cares only about what he gets in expectation rather than individual realizations of v, the intermediary could

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[27]Myerson and Satterthwaite (1983) are almost exclusively cited for their impossibility results. A notable exception is Spulber (1999, Ch.7), who compares the optimal direct mechanism of Myerson and Satterthwaite with price posting by the intermediary.

[28]A direct mechanism requires participants to report their valuations to the mechanism designer who will take actions for them rather than taking actions themselves. The idea of a dominant strategy implementation goes back to Vickrey (1961)'s analysis of second price auctions. It basically means that it is a dominant strategy (i.e. optimal independently of the other agent's actions) for every participant to report their types truthfully.

[29]The buyer gets a take-it-or-leave-it offer at price $P(c)$. It is clearly a dominant strategy to accept the offer iff $v \geq P(c)$. The same applies to the seller. This dominant strategy implementation is already mentioned in Myerson and Satterthwaite (1983) after Theorem 4.

just as well pay the seller the expected value as the net price $P(c) - \omega(P(c))$. Equating the net price $P(c) - \omega(P(c))$ with the seller's expected payoff and replacing $P(c)$ with p gives the optimal fee as stated in Proposition 1.[30]

Proposition 1. *An intermediary optimal fee setting mechanism exists and is essentially unique. The optimal fee function is*

$$\omega(p) = p - E_v[P^{-1}(v) \mid v \geq p] \tag{3}$$

for $p < \bar{v}$ and an arbitrary $\omega(p) \geq \bar{v} - p$ for $p \geq \bar{v}$. A seller with cost $c \leq P^{-1}(\bar{v})$ sets the price $p = P(c)$ and a buyer with valuation v accepts iff $p \leq v$. Moreover, the equilibrium is essentially unique.

A few remarks are in order. First, this fee setting mechanism is optimal in the general class of mechanisms, i.e. the intermediary cannot do better by any other mechanism than fee setting. Second, taking the derivative of (3) and rearranging by partial integration shows that the marginal fee ω' can never be higher than 100 percent.[31] This is of course what one would expect from incentive compatibility. Interestingly, $\omega'(p) < 0$ is not impossible a priori.[32] Third, consider two markets characterized by $P^{-1}(v)$ and $\hat{P}^{-1}(v)$ and by the same F and denote the associated intermediary optimal fees by $\omega(p)$ and $\hat{\omega}(p)$, respectively. If $P^{-1}(v) < \hat{P}^{-1}(v)$ for all v, then the total fee is higher in the former market, $\omega(p) > \hat{\omega}(p)$, since $E_v[P^{-1}(v) \mid v \geq p] < E_v[\hat{P}^{-1}(v) \mid v \geq p]$. If in addition $[P^{-1}(v)]' < [\hat{P}^{-1}(v)]'$ for all v, then the marginal fee is higher as well, $\omega'(p) > \hat{\omega}'(p)$ for all p.[33] Fourth, Proposition 1 implies that the intermediary can achieve his maximal expected profit without knowing or making use of the buyer's valuation when determining payments in case of trade. The buyer's valuation is only needed to determine whether the good is traded. However, the optimal mechanism depends in general on the distribution

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[30] Alternatively, one can equate the seller's utility under fee setting $[P(c) - \omega(P(c)) - c][1 - F(P(c))]$ with his information rent and solve first for $\omega(P(c))$ and then for $\omega(p)$ (see the formal proof in the Appendix).

[31] A similar observation in the context of income taxation is made by Mirrlees (1971).

[32] Notice that this does not violate incentive compatibility of the seller because the net price $p - \omega(p)$ is only one part of the seller's payoff, the probability of of sale $1 - F(p)$ being the other one.

[33] To see the latter, integrate $E_v[P^{-1}(v) \mid v \geq p]$ by parts (or go directly to equation (20) in the Appendix) and then take the derivative with respect to p.

of the buyer's valuation F. It is therefore rather striking that for a certain family of distributions of seller's types, namely all those that exhibit virtual costs that are linear in c, the optimal fee charged by the intermediary is independent of F and linear:[34]

Proposition 2 (Optimality of Linear Fee Mechanisms)**.** *The following are equivalent statements:*

(i) a linear fee mechanism is optimal, i.e. $\omega(p) = \xi p + \zeta$ is intermediary optimal,

(ii) c is drawn from a generalized power distribution $G(c) = \left(\frac{c-\underline{c}}{\bar{c}-\underline{c}} \right)^{\beta}$ with $\beta > 0$,

where $\xi = 1/(\beta + 1)$ and $\zeta = -\underline{c}/(1 + \beta)$ holds.

For example, if G is uniform on $[0, 1]$, then $\Gamma(c) = 2c$ and $\omega(p) = p/2$, which is obviously independent of F. As the optimal linear fee is fully determined by the two parameters β and $\underline{c}$ of the distribution of the seller's cost G, Corollary 1 follows directly from Proposition 2.

Corollary 1 (Invariance of Linear Fees)**.** *If a linear fee is intermediary optimal for some distributions F and G, then it will also be optimal for $\hat{F}$ and G, where $\hat{F}$ is an arbitrary regular distribution.*

The reverse implication of Corollary 1 also holds:

Proposition 3. *If a fee function ω is optimal for a given G and for an arbitrary regular F, then the fee has to be linear and G has to be a generalized power distribution.*

For an interpretation of these results see the application to real estate brokerage in Section 5.1.

Analogous results can be obtained for mechanisms where the buyer sets the price and the fee is conditioned on this price. It is for instance optimal for the intermediary to let the buyer set the price and charge the fee $\omega_B(p) = E_c[P(c)|c \leq p] - p$, which induces the buyer to set the price $p = P^{-1}(v)$. For $F(v) = 1 - [(v - \underline{v})/(\bar{v} - \underline{v})]^{\beta_B}$ the fee will be linear and independent of the seller's distribution.

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[34]Linearity of the optimal fee obtains under conditions that are similar to the ones under which **?** find that a double auction has a linear equilibrium.

Ex ante efficient Fee Setting Mechanism   In fact, not only the intermediary optimal allocation rule, but any monotone allocation rule[35] can be implemented as a fee setting mechanism. since nothing of substance in the construction of $\omega(p)$ requires the pricing function to implement the intermediary optimal allocation rule.

An allocation rule that has received some attention in the literature is the one that maximizes the ex ante expected gains from trade, subject to budget balance, and voluntary participation.

Corollary 2. *Assume increasing hazard rates, i.e. $(1 - F)/f$ decreasing and G/g increasing. The fee $\omega_{\alpha^*}(p) = p - E_v[P_{\alpha^*}^{-1}(v)|v \geq p]$ implements the ex-ante efficient allocation rule and induces an essentially unique equilibrium, where $P_\alpha(c) := \Phi_\alpha^{-1}(\Gamma_\alpha(c))$, $\Phi_\alpha(v) := \alpha\Phi(v) + (1 - \alpha)v$, and $\Gamma_\alpha(c) := \alpha\Gamma(c) + (1 - \alpha)c$ for some $\alpha \in [0, 1]$. α^* is the smallest α such that $\int\int(\Phi(v) - \Gamma(c))Q^\alpha(c, v)dcdv \geq 0$ where $Q^\alpha = 1$ if $\Phi_\alpha \geq \Gamma_\alpha$ and 0 else.*

For F and G uniform on [0,1], $\alpha^* = 1/3$ and $\omega_{\alpha^*}(p) = p/2 - 1/4$. Interestingly, for the case of uniforms the ex ante efficient fee charges the same percentage of the price as the intermediary optimal fee. But it also consists of a 'deductible' of 1/4 upon successful sale. Notice that this deductible may come in the form of services the intermediary provides to the seller.

The corollary complements the finding of Myerson and Satterthwaite (1983) that for F and G uniform the double-auction of Chatterjee and Samuelson (1983) has an equilibrium that implements the ex ante efficient mechanism. Notice also that the fee setting mechanisms of Proposition 1 and Corollary 2 do not only respect participation constraints *ad interim* but also *ex post* because in equilibrium a buyer never accepts a price exceeding her valuation and a seller never sets a price such that he would make a loss in case of trade. Therefore, these mechanisms are also within the more restrictive set of mechanisms that respect participation constraints *ex post*.[36]

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[35] A an allocation rule is said to be monotone if the probability that the buyer gets the good increases in v and the probability that the seller sells decreases in c.

[36] These arguments thus generalize, and simplify, the proof by **?** that a socially optimal incentive compatible mechanism exists that satisfies ex post participation constraints.

3.2   The Dynamic Game

We now turn to the analysis of the dynamic game with random matching introduced in Section 2. In every period every intermediary first announces a fee ω that is a function of the price the seller will set. Then the seller sets a price p. If the buyer accepts, he pays p, the seller gets the net price $p - \omega(p)$, and the intermediary the fee $\omega(p)$. If there is trade, the net utility of the seller with cost $\tilde{c}$ is $p - \omega(p) - \tilde{c}$, the buyer's net utility is $\tilde{v} - p$, and both traders leave the market. Throughout the analysis of the dynamic game, we focus on an equilibrium where the population of intermediaries choose time invariant mechanisms, though we do not restrict the individual intermediary to choose a time invariant mechanism.[37]

We first determine the equilibrium strategies of buyers and sellers, taking the mechanism(s) intermediaries use as fixed. Following a similar logic as Satterthwaite and Shneyerov (2007) we first consider the discounted utility of a buyer with static valuation $\tilde{v}$ who cannot commit to reject an offer below his dynamic valuation v

$$W_B(\tilde{v}, v) = \rho_B(v)(\tilde{v} - D_B(v)) + (1 - \rho_B(v))\delta W_B(\tilde{v}, v), \tag{4}$$

where $\rho_B(v)$ is the probability of trade implied by the mechanism chosen by the intermediaries and $D_B(v) := E_c[P(c)|P(c) \leq v]$ is the buyer's expected payment. Rearranging yields $W_B(\tilde{v}, v) = (\tilde{v} - D_B(v))P_B(v)$, where

$$P_B(v) := \frac{\rho_B(v)}{1 - (1 - \rho_B(v))\delta}$$

is "the discounted ultimate probability of trade" in Satterthwaite and Shneyerov (2007)'s terminology.

Assuming that the buyer plays a steady state strategy (i.e. the maximal price v that he is willing to accept is the same in each period), his "interim utility" is

$$W_B(\tilde{v}) = \sup_v (\tilde{v} - D_B(v))P_B(v) = (\tilde{v} - D_B(B(\tilde{v})))P_B(B(\tilde{v})).$$

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[37]However, given that all others are using time invariant mechanisms, an individual intermediary's best response mechanism will be time invariant as well.

By the same logic as Satterthwaite and Shneyerov (2007)'s Lemma 3 (i.e. using Milgrom and Segal (2002)'s generalized versions of the envelope theorem)

$$W_B(\tilde{v}) = W_B(\underline{\tilde{v}}) + \int_{\underline{\tilde{v}}}^{\tilde{v}} P_B(B(x))dx.$$

$W_B(\underline{\tilde{v}})$ turns out to be zero, since the lowest valuation buyer is just indifferent between participating and not. A buyer will accept an offer if the price is below his dynamic valuation

$$v = B(\tilde{v}) = \tilde{v} - \delta W_B(\tilde{v}).$$

Combining this with the previous result we get

$$B(\tilde{v}) = \tilde{v} - \delta \int_{\underline{\tilde{v}}}^{\tilde{v}} P_B(B(x))dx, \tag{5}$$

and the differential equation

$$B'(\tilde{v}) = 1 - \delta P_B(B(\tilde{v})). \tag{6}$$

Note that the dynamic valuation approaches the static valuation (i.e. $B(\tilde{v}) \to \tilde{v}$) as the waiting times between trading opportunities become infinitely long (i.e. as $\tau \to \infty$), which implies $\delta = e^{-(\eta+r)\tau} \to 0$ and basically means that the game reduces to the one-shot game analyzed in Section 3.1. Observe also that $B'(\tilde{v}) = (1-\delta)/(1-(1-\rho_B(v))\delta) \in [1 - \delta, 1]$. Therefore, the difference $\tilde{v} - B(\tilde{v})$ between static and dynamic type increases in $\tilde{v}$ for $\delta > 0$.

A similar analysis can be carried out for the seller. For expositional clarity assume for the moment that the intermediary uses the dominant strategy implementation described in Myerson and Satterthwaite (1983).[38] For this mechanism clearly the same logic applies for the seller as for the buyer: he accepts any offer that is above his dynamic costs. By the same procedure we get

$$S(\tilde{c}) = \tilde{c} + \delta W_S(\tilde{c}), \tag{7}$$

where $W_S(\tilde{c}) = \int_{\tilde{c}}^{\overline{\tilde{c}}} P_S(S(x))dx$, and

$$S'(\tilde{c}) = 1 - \delta P_S(S(\tilde{c})), \tag{8}$$

---

[38]That is, there is trade iff $\Phi(v) \geq \Gamma(c)$, in case of trade the buyer pays $\Phi^{-1}(\Gamma(c))$ and the seller gets $\Gamma^{-1}(\Phi(v)))$

where $P_S(c) := \rho_S(c)/(1 - (1 - \rho_S(c))\delta)$ is the ultimate discounted probability of trade for a seller. Observe that eq. (8) implies that the difference between dynamic and static type, $S(\tilde{c}) - \tilde{c}$, is increasing in $\tilde{c}$ for $\delta > 0$.[39]

Every intermediary has measure zero and takes the distribution F and G as exogenously given. Therefore, the probabilities of trade will be given by the intermediary optimal allocation rule (applied to the dynamic types):

$$\rho_B(v) = G(\Gamma^{-1}(\Phi(v))), \tag{9}$$

$$\rho_S(c) = 1 - F(\Phi^{-1}(\Gamma(c))). \tag{10}$$

Equations (6), (8), (9), (10), $F(v) = \tilde{F}(B^{-1}(v))$, and $G(c) = \tilde{G}(S^{-1}(c))$ characterize the dynamic types $v = B(\tilde{v})$, $c = S(\tilde{c})$ and their distributions F and G for any given steady state distributions $\tilde{F}$ and $\tilde{G}$. The relation of these distributions to the distributions of the entrants $\tilde{F}_0$ and $\tilde{G}_0$ is determined via eqs. (1) and (2).

In principle, this already allows us to derive the optimal fee function ω, by obtaining F and G from the underlying steady state static distributions and substituting into (1). This allows us to state the following corollary to Proposition 1:

Corollary 3. *For any strategies chosen by other intermediaries, choosing a fee setting mechanism is a best response for an intermediary. The optimal fee is given by $\omega(p)$ in Proposition 1 where F and G are the endogenous dynamic steady state distributions. Consequently, if there is a market equilibrium in any strategies, there is also an equilibrium with fee setting mechanisms.*

4   Steady State Comparative Statics

We now turn to a comparative statics analysis of the steady state equilibrium.

4.1   Model with Closed Form Solution

We now present a model that permits a closed form solution. We first assume that the steady state distributions of dynamic types $F(v)$ and $G(c)$ are uniform on $[1/2, 1]$ and

---

[39]If one interprets B and S as endogenous reservation prices, then the fact that $\tilde{v} - B(\tilde{v})$ and $S(\tilde{c}) - \tilde{c}$ are increasing functions obtains in many other models as well, including e.g. Gehrig (1993) and Rust and Hall (2003).

[0,1/2], respectively. That is, $F(v) = 2v - 1$ for $v \in [1/2, 1]$ and $G(c) = 2c$ for $c \in [0, 1/2]$. We then derive the conditions on the distributions of entrant types under which these assumptions are indeed correct. Since the dynamic types $v = B(\tilde{v})$ and $c = S(\tilde{c})$ are monotonically increasing functions of the static types, the distributions $\tilde{F}(\tilde{v})$ and $\tilde{G}(\tilde{c})$ can be inferred using $\tilde{F}(\tilde{v}) = F(B(\tilde{v}))$ and $\tilde{G}(\tilde{c}) = G(S(\tilde{c}))$.

Notice that the virtual valuation is $\Phi(v) = 2v - 1$ and the virtual cost is $\Gamma(c) = 2c$. The assumption on the supports implies $\Gamma(0) = \Phi(1/2)$ and $\Gamma(1/2) = \Phi(1)$. Observe also that under these conditions Corollary 3 implies that the equilibrium fee structure is $\omega(p) = p/2$. Equations (9) and (10) yield $\rho_B(v) = 2v - 1$ and $\rho_S(c) = 1 - 2c$. The ultimate probabilities of trade thus become

$$P_B(v) = \frac{2v - 1}{1 - 2\delta(1 - v)} \quad \text{and} \quad P_S(c) = \frac{1 - 2c}{1 - 2c\delta}.$$

This admits the closed form solutions

$$B(\tilde{v}) = \frac{1}{2\delta}\left(2\delta - 1 + \sqrt{(1 - \delta)(1 - 3\delta + 4\tilde{v}\delta)}\right)$$

and

$$S(\tilde{c}) = \frac{1}{2\delta}\left(1 - \sqrt{(1 - \delta)(1 + \delta - 4\tilde{c}\delta)}\right)$$

to the differential equations (6) and (8).

The function $c = S(\tilde{c})$ is plotted for in Fig. 2. It illustrates that the wedge $S(\tilde{c}) - \tilde{c}$ decreases in $\tilde{c}$, so that the sellers with the lowest static costs exhibit the largest differences between dynamic and static costs.

Replacing v by $B(\tilde{v})$ in $F(v)$ and c by $S(\tilde{c})$ in $G(c)$ yields

$$\tilde{F}(\tilde{v}) \quad = \quad \frac{\delta - 1 + \sqrt{(1 - \delta)(1 - 3\delta + 4\delta\tilde{v})}}{\delta} \tag{11}$$

$$\tilde{G}(\tilde{c}) \quad = \quad \frac{1 - \sqrt{(1 - \delta)(1 + \delta - 4\tilde{c}\delta)}}{\delta}. \tag{12}$$

The lower and upper bound of the support of the buyers' static types is found by respectively solving $\tilde{F}(\underline{\tilde{v}}) = 0$ and $\tilde{F}(\overline{\tilde{v}}) = 1$, yielding $\underline{\tilde{v}} = 1/2$ and $\overline{\tilde{v}} = (4 - 3\delta)/(4(1 - \delta))$. Similarly, the lower and upper bound of the static seller type support is found by equating $\tilde{G}(\underline{\tilde{c}}) = 0$ and $\tilde{G}(\overline{\tilde{c}}) = 1$, yielding $\underline{\tilde{c}} = -\delta/(4(1 - \delta))$ and $\overline{\tilde{c}} = 1/2$. On their supports,
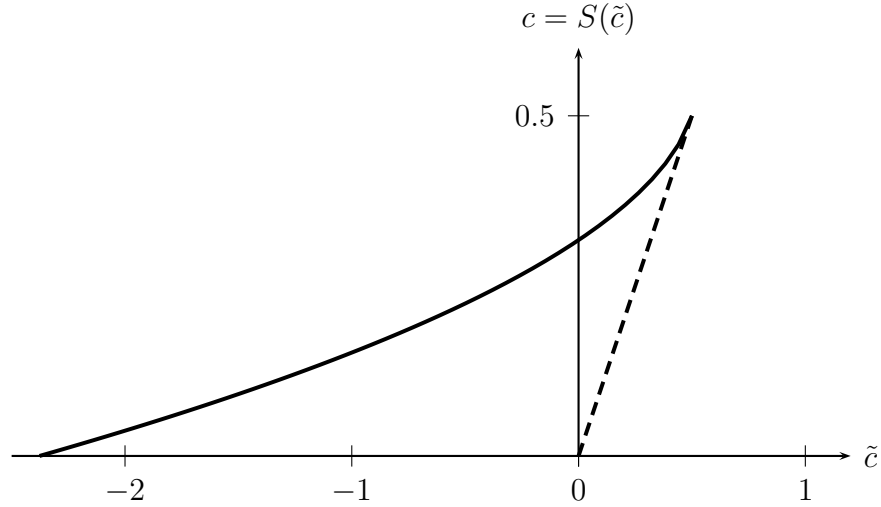
Figure 2: $S(\tilde{c})$ for $\eta = r = 0.1$ and $\tau = 1$. For $\tau = \infty$, $S(\tilde{c})$ is equal to the 45-degree line (dashed line).

the densities of the static types are

$$
\begin{aligned}
\tilde{f}(\tilde{v}) &= \frac{2(1-\delta)}{\sqrt{(1-\delta)(1-3\delta+4\delta\tilde{v})}} \\
\tilde{g}(\tilde{c}) &= \frac{2(1-\delta)}{\sqrt{(1-\delta)(1+\delta-4\tilde{c}\delta)}}.
\end{aligned}
$$

We are left to determine the static types distribution of entrants. We first make the following observations, which are true irrespective of the specific steady state distributions. Inspection of eq. (2) reveals that for every entering seller with dynamic type c there are $[\sigma(1-(1-\rho_S(c))e^{-\eta\tau})]^{-1}$ sellers with the same c in the market. Since $\rho_S(c)$ decreases in c, this is increasing in c, implying that sellers with *higher* dynamic costs accumulate more in the market. Similarly, eq. (1) implies that for every entering buyer of type v there are $[\sigma(1-(1-\rho_B(v))e^{-\eta\tau})]^{-1}$ buyers of the same type in the market. Since $\rho_B(v)$ increases in v, this mass is decreasing in v. Thus, buyers with *lower* dynamic types accumulate in the market. Both of these observations are very intuitive, of course, since buyers with lower valuations and sellers with higher costs have a lower probability of trading in any given period.

When the steady state distributions of traders in the market are uniform, substituting $S(\tilde{c})$ for c in (2) gives $\tilde{g}_0(\tilde{c}) = \sigma(1-(1-\rho_S(S(\tilde{c})))e^{-\eta\tau})\tilde{g}(\tilde{c})$. For $G(c)$ and $F(v)$ uniform

on $[0, 1/2]$ and $[1/2, 1]$, respectively, equation (2) simplifies to $g_0(c) = 2\sigma(1 - 2ce^{-\eta\tau})$. Since $\tilde{g}_0(\tilde{c}) = g_0(S(\tilde{c}))S'(\tilde{c})$, this yields

$$\tilde{g}_0(\tilde{c}) = 2\,\sigma\,\frac{1-\delta}{\delta}e^{-\eta\tau}\left(1 - \frac{1 - e^{-r\tau}}{\sqrt{(1-\delta)\,(1+\delta-4\tilde{c}\delta)}}\right). \tag{13}$$

Observe that $\tilde{g}_0(\tilde{c})$ decreases in $\tilde{c}$. Since the steady state distribution of seller types in the market is uniform, this means that less efficient sellers accumulate in the market in the sense that $\tilde{g}(\tilde{c}_1)/\tilde{g}_0(\tilde{c}_1) > \tilde{g}(\tilde{c}_2)/\tilde{g}_0(\tilde{c}_2) \Leftrightarrow \tilde{c}_2 < \tilde{c}_1$. Fig. 3 illustrates this.

Similarly, for buyers the density of the dynamic types of entrants is $f_0(v) = 2\sigma[1 - 2(1-v)e^{-\eta\tau}]$. Using the fact that $\tilde{f}_0(\tilde{v}) = f_0(B(\tilde{v}))B'(\tilde{v})$ we get

$$\tilde{f}_0(\tilde{v}) = 2\sigma\frac{1-\delta}{\delta}e^{-\eta\tau}\left[1 - \frac{e^{-r\tau}}{\sqrt{(1-\delta)(1-3\delta+4\delta\tilde{v})}}\right]. \tag{14}$$

Notice that $\tilde{f}_0(\tilde{v})$ increases in $\tilde{v}$. Since the steady state distribution of buyer types is uniform, this implies that buyer with lower static valuations tend to accumulate in the market insofar as $\tilde{f}(\tilde{v}_1)/\tilde{f}_0(\tilde{v}_1) > \tilde{f}(\tilde{v}_2)/\tilde{f}_0(\tilde{v}_2) \Leftrightarrow \tilde{v}_1 < \tilde{v}_2$.

We summarize these findings in the following proposition:

Proposition 4. *Assume that $\tilde{c}$ and $\tilde{v}$ have support $[-\delta/(4(1 - \delta)), 1/2]$ and $[1/2, (4 - 3\delta)/(4(1 - \delta))]$, respectively, and let the densities of entrant types be given by $\tilde{g}_0(\tilde{c})$ and $\tilde{f}_0(\tilde{v})$ as in eq. (13) and (14). Then the steady state distributions of static types are given by eqs. (11) and (12), and the distribution of dynamic types are uniform with $v \in [1/2, 1]$ and $c \in [0, 1/2]$, and the equilibrium fee of every intermediary is $\omega(p) = p/2$.*

Fig. 3 shows how $\tilde{g}(c)$ relates to $g(c)$. For $\delta = \epsilon = 0$, $\tilde{g}$ and g coincide.

4.2   First Order Effects – Perturbation Analysis

Even though an analytical solution cannot be found in general, we can describe the effects of infinitesimally small perturbations of an analytically tractable solution.
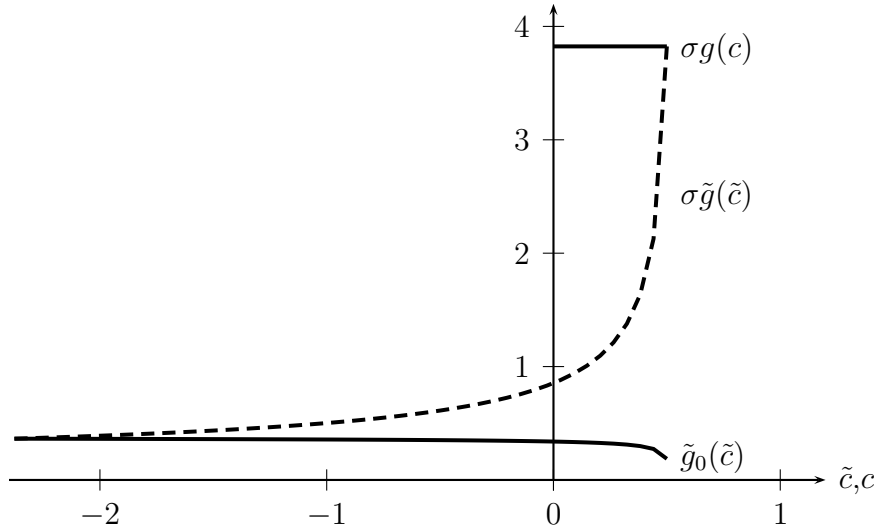
Figure 3: $\tilde{g}_0(\tilde{c})$, $\tilde{g}(\tilde{c})$, and $g(c)$ for $\eta = r = 0.1$ and $\tau = 1$.

We will start out with a case where we have an analytical solution: the static model. If traders discount the future with factor $\delta = e^{-(r+\eta)\tau} \to 0$ (equivalent to $r \to \infty$) and their probability of staying in the market $\epsilon := e^{-\eta\tau} \to 0$ (equivalent to $\eta \to \infty$), the solution of the dynamic game trivially coincides with the static game, we have $c = S(\tilde{c}) = \tilde{c}$ and $g = \tilde{g} = \tilde{g}_0$. As a next step we increase the probability of staying in the market ϵ infinitesimally, but keep the discount rate δ constant. This has the effect that static types and dynamic types still coincide, but entrant and steady state distributions become different since sellers accumulate. We will also assume that the change of ϵ only affects G and not F, because only the drop out rate of sellers changes. Another possible reason is that we are close to a power distribution, so that a change of F hardly has any effect.

The following analysis uses perturbation analysis to perturb a function infinitesimally and look at the first order effect of this perturbation on the system of differential equations. As long as second order effects are sufficiently small, this is a good approximation of the exact solution.

To simplify the exposition, we will denote in this subsection the entrant static type distributions F and G instead of $\tilde{F}_0$ and $\tilde{G}_0$; the distributions of the steady state dynamic types (which coincide with the static types) will be denoted by $\hat{F}$ and $\hat{G}$.

Now recall from (2)

$$\sigma(1 - (1 - \rho_S(c))\epsilon)\hat{g}(c) = g(c), \tag{15}$$

where $\rho_S(c) = 1 - F(\Phi^{-1}(\Gamma(c)))$ and σ is a constant such that the density function $\hat{g}$ adds up to one. In the following, we want to have a function γ that infinitesimally perturbs $\hat{g}$

$$\hat{g}(c) = (1 + \epsilon\gamma(c))g(c)$$

and ensures that $\hat{g}$ adds up to one, i.e. $\int_{\underline{c}}^{\overline{c}} \gamma g = 0$.

Increasing ϵ infinitesimally has the following first order effects.

Proposition 5. *The first-order effects of an increase of ϵ from $\epsilon = 0$ are the following:*

 (i) $(\ln \hat{G})'$ increases,

 (ii) $\hat{\Gamma}$ decreases and $\hat{\Gamma}^{-1}$ increases,

 (iii) the sign of the change of $(\hat{\Gamma}^{-1})'$ is ambiguous.

Since $\hat{G}$ enters $\hat{\omega}$ only through $\hat{\Gamma}^{-1}$ and $\hat{\omega}'$ only through $(\hat{\Gamma}^{-1})'$ (see equation (1) and the discussion after Proposition 1), this leads us immediately to the following Corollary.

Corollary 4. *As the waiting time between rematchings decreases (starting from an infinite waiting time and considering first-order effects)*

 (i) the overall fee $\omega(p)$ becomes lower,

 (ii) the sign of the change of the marginal fee $\omega'(p)$ is ambiguous.

That the overall fee is lower with more frequent rematching could be suspected by intuition for two reasons. First, a more frequent rematching resembles more competition and should therefore drive down fees. Second, we have seen for the case of uniform dynamic cost distributions that more frequent rematching makes the seller's cost distribution more convex, which corresponds for a power distribution to β increasing. A higher β means a lower fee. But a formal derivation of the first order effects shows that the intuition does not carry over further. An increase of β for a power distribution

also means a lower marginal fee. However, the first order effect for the marginal fee is ambiguous.

The role private information plays in our model is also worth a brief discussion in this context. If the valuations and costs of buyers and sellers were commonly known, then each intermediary would optimally leave zero net utility to each of them. Therefore, like in Diamond's model reducing the search friction, which in our setup corresponds to shortening the period length, would have no effect on equilibrium fees unless the search friction can be abolished completely.[40] With private information, however, shortening the period length does have a discernible effect on the equilibrium fee structure. Note further that in the dynamic setup, private information may be welfare improving, whereas in the static setup it is clearly welfare reducing. The reason is that with v and c being public knowledge, more trade happens than in the Walrasian equilibrium, hence reduction of trade with private information may be welfare enhancing.[41]

In Proposition 8 below we shall see that the present model's comparative statics with respect to the number of intermediaries are similar to those of a complete information model.

4.3   Numerical Results

We have shown that if the seller's cost is distributed according to a power distribution, linear fees are optimal for the intermediary. This of course raises the question whether, and why, one should observe power distributions empirically.

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[40]A simple illustration of the logic of the so called Diamond paradox (see e.g. Anderson and Renault, 1999) is to assume that, say, a buyer searches sequentially and discounts future payoffs with the factor $\delta \in (0, 1)$. Let $U_B(\tilde{v})$ be the payoff he gets when buying from the intermediary he is currently matched to. Since he has the option of continuing search, $U_B(\tilde{v}) = \delta U_B(\tilde{v})$ has to hold. But since $\delta < 1$ this implies $U_B(\tilde{v}) = 0$. With incomplete information, the buyer enjoys an informational rent of $\tilde{v} - D_B(v)$ as given in (4). Notice in particular that if it were the case that $\tilde{v} - D_B(v) = 0$, then $W_B(\tilde{v}, v) = 0$ would follow for exactly the same reason as in Diamond (1971).

[41]In the Walrasian equilibrium, buyers and sellers with $\tilde{v} \geq p^*$ and $\tilde{c} \leq p^*$ trade, where the Walrasian price is given by $1 - \tilde{F}_0(p^*) = \tilde{G}_0(p^*)$. With public information, a trader accepts the first offer he gets from an intermediary (giving him zero utility), since his option value of continuing is zero. Hence, all buyers and sellers with $v, c \in [\min\{\underline{\tilde{c}}, \underline{\tilde{v}}\}, \min\{\overline{\tilde{c}}, \overline{\tilde{v}}\}]$ trade, which is more than the Walrasian quantity. Whether a reduction of trade by private information increases or decreases welfare, depends on the specific distribution. See also Satterthwaite and Shneyerov (2007, 2008), Lauermann (2007), and Lauermann and Wolinsky (2008) for comparisons of effects arising in incomplete and complete information models with dynamic matching. We thank Stephan Lauermann for pointing this out to us.

Looking at the dynamics of the model leads to an intuitive explanation for power distributions. Sellers with high costs offer their goods at higher prices, hence it takes them longer to sell their goods and they cumulate more in the market. This changes the initial distribution to one with more weight on the right end and less on the left. Loosely speaking, this makes the distribution more similar to a power distribution (with $\beta > 1$ in $G(c) = c^\beta$). For distributions other than power distributions closed form solutions cannot be obtained for the system of differential equations that determine the market equilibrium. However, as we show in the following, numerical calculations confirm the intuitive explanation.

We approximate the functions in the system of differential equations with sixth degree Chebyshev polynomials and solve the system numerically. We will show that the more dynamic a market (i.e. the higher the rematching frequency) is, the "closer" the seller's distribution to a power distribution. We define "closeness" as the ratio of profits with the best linear fee to profits with the optimal fee function $\Pi_{\text{linear}}/\Pi_{\text{optimal}}$ with

$$\Pi_{\text{optimal}} = \int \int \max\{\Phi(v) - \Gamma(c), 0\} f(v) g(c) dv dc$$

and

$$\Pi_{\text{linear}} = \max_{\xi, \zeta} \int \left(\xi \Phi^{-1}\left(\frac{c + \zeta}{1 - \xi}\right) + \zeta \right) \left(1 - F\left(\Phi^{-1}\left(\frac{c + \zeta}{1 - \xi}\right)\right) \right) g(c) dc.$$

The expression for Π_{linear} stems from the fact that a seller facing a percentage fee ξ and a fixed fee ζ will set the price $p = \Phi^{-1}\left(\frac{c + \zeta}{1 - \xi}\right)$, which generates expected profits $(\xi p + \zeta)(1 - F(p))$ for the intermediary.

Since convex distributions are already more similar to power distributions and hence favorable for linear fees, we will take a distribution which works against linear fees as a numerical example, the concave distribution $\tilde{g}_0(\tilde{c}) = 6\tilde{c}(1 - \tilde{c})$ (and $\tilde{f}_0(\tilde{v}) = 6\tilde{v}(1 - \tilde{v})$), see the dotted lines in Fig. 4.[42]

Fig. 4 depicts the exogenous and endogenous distributions as the rematching frequency (and hence also δ and ϵ) increases. It illustrates graphically that as the rematching frequency increases the steady-state dynamic distribution looks "more similar" to a

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[42]To be precise, we take $\tilde{g}_0(x) = \tilde{f}_0(x) = [6x(1 - x) + 0.1]/1.1$ to avoid numerical problems with division by zero at $x = 0$ and 1.

power distribution. Fig. 5 shows that our measure of closeness to a power distribution increases indeed as the rematching frequency increases, i.e. for more dynamic markets linear fees do better. Actually, one can see that already for the static market, linear fees achieve more than 99.8% of what is obtainable with the optimal mechanism.
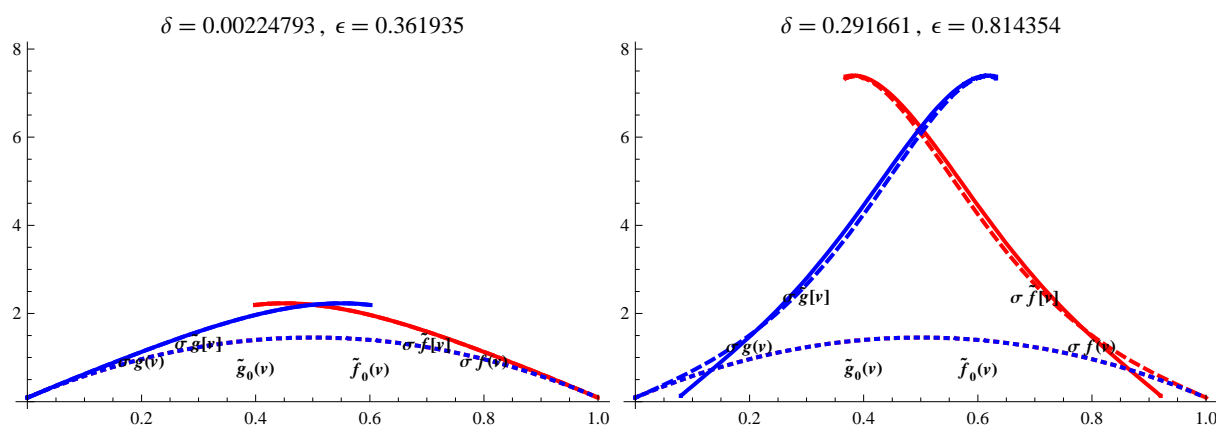
Figure 4: Exogenous (dotted: entrant static) and endogenous (dashed: steady-state static, solid: steady-state dynamic) densities with a low (left) and a high (right) rematching frequency. For the lowest rematching frequency ($\epsilon = \delta = 0$) we are back to the static model and the endogenous distributions coincide with the exogenous ones (dotted).

Figure 5: Ratio of best linear to optimal profits as the rematching frequency increases. The horizontal axis is the probability of remaining in the market ϵ with $\epsilon = e^{-0.1\tau}$, the discount rate δ is given by $\delta = e^{-0.2\tau}$.

5 Real Estate Brokers and Other Applications

We first discuss how our model can account for the stylized facts of real estate brokerage that we listed in the Introduction.

5.1 Stylized Facts for Real Estate Brokerage Revisited

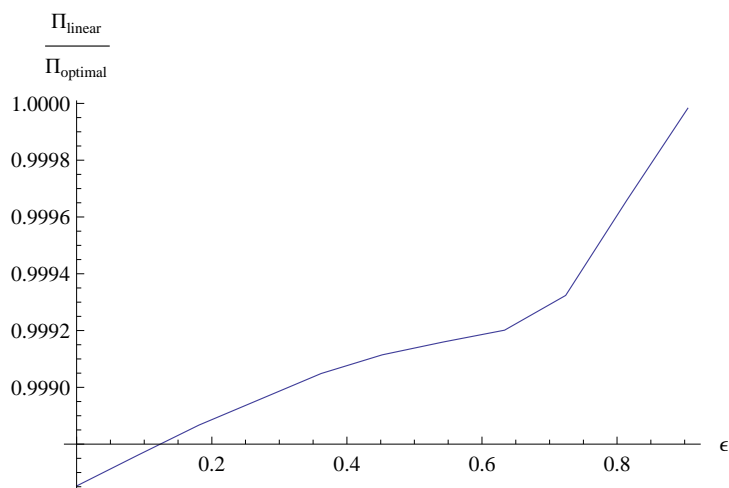We have partially dealt with the first stylized fact (6% fee) in the previous sections and will say more about it in the next subsection. The second (higher loan-to-value ratio leads to higher price) and the third (house originally bought during boom offered at higher price) are relatively easy to explain, the others will be dealt with later on. The second fact can be interpreted as the loan-to-value ratio being a proxy for the seller's cost. Note that in a Walrasian equilibrium, by the law of one price, the price of a house must only depend on its characteristics and not on the seller's preferences. Therefore, this fact is inconsistent with the law of one price. However, it is consistent with a setup with search frictions and incomplete information, since this leads to price dispersion. The third fact can, again, be explained by price dispersion of houses of the same quality. During a boom only buyers from the upper quantiles of the valuation distribution buy, for low valuation buyers prices are too high. When average prices are low, also buyers from the lower quantiles buy. Assuming that individuals that were high valuation buyers when they bought their houses are more likely to be high cost sellers when they sell later, we would expect the effect described in Genesove and Mayer (2001): buyers who bought during a boom ask for higher prices.[43,44]

In the following, besides giving explanations for the other stylized facts, we will show

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[43]One may worry that this argument relies on the valuations of buyers and sellers being differently distributed, even though in real estate markets most buyers are also sellers and vice versa. Note, however, that the explanation relies only on the *dynamic steady-state* distributions to be different for buyers and sellers (see solid line in Fig. **??**), the fundamentals, i.e. the *entrant static type* distributions (dotted line) can be the same for buyers and sellers. The transformation from the dotted to the solid line breaks the original symmetry for two reasons. First, low valuation buyers remain longer in the market, whereas for sellers it is the ones with high costs. Second, the option value of future trade has to be subtracted for a buyer, but added for a seller.

[44]Note that Genesove and Mayer (2001) observe more than the above-mentioned statement: the effect of the previous transaction price of a house on the current asking price depends on whether the price level has increased or decreased. Whether this can be explained in a setup similar to ours is a question for further research.

the empirical implications of our model. Of course, an empirical analysis would have to incorporate the specificities of the housing market.

Linear and Invariant Fees   As optimality of linear fees implies invariance of the fees with respect to the buyer's distribution, the empirical prediction of Proposition 2 is that whenever profit maximizing intermediaries choose linear fee setting as a mechanism, these fees will be invariant.

Of course, this raises the question whether the seller's distribution should vary if the buyer's varies as well. A first part of the answer is the upper part of the seller's cost distribution $[P^{-1}(\bar{v}), \bar{c}]$, i.e. those sellers who for sure cannot sell, is irrelevant for the intermediation problem at hand. Therefore, Proposition 2 means that a linear fee only implies a generalized power distribution in the relevant range $[\underline{c}, P^{-1}(\bar{v})]$. Above this range, G can have any shape, provided its virtual cost function is increasing.[45] Note also that the relevant range $[\underline{c}, P^{-1}(\overline{v})]$ can be say $[0; \$100,000]$ in the countryside and $[0; \$1,000,000]$ in a big city, but they both lead to the same fee if they have the same shape in this range.

Corollary 1 and the empirical prediction thus hold not only when the cost distribution is the same over time and across regions, but even if it has only the same shape in the relevant range. This is illustrated in Fig. 6 that shows two different distributions of the seller's cost that lead to the same fee.

The second part of the answer relies on how the distributions of steady state dynamic types are related to the distributions of entrant static types in the dynamic game with random matching. The dynamics of the market lets sellers with high costs cumulate in the market and drive their distribution towards a power distribution. While this is yet to be shown analytically, we already have numerical results pointing in this direction.

The widespread use of linear fees raises the question whether linear fees may perform well even when the seller's distribution does not exhibit linear virtual costs. Though a complete analysis of the performance of linear fees in such environments is beyond

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[45]For many markets, it is reasonable to assume that most sellers are above the relevant range, so that the sellers with a positive probability of trade are just the "tip of the iceberg". E.g. most house owners prefer staying in their houses rather than offer them for sale.
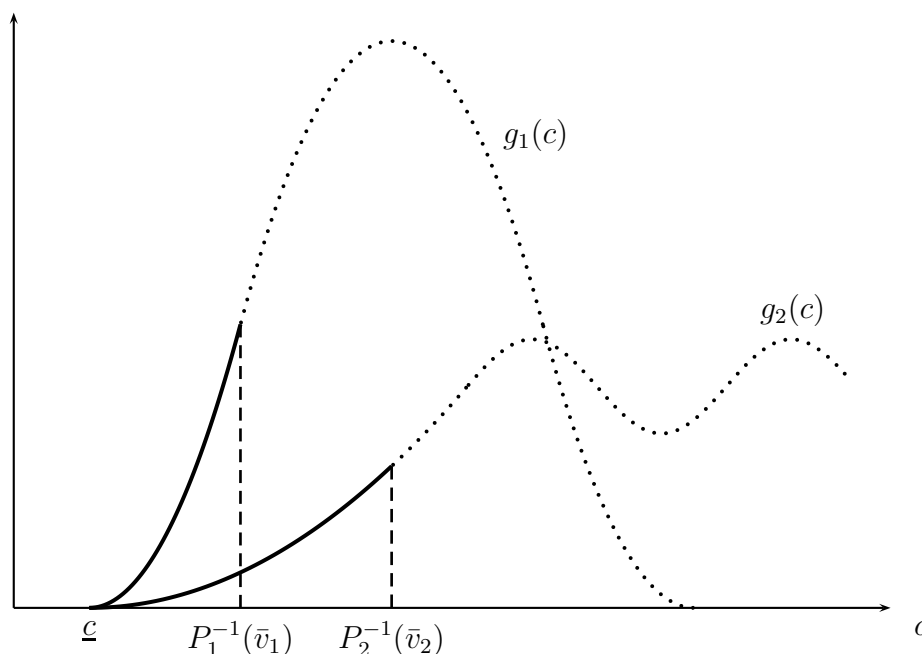
Figure 6: Two distributions of the seller's cost c leading to the same fee. The densities g_1 and g_2 have the same shape in their respective relevant ranges $[\underline{c}, P_1^{-1}(\bar{v}_1)]$ and $[\underline{c}, P_2^{-1}(\bar{v}_1)]$.

the scope of the present paper, we provide some numerical examples in Appendix B that suggest that linear fees are close to optimal for other distributions than power distributions. Here we restrict ourselves to the example $g(c) = 6c(1 - c)$ and $f(v) = 1$ as depicted is Fig. 7. Even though g is far from a power distribution, choosing a linear fee (equivalent to acting as if though having a power distribution $G(c) = c^{(1-\xi)/\xi}$ with $\xi = 0.4$) gives the intermediary 99.8 percent of the profits he would get with the optimal mechanism. Similar results were found for other distributions in Appendix B. This gives rise to the conjecture that power distributions are a useful approximation in many cases, even if the seller's distribution is of a different kind.[46]

Quality Adjusted Price and Time On Market   Thus far, we maintained the assumption that objects (say, houses) are homogeneous, which we now relax. We assume

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[46]One may further conjecture that the distributions for which linear fees are closer to optimality are also the ones that are more invariant to changes of the buyer's distribution. But these conjectures, of course, remain to be shown.
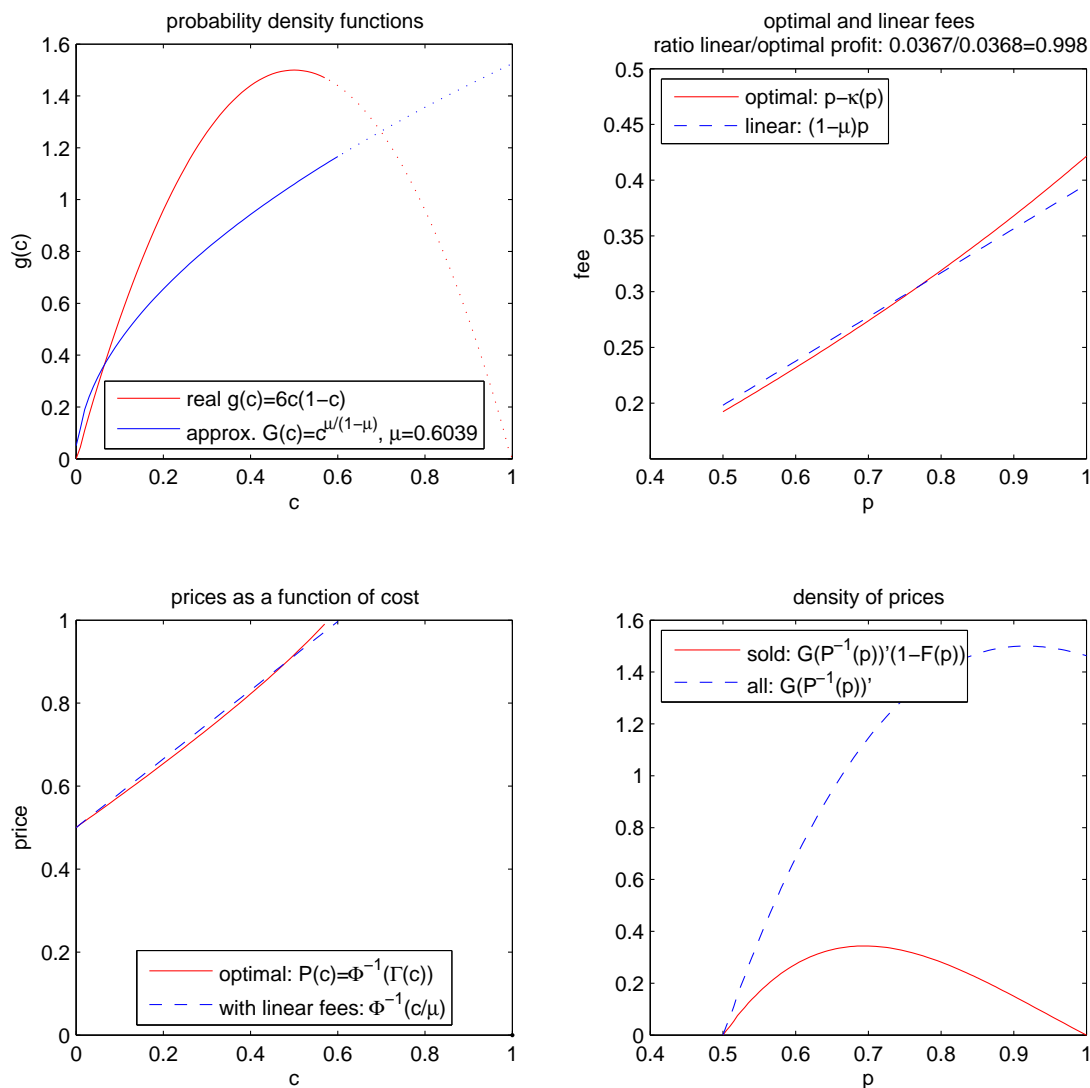
Figure 7: Optimal and linear fee for $g(c) = 6c(1 - c)$ and $f(v) = 1$.

that everyone agrees that the objective value of a certain house is θ and that an individual trader's valuation for the house is θt, where $t = v$ for a buyer and $t = c$ for a seller. Empirically observed prices $\hat{p}$ are given by $\hat{P}(c) = \theta P(c)$, where $P(c)$ is the quality adjusted price. Rutherford, Springer, and Yavas (2005) define the degree of overpricing (DOP) as DOP $= (\hat{P}(c) - \theta)/\theta$. The quality adjusted price is thus $P(c) = $ DOP $+ 1$. The price $P(c)$ in our model can be interpreted as a quality adjusted price.[47] Therefore, our model can be interpreted e.g. as consisting of several separate submarkets that differ only in their θ's, which are publicly observable.

There is a large empirical literature addressing the relation between the quality adjusted listing price and the time on market.[48] We now derive predictions on the relation between the quality adjusted price and the time a house is on the market before being taken off the list, which happens either because it is sold or because the seller leaves the market without selling. Interestingly, the baseline model predicts that the average time on market is the same for sold and unsold houses. The continuous time approximation of the discrete time geometric distribution is described in Proposition 6.[49]

Proposition 6 (Homogeneous Market). *For homogeneous houses the time on market of sold and unsold houses has the same distribution. The continuous time approximation of this distribution is exponential with the cumulative distribution function $1-\exp(-(\phi(p)+\eta)t)$ and mean*

$$T(p) = \frac{1}{\phi(p) + \eta}, \tag{16}$$

where ϕ is such that $e^{-\phi(p)\tau} = F(p)$. For a price p the ratio of houses ever sold, denoted $1 - F_\infty(p)$, is

$$1 - F_\infty(p) = \frac{\phi(p)}{\phi(p) + \eta}. \tag{17}$$

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[47]Since Rutherford, Springer, and Yavas (2005) estimate DOP as the average listing price a house with certain characteristics has, this means that DOP has mean 0. This corresponds to a quality adjusted price normalized to have mean 1.

[48]See e.g. Hendel, Nevo, and Ortalo-Magné (forthcoming), Rutherford, Springer, and Yavas (2005), Genesove and Mayer (1997, 2001), Kang and Gardner (1989), Yavas and Yang (1995) and Larsen and Park (1989).

[49]We provide the continuous time version of the distribution since it is more convenient for empirical purposes. The discrete time version is derived in the formal proof in the Appendix.

Consistent with our model the empirical literature (see e.g. Hendel, Nevo, and Ortalo-Magné, forthcoming) finds that the quality adjusted listing price and the time on market are positively correlated in cross-sectional data. One can also easily find an explanation in our framework for the negative correlation observed in longitudinal data.[50] In theory, observing the relation between listing price p and the time on market $T(p)$ on the one hand and the ratio of houses sold, $1 - F_{\infty}(p)$, on the other hand would be sufficient to estimate the steady state dynamic distribution F. The most straightforward approach is solving (16) and (17) for $\phi(p)$ and η which results in $\phi(p) = (1 - F_{\infty}(p))/T(p)$ and $\eta = F_{\infty}(p)/T(p)$.[51]

The following numerical example, depicted in Fig. 8, illustrates the predictions our model makes for dynamic type distributions $F(v) = v$ and $G(c) = c^{16}$. Note that the two subfigures on the left are our assumptions on F and G, the other four subfigures represent our predictions of the empirically observable functions.

Our results concerning the average time on market of sold and unsold houses can be used in two ways. If one does have data on the time on market of unsold houses, a comparison of these two variables can be used as an indicator of how well one has corrected for heterogeneity of house characteristics (see next paragraph). If one does not have this data for unsold houses, using the time on market of sold houses is enough for the analysis so far.

Consider now the case where a data set includes heterogeneous submarkets and heterogeneity is not controlled for.

Proposition 7 (Heterogeneous Submarkets). *If heterogeneous submarkets are in the observed sample, the time on market is lower for sold than unsold houses.*

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[50]I.e. during a boom period houses are sold faster and at higher prices than during a recession. In our framework this means that F and G change in booms such that both the average price $\int_{\underline{c}}^{P^{-1}(\bar{v})} P(c)dG(c)$ increases and the overall average time on market $\int_{\underline{c}}^{P^{-1}(\bar{v})} T(P(c))dG(c)$ decreases.

[51]An empirical analysis would of course have to overcome further issues not present in our theoretical framework.Some could be dealt with by straightforward extensions of our framework or standard econometric tools, such as seasonal differences, the fact that it takes some (random) time until advertisements for a house appear and potential buyers first view the house (this leads to the time on market not being exponentially distributed, but having a peak for some $t > 0$), or truncation of data. Others will require further thought.
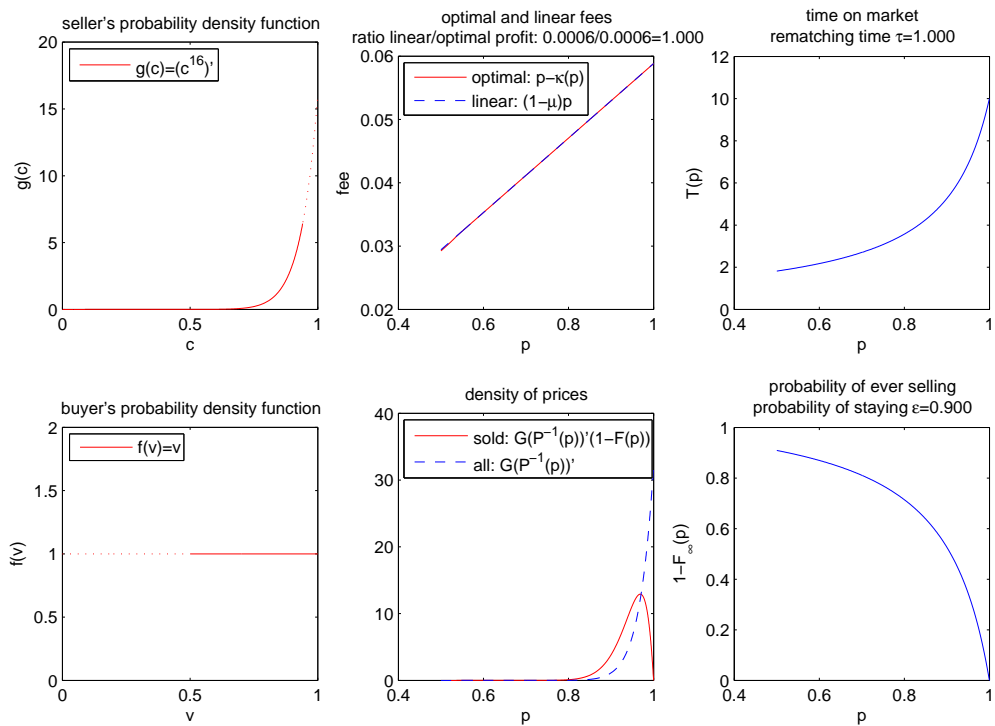
Figure 8: Predicted fee $\omega(p)$, density of prices, time on market, and probability of ever selling for $F(v) = v$ and $G(c) = c^{16}$.

Intuitively, the result stems from the fact that houses that have a higher probability of selling at the same price have a shorter time on market and are also relatively overrepresented in the set of sold houses. This result is consistent with Larsen and Park (1989)'s observation that failing to include unsold houses may lead to a bias in the estimation of time on market. Our analysis suggests that such a bias stems from heterogeneity. Prop. 7 naturally carries over to the case where times on market are estimated as averages over all prices rather than for a specific price.

Inefficient Free Entry by Intermediaries   Hsieh and Moretti (2003) perform an empirical analysis of entry into real estate brokerage. Making the (empirically validated) assumptions that real estate brokers use a fee setting mechanism and that the fee is 6 percent independently of the number of intermediaries in the industry, Hsieh and Moretti demonstrate that inefficiently many agents enter in equilibrium.[52] In particular, they find that the number of transactions per intermediary and year decreases during real estate booms and that booms are associated with significant new entry: the number of intermediaries increases in proportion to the overall profits made in real estate brokerage.

We now show that a simple extension of our dynamic model provides a concise explanation for both the findings of Hsieh and Moretti (2003) and for their assumptions. Let $\kappa > 0$ be an individual's per period opportunity cost of being a real estate broker. Assuming that there is free entry into the industry, intermediaries will enter until expected per period profits equal κ. Denote the mass of active intermediaries under free entry by ι^*.

Proposition 8. *If under free entry there are sufficiently many intermediaries to match every seller and every buyer, i.e. if $\iota^* > \sigma$, then the equilibrium mechanism is independent of the opportunity cost of entry κ and, consequently, independent of the number of active intermediaries ι^*.*

In the dynamic game we maintained the assumption that $\iota > \sigma$, which is obviously the analog to the assumption $\iota^* > \sigma$ under free entry. The proposition provides a simple

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[52]For an earlier theoretical work on the potential inefficiency of free entry conditions, see Salop (1979) and Mankiw and Whinston (1986).

explanation for the observation that the fees of real estate brokers do not vary with the number of active intermediaries or with profits per transaction (see e.g. Hsieh and Moretti, 2003): The number of active intermediaries only affects the probability of a match for an intermediary, but does not alter the optimal mechanism because it does not affect the probability of a match for a buyer or a seller. Proposition 8 shows that the comparative statics of our model with respect to the number of intermediaries are similar to those of a model with complete information such as Diamond (1971) because the equilibrium mechanisms are invariant with respect to this number in both types of models.[53]

Heterogeneity among brokers can be easily accomodated for. As a first step, consider heterogeneity with respect to the opportunity cost of entry. Index intermediaries with ι such that $\kappa(\iota)$ increases. With excess supply the marginal intermediary ι^* is given by $(\sigma/\iota^*)\Pi = \kappa(\iota^*)$, the mass of active brokers by ι^*, and brokers in the set $I = [0, \iota^*]$ are active. As a second step, introduce heterogeneity with respect to productivity, measured here as the relative matching probability $r(\iota)$. Intermediary i will be matched with probability $\sigma r(i)/R(I)$ with $R(I) := \int_{\iota \in I} r(\iota)d\iota$ in case of $\int_{\iota \in I} d\iota > \sigma$.

For r decreasing, we still have $I = [0, \iota^*]$. For other shapes of $r(\iota)$ one can get e.g. the phenomenon "middle class" doing real estate brokerage, described in Hsieh and Moretti Appendix A: brokers in $I^* = [\iota^*_{\min}, \iota^*_{\max}]$ participate, where $\iota^*_{\min} > 0$ and $\iota^*_{\max} < \infty$ are the marginal indifferent brokers.[54] People with very low skills would not earn enough as brokers, people with very high skills earn more in other jobs.

A rather simple explanation can be found for the empirically observed phenomenon of "star brokers" – a small fraction of brokers who capture a large fraction of transactions: $r(\iota)$ is large for a small mass of active brokers and small for a large mass.

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[53]Recall that in a model à la Diamond (1971) all sellers charge the monopoly price independent of the number of sellers.

[54]For $r' \leq 0$, an increase of Π will raise per period profits of the marginal intermediary i^*, and therefore, average industry profits by even more. Average industry profits can also decrease with per transaction profits as in the following example. Let initial per transaction profits be Π_0. Assume $r(\iota) = 1$ for $\iota < 1$ and $r(\iota) = 2$ for $\iota \geq 1$. Assume $\kappa(\iota) = \sigma\Pi_0/2 + \epsilon$ for $\iota < 2$ with some small ϵ and $\kappa(\iota) = \infty$ else. Initially, only agents in the interval $I^* = [1, 2]$ participate. Per year profits are $\sigma\Pi_0$. An increase to $\Pi_1 = (3/2)\Pi_0 - \epsilon$ results in the entry of low ability agents with $\iota \in [0, 1]$. Average per intermediary per period profits fall to $\sigma(3/4)\Pi_0$.

Observations of the distribution of per year profits among intermediaries would allow to make inferences about r and κ,[55] which would allow predictions about e.g. the effect of the entry test for realtors[56] or of a drop in real estate prices.

Brokers selling their own houses and FSBO   Rutherford, Springer, and Yavas (2005) and Levitt and Syverson (2008) observe that brokers selling their own houses get on average higher prices than independent sellers selling comparable houses. Note that from the perspective of our model, a broker selling his own house and houses for sale by owners (FSBO) are the same.[57] In this sense, similar evidence is provided by Hendel, Nevo, and Ortalo-Magné (forthcoming), who compare prices of houses sold via brokers and houses for sale by owners (FSBO) and find that prices achieved by FSBOs are higher, though not significantly. We call both the broker who owns the house he is selling and the owner who sells without a broker an *integrated seller* while an individual who sells via an intermediary is called an *independent seller*. We now show briefly how a simple extension of our dynamic model can shed new light on the price setting behavior of integrated sellers.

Assume that both independent and integrated sellers enter their respective pools in every period and are matched with a buyer with the same probability. While the equilibrium will be determined by a more complicated model and its derivation will require more details, we can make statements about the behavior of both types of sellers without having to compute the equilibrium. The following analysis is based on the assumption that both kinds of sellers face the same (endogenous) distribution of buyers F. This is e.g. the case if buyers join the FSBO market and the intermediary market

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[55]If the cumulative distribution function H of per year profits among intermediaries is observable, the following reasoning applies. Under the assumption that r is weakly increasing and $I^* = [\iota^*_{\min}, \iota^*_{\max}]$, the inverse of the cumulative distribution function is equal to $H^{-1}(x) = \sigma r((\iota^*_{\max} - \iota^*_{\min})x + \iota^*_{\min})/R(I^*)\Pi$, which should allow to recover r for the domain I^*. For the opportunity cost κ one can only infer that it is weakly less than per year profits in I^* and equal at the boundaries of the interval, $\iota^*_{\min}$ and $\iota^*_{\max}$. A change of I^* over time (e.g. because of a change of Π) would allow for further estimates.

[56]E.g. for r weakly decreasing a test that excludes brokers with the highest opportunity costs would be clearly welfare enhancing, a test that excludes the lowest opportunity cost intermediaries welfare reducing. Otherwise the effect will depend on the specific forms of r and κ.

[57]This is since we abstract away from agency issues and institutional details such as the multiple listings service (MLS), which only brokers can access.

with equal probabilities.[58]

In a static setup, the optimal mechanism of an integrated seller is posting a price (see Myerson (1981)). This integrated seller's profits will exceed the joint profits of an independent seller and an independent intermediary. However, its price $\Phi^{-1}(c)$ will be smaller than the price of an independent seller $\Phi^{-1}(\Gamma(c))$ because of the externality the intermediary's fee imposes, which is analogous to double marginalization. Interestingly, dynamics can reverse this result: The integrated seller will have a higher dynamic cost because his option value of not selling and staying in the market is higher than that for an independent seller with the same static cost $\tilde{c}$. This explanation for the observed price differences for houses sold by sellers via brokers and houses sold by independent sellers is quite different from the principal-agent based explanations advanced by Rutherford, Springer, and Yavas (2005) and Levitt and Syverson (2008). Note that our explanation does not leave the open question why sellers are willing to write contracts with a marginal fee of 6% (rather than close to 100%) if the agency problem is so severe.

Proposition 9. *(a) An independent seller at the lower end of the distribution ($\tilde{c} = \underline{\tilde{c}}$) sets a lower price than an integrated seller with the same static cost $\tilde{c}$.*

(b) An independent seller at the upper end of the distribution ($\tilde{c} = \overline{\overline{c}}$) sets a higher price than an integrated seller with the same static cost $\tilde{c}$.

(c) If G has a monotone hazard rate, then there is a $\hat{\tilde{c}} \in (\underline{\tilde{c}}, \overline{\overline{c}})$ such that an integrated seller sets a higher price than an independent seller with the same static cost iff $\tilde{c} < \hat{\tilde{c}}$.

(d) The average prices set by integrated sellers can be higher than those set by independent sellers even if their static entrant distributions $\tilde{g}_0$ are the same.

Concerning the empirically testable implications of (a) and (b) note that while $\tilde{c}$ are not (directly) observable, under the assumption that there is no selection effect (i.e. $\tilde{g}_0$

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[58]The argument is easiest if one assumes that overall the mass of owners is negligible for then F and G are as given in the main text. The assumption is not crucial, though: If one assumes that owners have positive measure and that buyers are randomly matched to an independent seller or an integrated seller in every period, then F and G will be different from the expressions in the text, but they will nonetheless determine P_{IS} and P_I as derived in the proof of the following proposition.

is the same for integrated and independent sellers), the distribution of realized prices corresponds to that of static costs $\tilde{c}$. Hence the statement can be taken to mean that (a) the lowest prices among integrated sellers are higher than the lowest prices among independent sellers and (b) the highest prices among integrated sellers are lower than the highest prices among independent sellers.[59] Part (d) of the Proposition shows that one may observe brokers getting higher prices when selling their own houses even without an agency problem and even without a selection effect.

Per Transaction Costs of Intermediation   Instead of or in addition to assuming a fixed cost of intermediation, let us now briefly consider the case where intermediation services involve a variable (as well). We show that under the assumption of a power distribution with $\underline{c} > 0$ of the seller's cost, all the previous results go through rather nicely. Recall first that a power distribution G with $\underline{c} > 0$ implies a negative fee $\xi < 0$, i.e. a subsidy from the intermediary to the seller which we typically do not observe. However, intermediaries also provide services (advertising, showing the house to potential buyers, legal advice) we do not account and that are costly. Thus, the free of charge provision of these services can be interpreted as a subsidy from the intermediary to the seller paid in resources.

This is also consistent with the following observation. In the U.K. real estate brokers typically charge a lower fee (2.5 percent) than in the U.S. but they do also provide less services. Recall that for a power distribution a larger β implies a lower percentage fee ... but also a smaller subsidy. Thus, assuming that the seller side both in the U.S. and the UK are characterized by power distributions, this is consistent with $\beta_{UK} > \beta_{US}$. This implies $\xi_{US} < \xi_{UK} < 0$, which is in turn consistent with the difference in service levels provided by brokers.

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[59]Note that (b) only holds if the same range of integrated and independent sellers enter the market (i.e. $[\underline{\tilde{c}}, \overline{\tilde{c}}]$ is the same). If free entry also applied for integrated sellers, one would expect more integrated sellers to enter at the upper end of the support $[\underline{\tilde{c}}_0, \overline{\tilde{c}}_0]$ since they do not have to pay fees.

5.2   Other Applications

We now briefly discuss two other applications, intermediaries in auctions and stock brokers.

Auction Houses and Auction Sites   Auction houses like Sotheby's and Christie's and auction internet sites like eBay set percentage fees[60] and the seller then sets a reservation price rather than a take-it-or-leave-it price. We now show that in a static setup with one seller and many buyers this is an optimal mechanism by extending the stage game to a setup with one intermediary, one seller, and many buyers.

As a preliminary, we first derive the intermediary optimal mechanism with many buyers and possibly many sellers.[61] Let N_B and N_S, respectively, be the number of buyers and sellers, whose valuations v_b and costs c_s are independent draws from distributions F_b with densities f_b and supports $[\underline{v}_b, \overline{v}_b]$ and distributions G_s with densities g_s and supports $[\underline{c}_s, \overline{c}_s]$. As before, we consider cases where virtual valuations $\Phi_b(v_b)$ and the virtual costs $\Gamma_s(c_s)$ are strictly increasing and we use b (s) exclusively to indicate a buyer (seller). Order and relabel the realized virtual valuations in decreasing and virtual costs in increasing order, i.e. $\Phi_1 > \Phi_2 > .. > \Phi_{N_B}$ and $\Gamma_1 < \Gamma_2 < .. < \Gamma_{N_S}$. Pair buyers and sellers with equal index. The case where $N_B \neq N_S$ can be easily dealt with by adding fictitious traders.[62] We define the Virtual-Walrasian allocation rule such that all pairs with $\Phi_k \geq \Gamma_k$ trade and all others do not. The Virtual-Walrasian quantity is the number of trading pairs, formally $K := \max\{k|\Phi_k \geq \Gamma_k\}$.

Lemma 1. *The intermediary optimal mechanism that respects individual rationality and incentive compatibility of buyers and sellers has a Virtual-Walrasian allocation rule and gives zero expected utility to buyers with $v_b = \underline{v}_b$ and sellers with $c_s = \overline{c}_s$.*

For $N_B = N_S = 1$ the Virtual-Walrasian allocation rule reduces, of course, to the

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[60]Prior to being convicted of collusion, this fee was 20 percent for Sotheby's and Christie's. Before they changed to a regressive fee structure, eBay used to charge a linear fee of 5 Percent.

[61]See also Baliga and Vohra (2003).

[62]If there are less buyers than sellers, fill up the ranks of buyers with fictitious buyers who do not trade for sure (i.e. $\Phi_k = -\infty$ for $N_B < k \leq N_S$). If there are less sellers, use fictitious sellers with $\Gamma_k = \infty$ for $N_S < k \leq N_B$.

intermediary optimal allocation rule of Myerson and Satterthwaite (1983).

Assume now that there is one seller (i.e. $N_S = 1$) and that the $N_B > 1$ buyers' valuations are independently drawn from the identical distribution F with support $[\underline{v}, \overline{v}]$. With many buyers and one seller, the intermediary optimal allocation rule requires the good to go to the buyer with the largest virtual valuation, provided this virtual valuation exceeds the seller's virtual cost.

Proposition 10. *Assume the intermediary faces N_B buyers whose valuations are i.i.d. draws from F and one seller whose cost is drawn from G. Then the following is an intermediary optimal mechanism. The intermediary sets the fee function $\omega(p_S) = p_S - E_v[P^{-1}(v) \mid v \geq p_S]$, where p_S is the final sale price. Then the seller sets the reserve price $p = P(c)$, and a standard auction ensues.*

Letting $\omega_\alpha = p_S - E_v[P_\alpha^{-1}(v) \mid v \geq p_S]$ (and assuming monotone hazard rates) where $P_\alpha^{-1}(v)$ is as defined above Corollary 2 one can also implement the ex ante efficient mechanism by setting $\alpha = \alpha^*(N_B)$, where $\alpha^*(N_B)$ solves

$$\int \int [\Phi(y) - \Gamma(c)] p^\alpha(y, c) N_B F(y)^{N_B - 1} f(y) g(c) dc dy = 0$$

with $Q^\alpha(y, c) = 0$ if $\Phi(y, \alpha) < \Gamma(c, \alpha)$ and $Q^\alpha(y, c) = 1$ otherwise.[63] For F and G uniform on $[0,1]$, $\alpha^*(1) = 0.25$, $\alpha^*(2) = 0.215$ and $\alpha^*(3) = 0.184$. Thus, for the case of one seller and many buyers an appropriately chosen fee setting mechanism followed by reserve price setting by the seller and a standard auction implements the ex ante efficient, direct mechanism Gresik and Satterthwaite (1989) analyze.

Collusion   An issue we have abstracted from is collusion. The U.S. Department of Justice uncovered commission fixing between Sotheby's and Christie's in the 1990s and is currently investigating whether real estate agents are colluding (DOJ, 2007). Analyzing how collusion would look like in our dynamic setup is therefore of particular interest and relevance. For the two extreme cases where τ is either zero or infinite, the answer is

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[63]See Gresik and Satterthwaite (1989) for the derivation of $\alpha^*(N_B)$.

simple. For $\tau \to \infty$ the probability of being rematched is zero. Therefore, the optimal collusive mechanism, i.e. the mechanism that maximizes joint profits, is given by the optimal mechanism in the static model derived in Section 3.1. For $\tau \to 0$ all the bilateral matchings essentially become one multilateral, simultaneous matching of all buyers and all sellers. An optimal mechanism for a monopolist, or equivalently for colluding firms, in this setup is to post bid and ask prices at which sellers are allowed to sell and buyers are allowed to buy, as we show in Proposition 11 below. Determining the optimal collusive mechanism for intermediate values of τ requires solving a rather complicated dynamic mechanism design problem, where the feasibility constraint is that only matched buyers and sellers can trade in any given period. This is left for future research.

Stock Brokers   Linear fees are widely used in stock brokerage. The empirical finance literature describes two kinds of fees used: flat fees and percentage fees (see Conrad, Johnson, and Wahal, 2001). One can think of stock markets where a broker willing to sell looks at one buyer's broker at at time. The short moment at which two brokers look at each other can be interpreted as the period τ between subsequent rematching (abstracting away from having both a buyer's and a seller's broker).[64]

An important reason why stock brokers may prefer percentage fees to (bid and ask) price posting (see Section 5.3) may be the highly volatile nature of the value of the goods (i.e. stocks) in the exchange of which they are engaged. This feature is arguably well captured by our model, where the buyer's and seller's valuations v and c are stochastic. Consequently, the market price exhibits substantial variability and price posting is not optimal for the intermediary, as shown in Proposition 11 below, whereas percentage fee setting is. Interestingly, the literature on market microstructure and intermediation mainly focuses on price posting by brokers. For example, Gehrig (1993) studies a static model with a monopolistic broker who quotes bid and ask prices. Similarly, Duffie, Garleau, and Peddersen (2005) analyze market making by price setting intermediaries in a dynamic setup, where the intermediaries' search intensity is determined endogenously and price posting is the exogenously given mechanism. An exception is the paper by

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[64]We thank Mark Satterthwaite for providing us this interpretation.

Duffie and Strulovici (2008), who provide a model with intermediaries in capital markets that charge percentage fees.[65] Another paper that departs from the assumption of price posting is Parlour and Rajan (2003) who study an infinite horizon model with market makers who quote bid-ask spreads rather than post bid and ask prices. This is obviously related yet distinct from the percentage fees used in practice and the ones studied in the present paper.

5.3   Alternative Mechanisms

We now consider in turn price posting and slotting allowances and relate them to the intermediary optimal mechanism.

Price Posting   Consider first *price posting*, which is widely used e.g. by used car dealers and currency exchange offices: the intermediary posts an ask (or buyer) price p^B and a bid (or seller) price p^S.[66] The quantity traded is the minimum of the number of buyers and sellers who are willing to trade at these prices, and the intermediary earns the bid-ask spread $p^B - p^S$ on each buyer-seller pair that trades.

Proposition 11. *For expositional simplicity assume $\Phi' > 1$ and $\Gamma' > 1$.*

(a) If the intermediary is matched with one buyer and one seller at a time, the optimal ask price p^B and bid price p^S are given by the equation system $p^B = \Gamma(p^S)$ and $p^S = \Phi(p^B)$, and price posting is never optimal for the intermediary.

(b) Consider price posting with N buyers and sellers with i.i.d. draws $v_i \sim F$ and $c_i \sim G$ and let prices be given by $\Phi(p^B) = \Gamma(p^S)$ and $1 - F(p^B) = G(p^S)$. As N goes to infinity (i) the optimal mechanism converges to a price posting mechanism with these prices and (ii) the price posting mechanism with these prices converges to optimality.

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[65]However, their focus is on capital mobility between markets and they assume the percentage fee mechanism as exogenously given. Moreover, capital owners who wish to trade do not have any private information, so that percentage fees can be charged directly on the surplus generated by a transaction. See also Yavas (1992) who makes a similar assumption.

[66]Riley and Zeckhauser (1983) and Peck (1996) also look at the optimality of price posting mechanisms, but in different setups and without intermediaries. The former relies on buyers' ability to wait without costs for a cheaper offer by the same seller, the latter assumes buyers with identical valuations for the good choosing simultaneously between offers from multiple sellers. Gresik and Satterthwaite (1989) compare the ex ante efficient mechanism with posted prices, which is also not an ex ante efficient mechanism.

(c) If the intermediary can store the good costlessly between subsequent rematchings,[67] price posting with prices as in (b) is optimal for the intermediary.[68]

Intuitively, with one buyer and one seller, having to post two fixed prices is not optimal since it takes away too much flexibility from the intermediary, e.g. price posting excludes the possibility of allowing trade when both the buyer and the seller have a high valuation. However, with many buyers and sellers, or with a storable good, the problem of the intermediary can be separated into two separate problems: buying from sellers and selling to buyers. For both problems, price posting is optimal Myerson (1981).

Prop. 11 (a) seems to stand in contradiction to the empirical observation that price posting is often used by intermediaries.[69] However, at a second glance, intermediaries that post prices are the ones that either face liquid markets (b) or can store the good (c).

For a seller and N_B buyers i.i.d. draws $v_i \sim F$ we get the following.[70]

Proposition 12. *As N_B approaches infinity, intermediary price posting and fee setting are equivalent and intermediary optimal.*

Slotting Allowances   Real world retailers often require upfront payments by sellers to allocate scarce shelf space and then charge a percentage fee on the revenue generated

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[67]In particulary, he has no liquidity constraints, no storage constraints, his discount factor is 1, he can have short positions of the good, and storage costs are zero. His only constraint is that he has to buy and sell with the same probability.

[68]Our assumption on storing the good is essentially the same as Garman's (1976) assumption of an intermediary with an infinite inventory of cash and stock and Glosten and Milgrom's (1985) assumption of "zero costs associated with all short positions in cash and stock". Both papers look at intermediaries facing sequentially arriving traders (in stock markets), but in a very different setup and with a very different focus: both assume the price posting mechanism as exogenously given, the former considers non-strategic traders arriving in continuous time, the latter the bid-ask spread set by zero profit intermediaries in the presence of insider trading.

[69]The result depends on Assumption 1. If one assumes instead that distributions do not overlap and that there is a sufficiently large gap between the distributions of buyers and sellers (more precisely $\Phi(\underline{v}) > \Gamma(\bar{c})$), then by Lemma 2 the intermediary always allows trade to occur and sets prices $p^B = \underline{v}$ and $p^S = \bar{c}$. However, this is not a sufficient explanation for the usage of price posting mechanisms: trade does not occur every time a buyer and a seller meet an intermediary.

[70]Note that for an infinite number of buyers and fee setting any ω satisfying $\omega(\bar{v}) = \bar{v} - \Gamma^{-1}(\bar{v})$ and inducing the seller to set the price $\bar{v}$ would do. For instance the intermediary could just as well charge a fixed fee $\bar{v} - \Gamma^{-1}(\bar{v})$.

by the seller.[71] We now show that this type of mechanism arises naturally as the intermediary optimal mechanism when the intermediary faces several competing sellers but cannot sell more than one unit. The latter may occur either because he only attracts one buyer or because he is capacity constrained. Consider a static model with one buyer and several sellers with i.i.d. draws $c_i \sim G$.

Proposition 13. *The following two-stage mechanism is intermediary optimal:*

   Stage 1*: The intermediary runs a standard auction (e.g. a second price) among the sellers for the right to participate in the second stage.*

   Stage 2*: The winner of stage 1 sets the price $p = P(c)$, facing the fee function $\omega(p)$ given in Proposition 1.*

Of course, if the intermediary attracts several buyers whose valuations are i.i.d. draws but is constrained by his physical capacity not to sell more than one unit, then the mechanism in Proposition 13 is still intermediary optimal, provided the price set by the seller in stage 2 is a reserve price in an optimal auction.

6   Conclusions

In this paper we study intermediation from an applied Bayesian mechanism design perspective, under the assumption that the intermediary has the bargaining power. We find that fee setting mechanisms, which are widely used in practice, but so far little understood in economics, are optimal for intermediaries in a wide array of settings, i.e. maximize the intermediaries' expected profits subject to buyers' and sellers' incentive and individual rationality constraints. We show that these mechanisms are optimal in a static model where one or more buyer(s) and one seller are matched to an intermediary, and they are optimal for intermediaries in a dynamic model where in every period a buyer, a seller and an intermediary are randomly matched, and where the valuations of buyers and sellers are endogenously determined. We show also that the dynamic model with random matching, in which each intermediary chooses an optimal mechanism, permits an analytical solution. Perturbation analysis reveals that the equilibrium fees become

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[71]See e.g. Sullivan (1997) or Marx and Schaffer (2007).

smaller when the rematching probability increases, which contrasts with models with complete information. Our baseline model can be applied to different industries with intermediaries. In particular, our model can explain several stylized facts observed in real estate brokerage. The dynamic model also provides a parsimonious explanation for why vertically integrated sellers, e.g. intermediaries who sell houses they own, may set higher prices than independent sellers, which is consistent with empirical evidence (see e.g. Rutherford, Springer, and Yavas, 2005; Levitt and Syverson, 2008). Further research on the optimal collusive mechanism in the dynamic model with random matching seems particularly fruitful given the widespread suspicion that intermediaries like real estate brokers set their fees collusively.

Appendix

A   Proofs

Since we will refer to the properties of intermediary optimal mechanisms often, we summarize Myerson and Satterthwaite (1983)'s Theorems 3 and 4 on intermediation in the following lemma.

Lemma 2 (Myerson-Satterthwaite). *An incentive compatible, interim individually rational mechanism is intermediary optimal if and only if it is such that (i) the good is transferred iff $\Phi(v) \geq \Gamma(c)$ and (ii) the seller with the highest cost $\bar{c}$ and the buyer with the lowest valuation $\underline{v}$ both have zero expected utility.*

Proposition 1:   Optimal fee.

Proof of Proposition 1. Even though a shorter proof can be obtained by first considering the dominant strategy implementation, we will derive our results through the incentive compatibility constraint because other proofs rely on the intermediate steps of this proof.

If through an appropriately chosen fee function $\omega(p)$ the seller with cost c can be induced to set the price $p = P(c)$, we know that we have an an intermediary optimal mechanism because the buyer will accept the offer iff $v \geq P(c) \Leftrightarrow \Phi(v) \geq \Gamma(c)$. We

first show that such a function $\omega(p)$ exists and is unique. Denote the expected payoff to the seller of type c who sets the 'prescribed' price $P(c)$ as $U(c) := (P(c) - \omega(P(c)) - c)(1 - F(P(c)))$. By the Envelope Theorem (see e.g. the proof of Theorem 1 in Myerson and Satterthwaite (1983) or Ch.5 in Krishna (2002)), for $p = P(c)$ to be an equilibrium strategy for the seller, $U(c)$ has to satisfy

$$U(c) = U(\bar{c}) + \int_c^{\bar{c}} [1 - F(P(t))]dt = \int_c^{\bar{c}} [1 - F(P(t))]dt, \tag{18}$$

where the second equality follows because $U(\bar{c}) = 0$. Moreover, since $1 - F(P(c)) = 0$ for all c such that $P(c) \geq \bar{v} \Leftrightarrow c \geq P^{-1}(\bar{v}) := \Gamma^{-1}(\Phi(\bar{v}))$, the upper limit of the integral can be written as $P^{-1}(\bar{v})$. Inserting the definition of $U(c)$ into (18) and rearranging yields

$$\omega(P(c)) = P(c) - c - \int_c^{P^{-1}(\bar{v})} \frac{1 - F(P(t))}{1 - F(P(c))}dt. \tag{19}$$

Substituting $p = P(c)$ into (19) and integrating using this substitution yields

$$\omega(p) = p - P^{-1}(p) - \frac{\int_p^{\bar{v}} (1 - F(v))[P^{-1}(v)]'dv}{1 - F(p)} = p - E_v[P^{-1}(v) \mid v \geq p], \tag{20}$$

where the second equality follows after integrating by parts ($E_v[P^{-1}(v) \mid v \geq p] = (\int_p^{\bar{v}} f(v)P^{-1}(v)dv)/(1 - F(p))$ being the expectation of $P^{-1}(v)$ taken with respect to the distribution F conditional on v being larger than p). Clearly, for given F and G the function $\omega(p)$ is unique.

Next we show that given $\omega(p)$ the Nash equilibrium is essentially unique. (The equilibrium is not unique merely because a seller of type $c > P^{-1}(\bar{v})$ can set any $p > \bar{v}$.) Since the buyer's unique best response is to accept whenever $p \leq v$, the problem of the seller with cost c, given $\omega(p) = p - E_v[P^{-1}(v)|v \geq p]$, is to choose p to maximize $(E_v[P^{-1}(v)|v \geq p] - c)(1 - F(p))$. By construction of $\omega(p)$, the first order condition is satisfied at $p = P(c)$. Since at the first order condition, the second order condition $0 > -f(p)[P^{-1}(p)]'$ is also satisfied, uniqueness follows. $\square$

Proposition 2:   Linear fee.

Proof of Proposition 2. By the same standard arguments leading to (18) we also get $U'(c) = -q(c)$ almost everywhere because of incentive compatibility. Equating this

with the derivative obtained from the definition $U'(c) = [(P(c) - \omega(P(c)) - c)q(c)]'$ and rearranging yields

$$\Phi(P(c)) = P(c) - \frac{P(c) - \omega(P(c)) - c}{1 - \omega'(P(c))}. \tag{21}$$

(i) implies (ii) Take $\omega(p) = \xi p + \zeta$. Then the right hand side of (21) becomes $(c + \zeta)/(1 - \xi)$. Equating this with $\Gamma(c)$ in order to achieve optimality according to Lemma 2 (i) gives the differential equation $g(c) = G(c)(1-\xi)/(\xi c+\zeta)$. With the condition $G(\underline{c}) = 0$ one obtains the expression in part (ii) of the proposition with $\beta = (1 - \xi)/\xi$ and $\underline{c} = -\zeta/\xi$. The upper bound of the support $\bar{c}$ remains arbitrary.

(ii) implies (i) Observe that with the distribution G specified in part (ii) one has $\Gamma^{-1}(p) = (1 - \hat{\xi})p - \hat{\zeta}$ with $\hat{\xi} := 1/(\beta+1)$ and $\hat{\zeta} := -\underline{c}/(\beta+1)$ and, therefore, $P^{-1}(p) = \Gamma^{-1}(\Phi(p)) = (1 - \hat{\xi})\Phi(p) - \hat{\zeta}$. Take (21) and replace $P(c)$ with p, c with $P^{-1}(p)$, and Φ by its definition. Rearranging leads to

$$(1 - F(p))(\kappa'(p) - (1 - \hat{\xi})) - f(p)(p - \omega(p) - ((1 - \hat{\xi})p - \hat{\zeta})) = 0. \tag{22}$$

Defining $l(p) := p - \omega(p) - ((1 - \hat{\xi})p - \hat{\zeta})$ equation (22) leads to $[l(p)(1 - F(p))]' = 0$. From part (ii) of Proposition 2 follows that $p - \omega(p)$ is not singular at $p = \bar{v}$ (actually $\omega(\bar{v}) = \bar{v} - P^{-1}(\bar{v})$). Since $1 - F(\bar{v}) = 0$ it follows that $l(p) \equiv 0$, i.e. $\omega(p) = \xi p + \zeta$ as in part (i) Proposition 2 is satisfied with $\xi = \hat{\xi}$ and $\zeta = \hat{\zeta}$. □

Proposition 3:   Invariance and linearity of fees.

Proof of Proposition 3. The optimality condition (i) of Lemma 2 implies $\Phi(P(c)) = \Gamma(c)$. If we want optimality to hold for arbitrary distributions F, and hence for arbitrary functions $P(c)$, equating the right hand side of (21) and $\Gamma(c)$ yields $\Gamma(c) = p - (p - \omega(p) - c)/(1 - \omega'(p))$ for arbitrary p. This differential equation in ω has the solution

$$\omega(p) = p - (1 - \xi)(p - \Gamma(c)) - c \tag{23}$$

defined up to a constant $1-\xi$. If we want this to hold for any c we need $c-(1-\xi)\Gamma(c) = -\zeta$ for some constant ζ, and hence $\Gamma(c) = (c + \zeta)/(1 - \xi)$. Substituting this back to (23) results in $\omega(p) = \xi p+\zeta$, i.e. a linear fee. This also implies a generalized power distribution G by Proposition 2. □

Corollary 2:   Fee setting mechanism implements α-allocation rule.

Proof. Under the assumption (which implies regularity) that $(1-F(v)/f(v)$ is decreasing and $G(c)/g(c)$ is increasing, the ex ante efficient allocation is as follows (see Myerson and Satterthwaite (1983)). For a given $\alpha \in [0,1]$ let $\Phi_\alpha(v) = \alpha\Phi(v) + (1-\alpha)v$ and $\Gamma_\alpha(c) = \alpha\Gamma(c) + (1-\alpha)c$. Call an allocation rule an α-allocation rule if it induces trade iff $\Phi_\alpha(v) \geq \Gamma_\alpha(c)$. The ex ante efficient mechanism has an α^*-allocation rule, where α^* is the smallest α such that $\int \int [\Phi(v) - \Gamma(c)]p^\alpha(v,c)f(v)g(c)dcdv \geq 0$, where $Q^\alpha(v,c) = 0$ if $\Phi_\alpha(v) < \Gamma_\alpha(c)$ and $Q^\alpha(v,c) = 1$ otherwise.

Since $\Phi_\alpha(v)$ is increasing in v, any α-allocation rule can be implemented via a fee setting mechanism if the seller with cost c can be induced to set the price p, where p is such that $\Phi_\alpha(p) = \Gamma_\alpha(c)$. Inverting yields $P_\alpha(c) := \Phi_\alpha^{-1}(\Gamma_\alpha(c))$. Accordingly, $P_\alpha^{-1}(p) := \Gamma_\alpha^{-1}(\Phi_\alpha(p))$. Denote by $\omega_\alpha(p)$ the fee set by the intermediary under some α-allocation rule, which leaves $p - \omega_\alpha(p)$ to the seller upon successful sale. The buyer accepts any price offer $p \leq v$ and simply pays p. The same arguments as for Proposition 1 then establish the corollary. $\square$

Proposition 5:   Effect of an infinitesimal perturbation.

Proof of Proposition 5. Since we only care about first-order effects, $\hat{g}(c) = (1-\epsilon\gamma(c))g(c)$ can be rewritten as

$$g = \frac{1}{1+\epsilon\gamma}\hat{g} = (1 - \epsilon\gamma + O(\epsilon^2))\hat{g}, \tag{24}$$

where $O(\epsilon^2)$ stands for the second order effect. Taking a constant α with $1 + \alpha\epsilon = \sigma$, this has to be equal to

$$(1 + \alpha\epsilon)(1 - \epsilon(1 - \rho_S(c)))\hat{g}(c) = (1 - \epsilon[(1 - \rho_S(c)) - \alpha] + O(\epsilon^2))\hat{g}(c). \tag{25}$$

α has to be chosen as $\alpha = \int(1-\rho_S)\hat{g}$ so that the density functions add up to one. Hence equating the right hand sides of (24) and (25) results in

$$\gamma(c) = F(\Phi^{-1}(\Gamma(c))) - \int_{\underline{c}}^{\overline{c}} F(\Phi^{-1}(\Gamma(t)))g(t)dt. \tag{26}$$

We know that γ is increasing, γ and g are orthogonal ($\int \gamma g = 0$), $\gamma(\underline{c}) < 0$, and $\gamma(\overline{c}) > 0$.

(i) We will first show that $(\ln G(c))'$ increases if ϵ increases:

$$\frac{\partial^2}{\partial\epsilon\partial c}\ln\hat{G} > 0. \tag{27}$$

Using

$$\frac{\partial}{\partial\epsilon}\hat{G} = \frac{\partial}{\partial\epsilon}\int(1+\epsilon\gamma)g = \int\gamma g, \tag{28}$$

we get

$$\frac{\partial}{\partial\epsilon}\ln\hat{G}\bigg|_{\epsilon=0} = \frac{1}{G}\int_{\underline{c}}^{c}g(c')\gamma(c')dc'. \tag{29}$$

Taking the derivative with respect to c yields

$$\frac{\partial}{\partial c}\frac{\partial}{\partial\epsilon}\ln\hat{G}\bigg|_{\epsilon=0} = \frac{g}{G^2}\int_{\underline{c}}^{c}g(c')\gamma(c')dc' + \frac{1}{G}g(c)\gamma(c),$$

the sign of which is to be determined. Multiplying by the positive expression G^2/g we get

$$G(c)\gamma(c) - \int_{\underline{c}}^{c}g(c')\gamma(c')dc' = \int_{\underline{c}}^{c}g(c')[\gamma(c) - \gamma(c')]dc'. \tag{30}$$

The expression in the brackets is positive since $\gamma'(c) > 0$ and $c > c'$, therefore, the whole expression is positive, which proves the statement

$$\frac{\partial^2}{\partial\epsilon\partial c}\ln G > 0. \tag{31}$$

(ii) As next we will prove that Γ is decreasing and Γ^{-1} increasing with ϵ. The following analysis can be simplified by defining a further function ψ, such that

$$\hat{G}(c) = (1 - \epsilon\psi(c))G(c), \tag{32}$$

The relation between ψ and γ is the following

$$g\gamma = -(G\psi)' \tag{33}$$

or

$$\psi = -\frac{1}{G}\int g\gamma \tag{34}$$

which is equal to the negative of the right hand side of (29). Therefore, $\psi' < 0$ by the argument in the previous section. We also know $\psi(\bar{c}) = 0$, $\psi \geq 0$.

The derivative is

$$\hat{g} = g - \epsilon(g\psi + G\psi') \tag{35}$$

By definition

$$\hat{\Gamma} \stackrel{\text{def}}{=} c + \frac{\hat{G}}{\hat{g}} = c + \frac{[1 - \epsilon\psi]G}{[(1 - \epsilon\psi) - \epsilon(G/g)\psi']g} = c + \frac{(G/g)[1 - \epsilon\psi]}{g - \epsilon(\psi + (G/g)\psi')}. \tag{36}$$

The Taylor expansion is

$$c + \frac{G}{g}[1 - \epsilon\psi]\left[1 + \epsilon(\psi + \frac{G}{g}\psi')\right] + O(\epsilon^2) = c + \frac{G}{g}\left[1 + \epsilon\frac{G}{g}\psi'\right] + O(\epsilon^2) \tag{37}$$

Using the definition of $\hat{\Gamma}$ this gives us

$$\hat{\Gamma} = \Gamma + \epsilon\left(\frac{G}{g}\right)^2 \psi' + O(\epsilon^2). \tag{38}$$

Since ψ' is negative $\hat{\Gamma}$ is decreasing with ϵ. The inverse of $\hat{\Gamma}$ is

$$\hat{\Gamma}^{-1} = \Gamma^{-1} - \epsilon\frac{(G/g)^2\psi'}{\Gamma'} + O(\epsilon^2). \tag{39}$$

The fraction is negative since $\psi' < 0$ and Γ is increasing by Myerson's regularity assumption. Therefore, for a perturbation with $\epsilon > 0$ we have $\hat{\Gamma}^{-1} > \Gamma^{-1}$.

(iii) Next, we will look at the change of $[\Gamma^{-1}]'$. Taking the derivative of (39) gives us

$$(\hat{\Gamma}^{-1})' = (\Gamma^{-1})' - \epsilon\left[\frac{(G/g)^2\psi'}{\Gamma'}\right]' + O(\epsilon^2) \tag{40}$$

$$= (\Gamma^{-1})' - \epsilon\frac{[2(G/g)\frac{g^2 - Gg'}{g^2}\psi' - (\frac{G}{g})^2\psi'']\Gamma' - (\frac{G}{g})^2\psi'\Gamma''}{(\Gamma')^2} + O(\epsilon^2) \tag{41}$$

The sign of the multiplier of ϵ is ambiguous. For instance, since $\psi' < 0$, for Γ'' sufficiently negative, $(\hat{\Gamma}^{-1})'$ is increasing with ϵ. However, for Γ'' sufficiently large, we have the opposite effect.

We can also make the analysis for $\Gamma'' = 0$ (or close to zero), which means that we have a power distribution and linear fees (or are close to it). After some algebra the expression in brackets in (41) can be transformed to

$$-3G\gamma(g^2 - Gg') + 3g\int g\gamma - 2\frac{Gg'}{g}\int g\gamma + G^2g\gamma' - Gg\gamma. \tag{42}$$

If this is negative then $[\Gamma^{-1}]'$ will be larger if ϵ increases, that is we have a flatter fee.

However, one can find examples of power distributions where this condition is not satisfied. Take $G = c^\beta$, $F = 1 - (1 - v)^\alpha$, which results in linear virtual valuation functions Γ and Φ. E.g. for $\alpha = 3$ and $\beta = 3$, the condition is not satisfied for certain values of c, the sign of

$$\frac{1}{4860} \frac{-225 + 1280\,c^4 - 3840\,c^3 - 363\,c^2 + 1920\,c}{c^2} \tag{43}$$

is different for different values of c as depicted in Fig. 9.
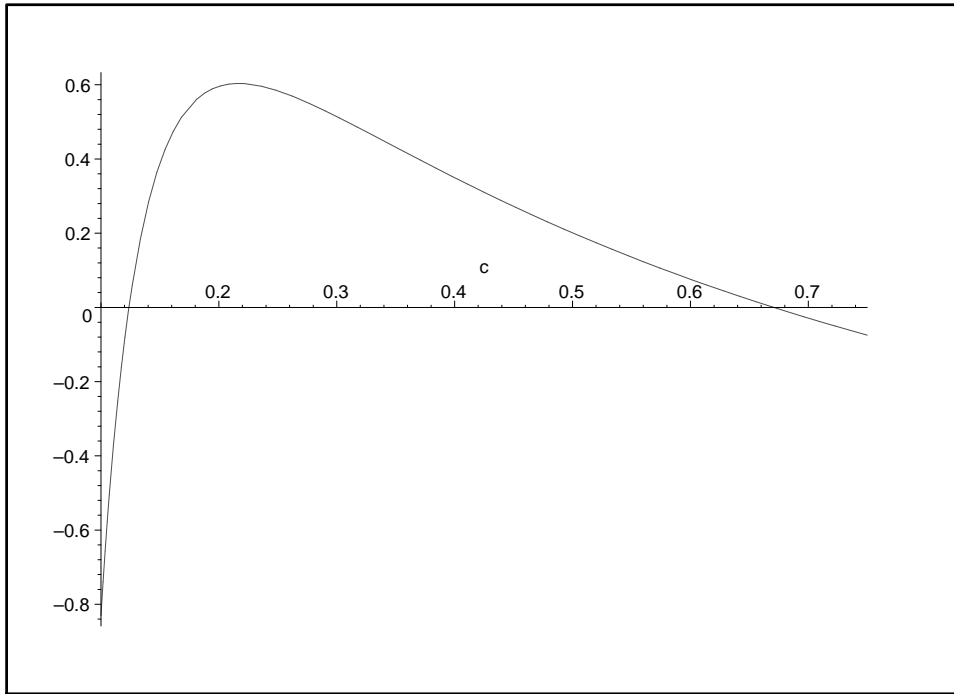
Figure 9: For $\alpha = 2$ and $\beta = 3$ the expression in Eq. (42) has different signs for different values of c.

$\square$

Proposition 6:   Time on market with one homogeneous good.

Proof of Proposition 6. Discrete Time. Consider first a cohort of sellers, who entered the market at some point t, normalized to $t = 0$, and offered their houses for some price p. Label the number of rematchings since $t = 0$ with $k := t/\tau$ and the expected number of sellers in the cohort staying in the market at the beginning with N_0 and in subsequent periods with N_k. The probability that a seller stays in the market until the

next rematching is the probability that he cannot sell times the probability that he does not drop out for exogenous reasons, i.e. $\epsilon F(p)$ with $\epsilon := e^{-\eta\tau}$. The number of sellers in period k is hence $N_k = (\epsilon F(p))^k N_0$. Time on market for the total population of both sold and unsold houses follows hence a geometric distribution with the cumulative distribution function $1 - (\epsilon F(p))^{t/\tau}$ and mean $T(p) = \tau/(1 - \epsilon F(p))$. Denote the number of sellers who leave the market in period k because they sell as N_k^s and those who leave with unsold houses as N_k^u. Clearly, $N_k^s = (1 - F(p))N_k$ and $N_k^u = \epsilon N_k$. Therefore, the ratio of sellers able to sell is $(1 - F(p))/(1 - \epsilon F(p))$. Now consider only the subsample of sellers who managed to sell their houses. Since N_k^s is just a constant factor smaller than N_k, the distribution of time on market of this subsample is the same as for the total population. Hence the cumulative distribution function is also $1 - (\epsilon F(p))^k$ and the mean time on market for sold houses is $T^s(p) = 1/(1 - \epsilon F(p))$. The same reasoning applies for sellers who did not sell their houses, so that $T^u(p) = 1/(1 - \epsilon F(p))$ is the mean time on market for unsold houses as well. Since we are looking at a market in a stationary equilibrium, in every period the same number of N_0 sellers enters and the previous argument carries over to a setup where cohorts of sellers enter every period rather than only one cohort entering at $t = 0$.

Continuous Time. The same logic applies to the continuous time approximation of the distribution. Denote the mass of sellers in the cohort at period $t = 0$ as $N(0)$. The number of sellers remaining in the market in period t is $N(t) = N(0)e^{-(\phi+\eta)t}$ dropping the argument p in $\phi(p)$. In each period $dN^s(t) = N(t)\phi dt$ houses are sold and $dN^u(t) = N(t)\eta dt$ drop out unsold. Cumulatively, we have $N^s(t) = \int_0^t dN^s(t') = (\phi/(\phi + \eta))[N(0) - N(t)]$ and $N^u(t) = \int_0^t dN^u(t') = (\eta/(\phi + \eta))[N(0) - N(t)]$. After infinitely many periods, fraction $1 - F_\infty := N^s(\infty)/N(0) = \phi/(\phi + \eta)$ of houses have been sold. The average time on market for sold houses is

$$T^s = \frac{\int_0^\infty t\, dN^s(t)}{\int_0^\infty dN^s(t)} = -\frac{\partial}{\partial\phi}\ln\int_0^\infty e^{-(\phi+\eta)t}dt = -\frac{\partial}{\partial\phi}\ln\frac{1}{\phi+\eta} = \frac{1}{\phi+\eta}.$$

By the same logic, the average time on market of unsold houses is $T^u = 1/(\phi + \eta)$.

$\square$

Proposition 7:   Time on market in heterogeneous submarkets.

Proof of Proposition 7. Consider multiple submarkets, indexed by i, with different probabilities of sale $\phi_i(p)$. Houses of each submarket are represented with weight w_i in the total sample. Taking averages over submarkets, the mean time on market for sold $T^s(p)$ and unsold $T^u(p)$ houses is

$$T^s = \left(\sum_i w_i \frac{\phi_i}{\phi_i + \eta} \frac{1}{\phi_i + \eta} \right) \left(\sum_i w_i \frac{\phi_i}{\phi_i + \eta} \right)^{-1},$$

$$T^u = \left(\sum_i w_i \frac{\eta}{\phi_i + \eta} \frac{1}{\phi_i + \eta} \right) \left(\sum_i w_i \frac{\eta}{\phi_i + \eta} \right)^{-1},$$

the parameter p being dropped. The ratio of the two means is

$$\frac{T^u}{T^s} = \frac{\sum_i w_i \frac{\eta}{(\phi_i+\eta)^2} \sum_j w_j \frac{\phi_j}{\phi_j+\eta}}{\sum_i w_i \frac{\phi_i}{(\phi_i+\eta)^2} \sum_j w_j \frac{\eta}{\phi_j+\eta}} =: \frac{N}{D} \, .$$

The difference between the numerator N and the denominator D is

$$N - D = \eta \sum_{ij} w_i w_j \frac{\phi_j - \phi_i}{(\phi_i + \eta)^2 (\phi_j + \eta)},$$

$$= -\eta \sum_{ij} w_i w_j \frac{\phi_j - \phi_i}{(\phi_i + \eta)(\phi_j + \eta)^2} \, ,$$

where the second equation comes from interchanging the summation variables. Adding the two expressions for $N - D$ one gets

$$2(N - D) = \eta \sum_{ij} w_i w_j \frac{(\phi_j - \phi_i)^2}{(\phi_i + \eta)^2 (\phi_j + \eta)^2} \geq 0 \, ,$$

hence $T^u \geq T^s$. The inequality is strict for heterogeneous submarkets. $\square$

Proposition 8:   Invariant Mechanisms.

Proof of Proposition 8. Denote an intermediary's expected profit in a given period if matched to a buyer and a seller by $\Pi := \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \max\{0, \Phi(v) - \Gamma(c)\} f(v) g(c) dc dv > 0$. (If unmatched, then his per period profit is 0.) Provided $\iota^* > \sigma$, each active intermediary's expected per period probability of being matched with a buyer and a seller is σ / ι^*

and his per period profit is $(\sigma/\iota^*)\Pi$. The equilibrium number of intermediaries in the market is given by $(\sigma/\iota^*)\Pi = \kappa$ under free entry. Note that changes in ι^* only affect, adversely, the volume of transactions per intermediary and thereby expected equilibrium profit per intermediary. Since every buyer and seller is matched with probability 1 in every period regardless of whether the number of intermediaries is, this leaves also every intermediary's mechanism design problem in every period unaffected. Therefore, the equilibrium mechanisms intermediaries employ do not vary as ι^* changes, provided only ι^* is not smaller than σ.[72] Therefore, business stealing is the only effect of entry, so that free entry leads to excessive entry.   □

Proposition 9:   Integrated Intermediary-Seller and Expected Prices.

Proof of Proposition 9. Denote, respectively, by $W_S(\tilde{c})$ and $W_I(\tilde{c})$ the expected continuation payoff of staying in the market (i.e. when not selling) of a seller with static type $\tilde{c}$ who sells via an intermediary and of an integrated seller of the same type who sells directly. Their dynamic types are $S(\tilde{c}) = \tilde{c} + \delta W_S(\tilde{c})$ and $S_I(\tilde{c}) := \tilde{c} + \delta W_I(\tilde{c})$. Now $W_I(\tilde{c}) > W_S(\tilde{c})$ holds for all $\tilde{c} \in [\underline{\tilde{c}}, \overline{\tilde{c}}]$. To see this, notice that the integrated seller always has the option of setting the same price as the independent seller, in which case they both sell with the same probability. Because the integrated seller does not have to pay a fee, it follows (via a revealed preference argument if their prices differ) that in every period the integrated seller's expected payoff exceeds the expected payoff of an independent seller. Consequently, $W_I(\tilde{c}) > W_S(\tilde{c})$.

(a) The independent seller sets the price $\tilde{P}(\tilde{c}) := P(S(\tilde{c})) = \Phi^{-1}(\Gamma(\tilde{c} + \delta W_S(\tilde{c})))$ and the integrated seller sets the price $\tilde{P}_I(\tilde{c}) := \Phi^{-1}(\tilde{c} + \delta W_I(\tilde{c}))$, where Φ and Γ are derived from the distributions of the dynamic types of buyers and sellers F and G that are endogenous to the model. Since $\Gamma(\underline{c}) = \underline{c}$, $\Gamma(\underline{\tilde{c}} + \delta W_S(\underline{\tilde{c}})) = \underline{\tilde{c}} + \delta W_S(\underline{\tilde{c}}) < \underline{\tilde{c}} + \delta W_I(\underline{\tilde{c}})$. Since Φ^{-1} is increasing, this implies $\tilde{P}_I(\underline{\tilde{c}}) > \tilde{P}(\underline{\tilde{c}})$.

(b) In equilibrium, the mechanism is such that an independent seller at the upper end

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[72]Note that if $\iota^* < \sigma$ were the case, then the equilibrium mechanisms would vary with ι^* since buyers and sellers are now matched with probability $\iota^*/\sigma < 1$. The equilibrium number of intermediaries is then given by $(\sigma/\iota^*)\hat{\Pi}(\iota^*) = \kappa$ where $\hat{\Pi}(\iota)$ is some decreasing function that determines the equilibrium per transaction profits of an intermediary.

of the distribution $\overline{\overline{c}}$ has utility zero and sells with probability zero. Hence, $W_S(\overline{\overline{c}}) = 0$. Therefore, an integrated seller with cost $\overline{\overline{c}}$ will sell with probability zero if he sets a price $\tilde{P}_I(\overline{\overline{c}}) \geq \tilde{P}(\overline{\overline{c}})$, in which case his option value $W_I(\overline{\overline{c}})$ would be equal to 0. But then his price would have to be lower than that of a seller, since $\Phi^{-1}(\overline{\overline{c}} + W_I(\overline{\overline{c}})) = \Phi^{-1}(\overline{\overline{c}}) < \tilde{P}(\overline{\overline{c}})$, which is a contradiction. Hence $\tilde{P}_I(\overline{\overline{c}}) < \tilde{P}(\overline{\overline{c}})$ follows.

(c) Parts (a) and (b) together with continuity imply that there will be at least one point of intersection between $\tilde{P}_I(\tilde{c})$ and $\tilde{P}(\tilde{c})$. Let $\hat{\tilde{c}}_i$ be some such point. We now show that $\Gamma' > 1$ implies that $\tilde{P}'(\hat{\tilde{c}}_i) > \tilde{P}_I'(\hat{\tilde{c}}_i)$ at every point of intersection $\hat{\tilde{c}}_i$. This in turn implies that $\tilde{P}(\tilde{c}) \geq \tilde{P}_I(\tilde{c})$ for all $\tilde{c}$ above the first (i.e. smallest) point of intersection.

Notice first that $\tilde{P}'(\hat{\tilde{c}}_i) = \Phi^{-1'}(\Gamma(\hat{\tilde{c}}_i + \delta W_S(\hat{\tilde{c}}_i)))\Gamma'(\hat{\tilde{c}}_i + \delta W_S(\hat{\tilde{c}}_i))(1 + \delta W_S'(\hat{\tilde{c}}_i))$ and $\tilde{P}_I'(\hat{\tilde{c}}_i) = \Phi^{-1'}(\hat{\tilde{c}}_i + \delta W_I(\hat{\tilde{c}}_i))(1 + \delta W_I'(\hat{\tilde{c}}_i))$. Since prices are the same, the arguments of $\Phi^{-1'}$ in both expressions, and hence $\Phi^{-1'}$ in both expressions, must be the same. Given the assumption $\Gamma' > 1$, the result thus follows if we can show that $1 + \delta W_S'(\hat{\tilde{c}}_i) \geq 1 + \delta W_I'(\hat{\tilde{c}}_i)$. Recall that $W_S'(\hat{\tilde{c}}_i) = -\rho_S(\hat{\tilde{c}}_i)/(1 - (1 - \rho_S(\hat{\tilde{c}}_i))\delta)$, where $\rho_S(\hat{\tilde{c}}_i) = 1 - F(\tilde{P}(\hat{\tilde{c}}_i))$. By analogous arguments, $W_I'(\hat{\tilde{c}}_i) = -\rho_I(\hat{\tilde{c}}_i)/(1 - (1 - \rho_I(\hat{\tilde{c}}_i))\delta)$, where $\rho_I(\hat{\tilde{c}}_i) = 1 - F(\tilde{P}_I(\hat{\tilde{c}}_i))$. Since $\tilde{P}(\hat{\tilde{c}}_i) = \tilde{P}_I(\hat{\tilde{c}}_i)$ at every point of intersection, $\rho_S(\hat{\tilde{c}}_i) = \rho_I(\hat{\tilde{c}}_i)$ and $\delta W_S'(\hat{\tilde{c}}_i) = \delta W_I'(\hat{\tilde{c}}_i)$ follows. This completes the proof that $\tilde{P}(\tilde{c}) \geq \tilde{P}_I(\tilde{c})$ for all $\tilde{c}$ above the first (i.e. smallest) point of intersection.

(d) Consider the model of Subsection 4.1, where $F(v) = 2v - 1$ for $v \in [1/2, 1]$ and $G(c) = 2c$ for $c \in [0, 1/2]$. Consequently in equilibrium the independent seller with dynamic cost c sets the price $P(c) = c + 1/2$. Since $c = S(\tilde{c})$, the independent seller with the static cost $\tilde{c}$ sets the price $\tilde{P}(\tilde{c}) = S(\tilde{c}) + \frac{1}{2} = \frac{1}{2\delta}\left(1 + \delta - \sqrt{(1-\delta)(1+\delta-4\tilde{c}\delta)}\right)$. The value function for an integrated seller is $V(\tilde{c}) = \max_p\{(1 - F(p))(p - \tilde{c}) + F(p)\delta V(\tilde{c})\}$, where the first (second) term on the right hand side is the payoff in case of a sale (no sale). Rearranging yields $V(\tilde{c}) = \max_p\left\{\frac{1 - F(p)}{1 - \delta F(p)}(p - \tilde{c})\right\} = \max_p\left\{\frac{2(1 - p)}{1 + \delta - 2\delta p}(p - \tilde{c})\right\}$, where the second equality follows from F being uniform. This is maximized at $\tilde{P}_I(\tilde{c}) = \frac{1 + \delta - \sqrt{(1-\delta)(1+\delta-2\tilde{c}\delta)}}{2\delta}$. In what follows we assume that the distribution of all integrated sellers' static costs is the same as that of independent sellers, so that its density is (13), but that the aggregate measure of all integrated sellers is still negligible. This permits a comparison of expected prices. Using $\tilde{g}_0(\tilde{c})$ as given in (13), the difference in expected

values $E[\tilde{P}_I - \tilde{P}]$ can be computed. It is straightforward to find conditions such that $E[\tilde{P}_I - \tilde{P}] > 0$. For example, $E[\tilde{P}_I - \tilde{P}] > 0$ if $\tau = 1, \eta = 1/10$ and $\delta = 4/5 (\Leftrightarrow r = -\ln(4/5) - 1/10)$. Numerical simulations reveal that $E[\tilde{P}_I - \tilde{P}] > 0$ for any $\delta \geq 0.7$.

$\square$

Lemma 1:   Optimal mechanism with many buyers and sellers.

Sketch of the Proof of Lemma 1. A direct mechanism asks buyers and sellers to report their valuations and costs. Denoting by $(\boldsymbol{v}, \boldsymbol{c})$ a collection of such reports with $\boldsymbol{v} = (v_1, .., v_{N_B})$ and $\boldsymbol{c} = (c_1, .., c_{N_S})$, the direct mechanism is then characterized by the probability $Q_b(\boldsymbol{v}, \boldsymbol{c})$ that b gets a unit of the good and $Q_s(\boldsymbol{c}, \boldsymbol{v})$ that s produces a unit of the good for $b = 1, .., N_B$ and $s = 1, .., N_S$ and by the payments $M_b(\boldsymbol{v}, \boldsymbol{c})$ it asks from buyers and the payments $M_s(\boldsymbol{c}, \boldsymbol{v})$ it makes to sellers. Clearly, a mechanism is only feasible if for all $(\boldsymbol{v}, \boldsymbol{c})$, $\sum_{b=1}^{N_B} Q_b(\boldsymbol{v}, \boldsymbol{c}) \leq \sum_{s=1}^{N_S} Q_s(\boldsymbol{c}, \boldsymbol{v})$. Let $\boldsymbol{Q}$ be the collection of these probabilities. We refer to $\boldsymbol{Q}$ as the allocation rule of the mechanism.

We only sketch the proof, a fully detailed version of which is available upon request. Lengthy, though completely standard arguments (see e.g. Krishna, 2002) can be applied to show that a revenue (or payoff) equivalence theorem holds. Formally, $m_b(v_b) = m_b(\underline{v}_b) + q_b(v_b)v_b - \int_{\underline{v}_b}^{v_b} q_b(t)dt$ and $m_s(c_s) = m_s(\underline{c}_s) + q_s(c_s)c_s - \int_{c_s}^{\bar{c}_s} q_s(t)dt$ for all c, v, lower case functions standing for expectations about all others' valuations and costs (e.g. $m_b(v_b) := E_{\boldsymbol{v}_{-b}, \boldsymbol{c}}[M_b(\boldsymbol{v}, \boldsymbol{c})]$). Again, by standard arguments, this implies $E[m_b(v_b)] = m_b(\underline{v}_b) + E[\Phi_b(v_b)q_b(v_b)]$ and $E[m_s(c_s)] = m_s(\bar{c}_s) + E[\Gamma_s(c_s)q_s(c_s)]$. A profit maximizing intermediary will make the individual rationality constraint just binding, therefore, his expected profit $\sum_{b=1}^{N_B} E[m_b(v_b)] - \sum_{s=1}^{N_S} E[m_s(c_s)]$ is

$$\int_X \left\{ \sum_{b=1}^{N_B} \Phi_b(v_b)Q_b(\boldsymbol{v}, \boldsymbol{c}) - \sum_{s=1}^{N_S} \Gamma_s(c_s)Q_s(\boldsymbol{c}, \boldsymbol{v}) \right\} f(\boldsymbol{v})g(\boldsymbol{c})d\boldsymbol{v}d\boldsymbol{c}, \tag{44}$$

where $f(\boldsymbol{v})$ and $g(\boldsymbol{c})$ are the joint densities of all buyers and sellers, respectively, and X is the product set containing all $(\boldsymbol{v}, \boldsymbol{c})$. Inspection of the term in curly brackets reveals that the profit can be maximized point by point by implementing the Virtual-Walrasian allocation for each realization $(\boldsymbol{v}, \boldsymbol{c})$.

$\square$

Proposition 10: Intermediary optimal auction.

Proof of Proposition 10. It is sufficient to prove our statement for a second price auction, since by the revenue equivalence theorem it then also holds for any standard auction.[73] So consider a second price auction where the seller faces the fee function $\omega(p_S)$ levied on the sale price p_S. The seller reports his cost as $\hat{c}$ and the intermediary sets the reservation price $P(\hat{c})$. This seller's expected profit is

$$N_B \left\{ (P(\hat{c}) - \omega(P(\hat{c})))(1 - F(P(\hat{c})))F(P(\hat{c}))^{N_B - 1} \right.$$
$$\left. + \int_{P(\hat{c})}^{\overline{v}} (y - \omega(y))(1 - F(y))(N_B - 1)F(y)^{N_B - 2} f(y) dy \right\} + cF(P(\hat{c}))^{N_B}$$

because if the reserve price $P(\hat{c})$ is binding, the sale price is $p_S = P(\hat{c})$, which explains the first $\omega(.)$ term. If the reserve is not binding, the sale price is the second highest bid y, and this explains the second $\omega(.)$; see also Krishna (2002, p.25). Note that the good is sold to the buyer with the largest virtual valuation, provided this is larger than the reserve p.

For truth telling to be an equilibrium, the first order condition with respect to $\hat{c}$ has to be satisfied at $\hat{c} = c$. With some algebra, the first order condition can be rearranged to

$$(1 - \omega'(P(c)))(1 - F(P(c))) - (P(c) - \omega(P(c)) - c)f(P(c)) = 0.$$

As (19) is the solution to this differential equation, it follows from the proof of Proposition 1 that the fee structure with the fee function $\omega(p_S) = p - E_v[P^{-1}(v) \mid v \geq p_S]$ induces the seller to set the intermediary the reserve in the intermediary optimal way. Thus, the mechanism described in Proposition 10 is the intermediary optimal allocation rule. □

Proposition 11: Non-optimality of price posting mechanisms.

Proof of Proposition 11. Part (a): The intermediary's expected profit with price posting is $(p^B - p^S)(1 - F(p^B))G(p^S)$. The assumptions about the inverse hazard rates ensure

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[73]It follows from Lemma 3 in Myerson (1981) that all standard auction formats will have the same expected revenue and indeed the same reserve price; see also Milgrom (2004, Ch.3) or Krishna (2002, Ch.5).

concavity of the profit function. Therefore, the unique maximum is given by the first order conditions. Taking derivatives with respect to p^B and p^S yields $p^S = \Phi(p^B)$ and $p^B = \Gamma(p^S)$.

We complete the proof by showing that trade with price posting neither implies nor is implied by trade in the intermediary optimal mechanism of Myerson and Satterthwaite for arbitrary distributions F and G.

Trade with price posting, no trade with the intermediary optimal mechanism. Take a buyer and a seller for whom trade just occurs with price setting, i.e. valuation p^B and cost p^S. We know that a profit maximizing intermediary will always set $p^B > p^S$. Combining this with the first order conditions we get $\Phi(p^B) = p^S < p^B = \Gamma(p^S)$. This implies by Lemma 2 (i) that no trade occurs with the optimal mechanism for valuation p^B and cost p^S.

Trade with the intermediary optimal mechanism, no trade with price posting. Take the lowest cost seller with cost $\underline{c}$ and a buyer with valuation v' such that trade just occurs with the optimal mechanism, i.e. $\Phi(v') = \Gamma(\underline{c})$. As $p^S > \underline{c}$ must hold for positive probabilities of trade with price posting, we have $\Phi(v') = \Gamma(\underline{c}) = \underline{c} < p^S = \Phi(p^B)$. This implies $v' < p^B$ and hence no trade with price posting.

Part (b): Index the realized valuations in decreasing and costs in increasing order. To avoid dealing separately with the special case where all buyers and sellers trade, add a fictitious buyer who will never trade with $v_{N+1} = -\infty$ and a fictitious seller with $c_{N+1} = \infty$.[74] Denote the Virtual-Walrasian quantity as defined before Lemma 1 as $K := \max\{i|\Phi(v_i) \geq \Gamma(c_i)\}$.

(i) We consider the case where $\text{plim}_{N\to\infty} K/N < 1$, i.e. not all buyers and sellers trade in the limit. For $\text{plim}_{N\to\infty} K/N = 1$ the proof is similar and therefore omitted.

It can be easily shown that for a finite number of buyers and sellers a dominant strategy implementation of the Virtual-Walrasian allocation rule is optimal: everyone reports their valuations/costs, a buyer pays the minimal valuation which would have been sufficient for him to get the good, the seller gets analogously the maximal cost.

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[74]The number of buyers and sellers being unequal could be dealt with in a similar fashion. If there are e.g. less sellers than buyers, the missing sellers can be filled up with fictitious sellers.

Formally, a buyer pays $\max\{v_{K+1}, \Phi^{-1}(\Gamma(c_K))\}$ and a seller gets $\min\{c_{K+1}, \Gamma^{-1}(\Phi(v_K))\}$.

The valuation of the marginal trading and non-trading buyers and the marginal seller's cost plus the spread charged by the intermediary converge in probability to the same value, which we denote as p^B:

$$\plim_{N\to\infty} v_K = \plim_{N\to\infty} v_{K+1} = \plim_{N\to\infty} \Phi^{-1}(\Gamma(c_K)) =: p^B. \tag{45}$$

Similarly,

$$\plim_{N\to\infty} c_K = \plim_{N\to\infty} c_{K+1} = \plim_{N\to\infty} \Gamma^{-1}(\Phi(v_K)) =: p^S. \tag{46}$$

For the fraction of buyers and sellers who trade we have

$$\plim_{N\to\infty} \frac{K}{N} = \plim_{N\to\infty} \frac{\max\{i|v_i \geq p^B\}}{N} = 1 - F(p^B), \tag{47}$$

$$\plim_{N\to\infty} \frac{K}{N} = \plim_{N\to\infty} \frac{\max\{i|c_i \geq p^S\}}{N} = G(p^S). \tag{48}$$

(45), (46), (47), and (48) imply that the optimal mechanism converges to price posting with p^B and p^S that satisfy $\Phi(p^B) = \Gamma(p^S)$ and $1 - F(p^B) = G(p^S)$.

(ii) Define the number of buyers willing to trade as $k_b := \max\{i|v_i \geq p^B\}$, and for the sellers $k_s := \max\{i|c_i \leq p^S\}$. By $1 - F(p^B) = G(p^S)$

$$\plim_{N\to\infty} \frac{k_b}{N} = 1 - F(p^B) = G(p^S) = \plim_{N\to\infty} \frac{k_s}{N}.$$

By $\Phi(p^B) = \Gamma(p^S)$ we have $\Phi(v_{k_b}) \geq \Phi(p^B) = \Gamma(p^S) \geq \Gamma(c_{k_s})$ and by analogy $\Phi(v_{k_b+1}) < \Gamma(c_{k_s+1})$. Therefore, the fraction of traded quantity is in the limit

$$\plim_{N\to\infty} \frac{\min\{k_b, k_s\}}{N} = \plim_{N\to\infty} \frac{\max\{i|\Phi(v_i) \geq \Gamma(c_i)\}}{N} \stackrel{\text{def}}{=} \plim_{N\to\infty} \frac{K}{N},$$

which is the fraction of the Virtual-Walrasian quantity. Further, it is easy to show that this mechanism is incentive compatible and gives zero utility to the most inefficient agents. Therefore, by Lemma 1 it maximizes the intermediary's profit.

Part (c): The intermediary can store the good costlessly in an infinite horizon setup and only has to make sure that he buys and sells with the same probability. Hence his problem is the same as if all buyers and sellers arrived at the same time. Therefore, by (b), price posting is optimal with the same prices as given in (b).   □

Proposition 12:   Fee setting and price posting mechanism for one seller and infinite number of buyers.

Proof of Proposition 12. As N_B converges to infinity, the highest bid almost surely converges to $\bar{v}$. Hence we are back to the one sided incomplete information problem. By Myerson (1981) the optimal mechanism is to post a price $p^S = \Gamma^{-1}(\bar{v})$ for the seller and extract full rents from the buyer with $p^B = \bar{v}$. This can also be represented as a fee setting mechanism with $\omega(p) = p[1 - \Gamma^{-1}(\bar{v})/\bar{v}]$, which induces the seller to set $P(c) = \bar{v}$. $\square$

Proposition 13:   Slotting Allowances.

Proof of Proposition 13. In optimum, trade occurs between the lowest cost seller and the buyer if and only if this seller's virtual cost is less than the buyer's virtual valuation (see Lemma 1). The fee $\omega(p)$ of Proposition 1 makes sure that the seller active in stage 2 sets the price in exactly such a way that the buyer buys if and only if $\Phi(v) \geq \Gamma(c)$. Denote a seller's stage 2 expected payoff as $U_S(c)$ (under the fee ω). It is a dominant strategy in a second-price auction for a seller to bid $U_S(c)$ in stage 1. Since $U_S'(c) > 0$ whenever $U_S(c) > 0$, a second price auction (or in fact any standard auction) in stage 1 allocates the right to set the price efficiently, i.e. to the lowest virtual cost seller. (Notice that all those types of sellers for whom $U_S(c) = 0$ will set such a high price in stage 2 that they will never sell. Therefore, it is immaterial whether a winner is determined or not if every seller bids zero in stage 1.) Consequently, the mechanism implements the intermediary optimal allocation rule. Moreover, the expected payoffs of sellers with costs $\bar{c}$ and the buyer with valuation $\underline{v}$ is zero. $\square$

B   Robustness of Linear Fees

In the following, we will make a semi-formal argument in favor of the robustness of linear fees, i.e. that they perform well even if the seller's cost is not drawn from a power distribution. Alternatively, this can be interpreted as arguing in favor of using a power distribution as an approximation of another distribution G. For analytical convenience,

we assume that the buyer's valuation is drawn from the uniform on $[0,1]$, so that given the linear fee ξ the seller's optimal price is $p(\xi, c) = 1/2 + c/(2\xi)$ independently of the distribution G_k. We consider the following four different distributions of c on $[0,1]$:

(i) $G_H(c) = c^2(3 - 2c)$ with $g_H(c) = 6c(1 - c)$ and $\Gamma_H(c) = \frac{c}{1-c}\frac{9-8c}{6}$

(ii) $G_T(c) = 2c - c^2$ with $g_T(c) = 2(1 - c)$ and $\Gamma_T(c) = \frac{c}{1-c}\frac{4-2c}{2}$

(iii) $G_C(c) = 3(c - c^3/3)/2$ with $g_C(c) = 3(1 - c^2)/2$ and $\Gamma_C(c) = \frac{2c(3-2c^2)}{1-c^2}$

(iv) $G_O(c) = 2c^{3/2}/3 - 2c^{5/2}/5$ with $g_O(c) = 15c^{1/2}(1 - c)/4$ and $\Gamma_O(c) = \frac{c}{1-c}\frac{25-21c}{15}$.

Observe that all of these satisfy increasing virtual costs. Denote by $\Pi_{G_k}^* := E_{v,c}[\Phi - \Gamma_k \mid \Phi - \Gamma_k > 0]$ the intermediary's expected profit under the optimal mechanism when the distribution is G_k and by $\Pi_{G_k}^L = \max_\xi (1 - \xi) \int_0^\xi p(\xi, c)(1 - p(\xi, c))g_k(c)dc$ the intermediary's expected profit under the same distribution when using the linear fee ξ optimally with $k \in \{H, T, C, O\}$. Quite interestingly, for all examples $\Pi_{G_k}^L > 0.9979\Pi_{G_K}^*$.

For these examples, and it seems reasonable to conjecture also for others, even if the intermediary merely uses an optimal linear mechanism he only loses very little of the profit he could achieve using an optimal mechanism. Though farther analysis is certainly warranted, we find this result quite remarkable.

C   The General Mechanism Design Problem

Preliminaries.   We first define the ironed virtual valuations $\overline{\Phi}$ and $\overline{\Gamma}$, following the procedure in Myerson (1981, p.68). For the buyer define

$$J_b(q) := \text{conv} \int_0^q \Phi(F^{-1}(r))dr,$$

where conv stands for the convex hull of a function. The ironed virtual valuation function of the buyer can now be defined as $\overline{\Phi}(v) = J_b'(F(v))$. It is easy to check that $\overline{\Phi}(v) = \Phi(v)$ wherever $\overline{\Phi}$ is strictly increasing.

For the seller, construct the ironed virtual cost function in a similar way by defining

$$J_s(q) := \text{conv} \int_0^q \Gamma(G^{-1}(r))dr,$$

and the ironed virtual cost function as $\overline{\Gamma}(c) = J_s'(G(c))$.

Lemma 3. *In the general case (without Myerson's regularity assumption), it is optimal for the intermediary to allow trade iff $\overline{\Phi}(v) \geq \overline{\Gamma}(c)$.*

Proof. By the same logic as Myerson's (1981) theorem on p. 69. $\square$

Optimal Fee Setting Mechanism. The easiest way to derive the optimal fee is by a brief detour through the dominant strategy implementation of the optimal allocation rule. Similarly to Myerson and Satterthwaite (1983, p.280), we can implement the allocation rule with dominant strategies in the general case by letting the buyer pay $\overline{P}(c) := \overline{\Phi}^{-1}(\overline{\Gamma}(c))$ and the seller get $\overline{P}^{-1}(v) := \overline{\Gamma}^{-1}(\overline{\Phi}(v))$ in case of trade, where the inverses are redefined as

$$\overline{\Phi}^{-1}(x) := \min\{v|v \geq \underline{v} \text{ and } \overline{\Phi}(v) \geq x\}, \tag{49}$$

$$\overline{\Gamma}^{-1}(x) := \max\{c|c \leq \overline{c} \text{ and } \overline{\Gamma}(c) \leq x\}. \tag{50}$$

Without trade, both get 0. We can now use the same logic as in the main text to derive the optimal fee setting mechanism. The dominant strategy implementation already gives us the right price for the buyer, $\overline{P}(c)$. To make sure that the seller's payment only depends on c, we have to take expectation of $\overline{P}^{-1}(v)$, which he gets in the dominant strategy implementation. The net price the seller gets is $E_v[\overline{P}^{-1}(v)|v \geq \overline{P}(c)]$, since we have to take expectations conditional on trade taking place. The optimal fee is the gross minus the net price, in the indirect mechanism representation (using p rather than $\overline{P}(c)$) this is

$$\omega(p) = p - E_v[\overline{P}^{-1}(v)|v \geq p] \tag{51}$$

for $p \leq \overline{v}$ and any $\omega(p) \geq \overline{v} - p$ else. This is the same as in the main text, except that we are using ironed virtual valuations. If $\overline{\Phi}(\overline{v}) > \overline{\Gamma}(\overline{c})$ (Assumption 1 violated), the fee becomes

$$\omega(p) = p - E_v[\overline{P}^{-1}(v)|v \geq p] + \left\{E[\overline{P}^{-1}(v)|v \geq P(\overline{c})] - P(\overline{c})\right\} \tag{52}$$

where the expression in braces makes sure that $\overline{c}$ sellers have no informational rents (see Lemma 2 (ii)).

The price paid by the buyer, $\overline{P}(c)$ is a left continuous function (the discontinuities being determined by the discontinuities of $\overline{\Phi}^{-1}$) with horizontal intervals where $\overline{\Gamma}$ is horizontal. The inverse $\overline{P}^{-1}(v)$ is a right continuous function (discontinuities given by $\overline{\Gamma}^{-1}$) with horizontal intervals where $\overline{\Phi}$ is horizontal

References

ANDERSON, S., AND R. RENAULT (1999): "Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model," *RAND Journal of Economics*, 30(4), 719–735.

ANGLIN, P., AND R. ARNOTT (1991): "Residential real estate brokerage as a principal-agent problem," *The Journal of Real Estate Finance and Economics*, 4(2), 99–125.

ARNOLD, M. (1992): "The Principal-Agent Relationship in Real Estate Brokerage Services," *Real Estate Economics*, 20(1), 89–106.

ATAKAN, A. E. (2006a): "Assortative Matching with Explicit Search Costs," *Econometrica*, 74(3), 667–680.

——— (2006b): "Competitive Equilibria in Decentralized Matching with Incomplete Information," Discussion Papers 1437, Northwestern University, Center for Mathematical Studies in Economics and Management Science.

BAGNOLI, M., AND N. KHANNA (1991): "Buyers'and sellers' agents in the housing market," *The Journal of Real Estate Finance and Economics*, 4(2), 147–156.

BALIGA, S., AND R. VOHRA (2003): "Market Research and Market Design," *Advances in Theoretical Economics*, 3(1), Article 5.

BULOW, J., AND J. ROBERTS (1989): "The Simple Economics of Optimal Auctions," *Journal of Political Economics*, 97(5), 1060–90.

BURANI, N. (2008): "Matching, search and intermediation with two-sided heterogeneity," *Review of Economic Design*, 12, 75–117.

CHATTERJEE, K., AND W. SAMUELSON (1983): "Bargaining under Asymmetric Information," *Operations Research*, 31, 835–851.

CHEN, H., AND J. RITTER (2000): "The Seven Percent Solution," *Journal of Finance*, 55(3), 1105–1131.

CONRAD, J. S., K. M. JOHNSON, AND S. WAHAL (2001): "Institutional trading and soft dollars," *Journal of Finance*, 56, 397–416.

DAMIANOV, D. (2005): "Competition among Sellers by Mechanism Design," .

DE FRAJA, G., AND J. SAKOVICS (2001): "Walras Retrouve: Decentralized Trading Mechanisms and the Competitive Price," *Journal of Political Economy*, 109(4), 842–863.

DIAMOND, P. (1971): "A Model of Price Adjustment," *Journal of Economic Theory*, 3(2), 156–168.

DOJ (2007): "Competition in the Real Estate Brokerage Industry," *Report by the Federal Trade Commission and the U.S. Department of Justice*.

DUFFIE, D., N. GARLEAU, AND L. H. PEDDERSEN (2005): "Over-the-Counter Markets," *Econometrica*, 73(6), 1815–47.

DUFFIE, D., AND B. STRULOVICI (2008): "Capital Mobility and Asset Pricing," *Mimeo*.

GALE, D. (2000): *Strategic Foundations of General Equilibrium: Dynamic Matching and Bargaining Games*. Cambridge University Press.

GARMAN, M. B. (1976): "Market microstructure," *Journal of Financial Economics*, 3(3), 257–275.

GEHRIG, T. (1993): "Intermediation in Search Markets," *Journal of Economics & Management Strategy*, 2(1), 97–120.

GENESOVE, D., AND C. J. MAYER (1997): "Equity and Time to Sale in the Real Estate Market," *American Economic Review*, 87(3), 255–269.

———— (2001): "Loss Aversion and Seller Behavior: Evidence from the Housing Market," *Quarterly Journal of Economics*, 116(4), 1233–1260.

GLOSTEN, L. R., AND P. R. MILGROM (1985): "Bid, ask and transaction prices in a specialist market with heterogeneously informed traders," *Journal of Financial Economics*, 14(1), 71–100.

GRESIK, T., AND M. SATTERTHWAITE (1989): "The Rate at Which a Simple Market Converges to Efficiency as the Number of Traders Increases: An Asymptotic Result for Optimal Trading Mechanisms," *Journal of Economic Theory*, 48, 304.

HACKETT, S. (1992): "A comparative analysis of merchant and broker intermediation," *Journal of Economic Behavior and Organization*, 18(3), 299–315.

HAGIU, A. (2006): "Merchant or Two-Sided Platform?," *Working Paper*.

HENDEL, I., A. NEVO, AND F. ORTALO-MAGNÉ (forthcoming): "The Relative Performance of Real Estate Marketing Platforms: MLS versus FSBOMadison.com," *American Economic Review*.

HSIEH, C., AND E. MORETTI (2003): "Can Free Entry Be Inefficient? Fixed Commissions and Social Waste in the Real Estate Industry," *Journal of Political Economy*, 111(5), 1076–1122.

KANG, H. B., AND M. J. GARDNER (1989): "Selling Price and Marketing Time in the Residential Real Estate Market," *Journal of Real Estate Research*, 4(1), 21–35.

KRISHNA, V. (2002): *Auction Theory*. Elsevier Science, Academic Press.

LARSEN, J., AND W. PARK (1989): "Non-Uniform Percentage Brokerage Commissions and Real Estate Market Performance," *Real Estate Economics*, 17(4), 422–438.

LAUERMANN, S. (2007): "Dynamic Matching and Bargaining Games: A General Approach," *University of Michigan mimeo*.

LAUERMANN, S., AND A. WOLINSKY (2008): "Search with Adverse Selection," Presentation at the Midwest Economic Theory Meeting Fall 2008.

LEVITT, S. D., AND C. SYVERSON (2008): "Market Distortions When Agents Are Better Informed: The Value of Information in Real Estate Transactions," *Review of Economics and Statistics*, 90(4), 599–611.

LEWIS, T., AND M. OTTAVIANI (2008): "Search Agency," mimeo, Northwestern Univeristy.

LOERTSCHER, S. (2007): "Horizontally Differentiated Market Makers," *Journal of Economics & Management Strategy*, 16(4), 793–825.

MANKIW, N. G., AND M. D. WHINSTON (1986): "Free Entry and social efficiency," *Rand Journal of Economics*, 17(1), 48–58.

MARX, L., AND G. SCHAFFER (2007): "Upfront Payments and Resale Price Maintenance: A Comparison of Facilitating Practices," *Rand Journal of Economics*, 22, 120–135.

MATROS, A., AND A. ZAPECHELNYUK (2006): "Optimal Mechanisms for an Auction Mediator," Discussion Paper Series dp424, Center for Rationality and Interactive Decision Theory, Hebrew University, Jerusalem.

MCAFEE, R. P. (1993): "Mechanism Design by Competing Sellers," *Econometrica*, 61(6), 1281–1312.

MILGROM, P. (2004): *Putting Auction Theory to Work*. Cambridge University Press.

MILGROM, P., AND I. SEGAL (2002): "Envelope Theorems for Arbitrary Choice Sets," *Econometrica*, 70(2), 583–601.

MIRRLEES, J. A. (1971): "An Exploration in the Theory of Optimum Income Taxation," *Review of Economic Studies*, 38, 175–208.

MORENO, D., AND J. WOODERS (2002): "Prices, Delay, and the Dynamics of Trade," *Journal of Economic Theory*, 104(2), 304–339.

MORTENSEN, D., AND R. WRIGHT (2002): "Competitive Pricing and Efficiency in Search Equilibrium," *International Economic Review*, 43(1), 1–20.

MYERSON, R. (1981): "Optimal auction design," *Mathematical Operations Research*, 6(1), 58–73.

MYERSON, R., AND M. SATTERTHWAITE (1983): "Efficient Mechanisms for Bilateral Trading," *Journal of Economic Theory*, 29(2), 265–281.

NEW YORK TIMES (2008): "Hirsts Art Auction Attracts Plenty of Bidders, Despite Financial Turmoil, September 16, 2008," http://www.nytimes.com/2008/09/16/arts/design/16auct.html.

PARLOUR, C. A., AND U. RAJAN (2003): "Payment for order flow," *Journal of Financial Economics*, 68, 379–411.

PECK, J. (1996): "Competition in Transactions Mechanisms: The Emergence of Price Competition," *Games and Economic Behavior*, 16(1), 109–123.

PETERS, M., AND S. SEVERINOV (1997): "Competition Among Sellers who offer Auctions instead of Prices," *Journal of Economic Theory*, 75(1), 141–179.

——— (2006): "Internet Auctions with Many Traders," *Journal of Economic Theory*, pp. 220–245.

RILEY, J., AND R. ZECKHAUSER (1983): "Optimal Selling Strategies: When to Haggle, When to Hold Firm ," *Quarterly Journal of Economics*, pp. 267–89.

RUST, J., AND G. HALL (2003): "Middlemen versus Market Makers: A theory of Competitive Exchange," *Journal of Political Economy*, 111(2), 353–403.

RUTHERFORD, R., T. SPRINGER, AND A. YAVAS (2005): "Conflicts between principals and agents: evidence from residential brokerage," *Journal of Financial Economics*, 76(3), 627–665.

SALOP, S. (1979): "Monopolistic Competition with outside goods," *Bell Journal of Eonomics*, 10(1), 141–56.

SATTERTHWAITE, M., AND A. SHNEYEROV (2007): "Dynamic Matching, Two-Sided Information, and Participation Costs: Existence and Convergence to Perfect Competition," *Econometrica*, 75(1), 155–200.

SATTERTHWAITE, M., AND A. SHNEYEROV (2008): "Convergence to perfect competition of a dynamic matching and bargaining market with two-sided incomplete information and exogenous exit rate.," *Games and Economic Behavior*, 63(2), 435–467.

SERRANO, R. (2002): "Decentralized information and the Walrasian outcome: a pairwise meetings market with private values," *Journal of Mathematical Economics*, 38(1-2), 65–89.

SPULBER, D. F. (1988): "Bargaining and regulation with asymmetric information about demand and supply.," *Journal of Economic Theory*, 44(2), 251–268.

———— (1996): "Market Making by Price-Setting Firms," *Review of Economic Studies*, 63(4), 559–580.

———— (1999): *Market Microstructure: Intermediaries and the Theory of the Firm.* Cambridge University Press, Cambridge.

———— (2008): *The Theory of the Firm: Microeconomics With Endogenous Entrepreneurs, Firms, Markets, and Organizations.* Cambridge University Press: Cambridge.

SULLIVAN, M. (1997): "Slotting Allowances and the Market for New Products," *Journal of Law and Economics*, 40(2), 461–493.

VICKREY, W. (1961): "Counterspeculation, auctions and competitive sealed tenders," *Journal of Finance*, 16(1), 8–37.

WILLIAMS, J. (1998): "Agency and brokerage of real assets in competitive equilibrium," *Review of Financial Studies*, 11(2), 239–280.

WOLINSKY, A. (1988): "Dynamic Markets with Competitive Bidding," *Review of Economic Studies*, 55(1), 71–84.

WOODERS, J. (1997): "Equilibrium in a market with intermediation is Walrasian," *Review of Economic Design*, 3(1), 75–89.

YAVAS, A. (1992): "Marketmakers versus Matchmakers," *Journal of Financial Intermediation*, 2, 33–58.

———— (1996): "Search and Trading in Intermediated Markets," *Journal of Economics & Management Strategy*, 5, 195–216.

YAVAS, A., AND S. YANG (1995): "The Strategic Role of Listing Price in Marketing Real Estate: Theory and Evidence," *Real Estate Economics*, 23(3), 347–368.