Merger Policy with Merger Choice[*]

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Abstract

We analyze the optimal policy of an antitrust authority towards horizontal mergers when merger proposals are endogenous and firms choose which of several mutually exclsuive mergers to propose.

1 Introduction

The evaluation of proposed horizontal mergers involves a basic trade-off: mergers may increase market power, but may also create efficiencies. Whether a given merger should be approved depends, as first emphasized by Williamson (1968), on a balancing of these two effects.

In most of the literature discussing horizontal merger evaluation, the assumption is that a merger should be approved if and only if it improves welfare, whether that be aggregate surplus or just consumer surplus, as is in practice the standard adopted by most antitrust authorities [see, e.g., Farrell and Shapiro (1993), McAfee and Williams (1992)]. This paper contributes to a small literature that formally derives optimal merger approval rules. This literature started with Besanko and Spulber (1993), who discussed the optimal rule for an antitrust authority who cannot directly observe efficiencies but who recognizes that firms know this information and decide whether to propose a merger based on this knowledge. Other recent papers in this literature include Nocke and Whinston (2008) Ottaviani and Wickelgren (2009), and Armstrong and Vickers (2010).

In this paper, we focus on a setting in which one firm may merge with one of a number of other firms. These mergers are mutually exclusive, and each may result in a different post-merger cost level. The merger that is proposed is the result of a bargaining process among the

firms. The antitrust authority observes the characteristics of the merger that is proposed, but neither the feasibility nor the characteristics of any mergers that are not proposed. We focus in the main part on an antitrust authority who wishes to maximize expected consumer surplus. Our main result characterizes the form of the antitrust authority's optimal policy, which we show should impose a tougher standard on mergers involving larger acquirers (in terms of their pre-merger share). Specifically, the minimal acceptable level of increase in consumer surplus is strictly positive for all but the smallest acquirer, and is larger the larger is the acquirer's premerger share.

The closest papers to our are Lyons (2003) and Armstrong and Vickers (2010). Lyons first identifies the issue that arises when firms may choose which merger to propose. Armstrong and Vickers (2010) provide an elegant characterization of the optimal policy when mergers (or, more generally, projects that may be proposed by an agent) are ex ante identical in terms of their distributions of possible outcomes. Our paper differs from Armstrong and Vickers (2010) primarily in its focus on the optimal differential treatment of mergers that differ in this ex ante sense.

The paper is also related to Nocke and Whinston (2008). That paper established conditions under which the optimal dynamic policy for an antitrust authority who wants to maximize discounted expected consumer surplus is a completely myopic policy, in which a merger is approved if and only if it does not lower consumer surplus at the time it is proposed. One of the important assumptions for that result was that potential mergers were "disjoint," in the sense that the set of firms involved in different possible mergers do not overlap. The present paper explores, in a static setting, the implications of relaxing that disjointness assumption, focusing on the polar opposite case in which all potential mergers are mutually exclusive.

The paper proceeds as follows: We describe the model in Section 1. Section 2 derives our main result, which characterizes the optimal policy in the case in which the proposed merger maximizes industry aggregate profit due to efficient bargaining among the firms. In Section 4 we show that our main characterization extends to some cases in which that bargaining is not efficient, including the case of the Segal (1999) offer game. Section 5 discusses some other extensions of our results, and Section 6 concludes.

2   The Model

We consider a homogeneous goods industry in which firms compete in quantities (Cournot competition). Let $\mathcal{N} = \{0, 1, 2, ..., N\}$ denote the (initial) set of firms. All firms have constant returns to scale; firm i's marginal cost is denoted c_i. Inverse demand is given by $P(Q)$. We impose standard assumptions on demand:

Assumption 1. *For all Q such that $P(Q) > 0$, we have:*

(i) $P'(Q) < 0$;

(ii) $P'(Q) + QP''(Q) < 0$;

(iii) $\lim_{Q \to \infty} P(Q) = 0$.

It is well known that under these conditions there exists a unique Nash equilibrium in quantities. Moreover, this equilibrium is "stable" (each firm i's best-response function $b_i(Q_{-i}) \equiv \arg\max_{q_i}[P(Q_{-i} + q_i) - c_i]q_i$ satisfies $b_i'(Q_{-i}) \in (-1, 0)$, where $Q_{-i} \equiv \sum_{j \neq i} q_j$) so that comparative statics are "well behaved" (if a subset of firms jointly produce less [more] because of a change in their incentives to produce output, then equilibrium industry output will fall [rise]). The vector of output levels in the pre-merger equilibrium is given by $q^0 \equiv (q_0^0, q_1^0, ..., q_n^0)$, where q_i^0 is firm i's quantity. For simplicity, we assume that pre-merger marginal costs are such that all firms in $\mathcal{N}$ are "active" in the pre-merger equilibrium, i.e., $q_i^0 > 0$ for all i. Aggregate output, price, consumer surplus, firm i's profit and aggregate profit in the pre-merger equilibrium are denoted $Q^0 \equiv \sum_i q_i^0$, $P^0 \equiv P(Q^0)$, CS^0, $\pi_i^0 \equiv [P(Q^0) - c_i]q_i^0$, and $\Pi^0 \equiv \sum_{i \in \mathcal{N}} \pi_i^0$, respectively.

Suppose that there is a set of K potential mergers, each between firm 0 (the "target") and a single merger partner (an "acquirer") $k \in \mathcal{K} \subseteq \mathcal{N}$. There is a random variable $\phi_k \in \{0, 1\}$ that determines the feasibility of the merger between firm 0 and firm k. If $\phi_k = 1$, the merger is feasible. A feasible merger is described by $M_k = (k, \bar{c}_k)$, where k is the identity of the acquirer and $\bar{c}_k$ the (realized) post-merger marginal cost, which is drawn from distribution function G_k with support $[l, h_k]$ and no mass points. The random draws of ϕ_k and $\bar{c}_k$ are independent across mergers. The set of feasible mergers is denoted $\mathfrak{F} \equiv \{M_k : \phi_k = 1\} \cup M_0$, where M_0 denotes the status quo (or "null merger") resulting in outcome $q(M_0) \equiv q^0$, $s_i(M_0) \equiv q_i^0/Q^0$, $\Pi(M_0) \equiv \Pi^0$, and $\Delta CS(M_0) = \Delta\Pi(M_0) = 0.$. If merger M_k for $k \geq 1$ is implemented, the vector of outputs in the resulting post-merger equilibrium is denoted $q(M_k) \equiv (q_1(M_k), ..., q_N(M_k))$, where $q_k(M_k)$ is the output of the merged firm, aggregate output is $Q(M_k) \equiv \sum_i q_i(M_k)$, and firm i's market share is $s_i(M_k) \equiv q_i(M_k)/Q(M_k)$. The post-merger profit of non-merging firm i is given by $\pi_i(M_k) \equiv [P(Q(M_k)) - c_i]q_i(M_k)$, the merged firm's profit by $\pi_k(M_k) \equiv [P(Q(M_k)) - \bar{c}_k]q_k(M_k)$, and aggregate profit by $\Pi(M_k) \equiv \sum_{i \in \mathcal{N} \setminus \{0\}} \pi_i(M_k)$. The induced change in consumer surplus is

$$\Delta CS(M_k) \equiv \left\{ \int_0^{Q(M_k)} P(s)ds - P(Q(M_k))Q(M_k) \right\} - CS^0,$$

and the induced change in aggregate profit is $\Delta\Pi(M_k) \equiv \Pi(M_k) - \Pi^0$.

As these mergers are mutually exclusive, at most one merger can be proposed to the antitrust authority. If merger M_k, $k \in \mathfrak{F}$, is proposed, the antitrust authority can observe all aspects of that merger. We assume that the antitrust authority can commit ex ante to a merger-specific approval policy by specifying an approval set $\mathcal{A} \equiv \{M_k : \bar{c}_k \in \mathcal{A}_k\} \cup M_0$, where $\mathcal{A}_k \subseteq [l, h_k]$ for $k \in \mathcal{K}$ are the post-merger marginal cost levels that would lead to approval of merger k. Because of our assumption of full support and no mass points, we can without loss of generality restrict attention to the case where each $\mathcal{A}_k$ is a (finite or infinite) union of closed intervals, i.e., $\mathcal{A}_k \equiv \cup_{r=1}^R [l_k^r, h_k^r]$, where $l \leq l_k^r < h_k^r \leq h_k$ (R can be infinite). Note that the status quo M_0 is always "approved."

Given the set of feasible mergers $\mathfrak{F}$ and the antitrust authority's approval set $\mathcal{A}$, the set of feasible mergers that would be approved if proposed is given by $\mathfrak{F} \cap \mathcal{A}$. A bargaining process amongst the firms determines which feasible merger is actually proposed. Note that this bargaining problem involves externalities as firms' payoffs depend on the identity of the acquirer. There are various ways in which one could model this situation. In much of the

paper, we focus on the benchmark case in which bargaining is efficient from the viewpoint of the industry. That is, we assume for now that firms propose the merger $M^*(\mathfrak{F}, \mathcal{A})$ from the set of feasible and approvable mergers that maximizes aggregate profit, where for any set of mergers $\mathfrak{F} \cap \mathcal{A}$,

$$M^*(\mathfrak{F}, \mathcal{A}) \equiv \arg \max_{M_k \in \mathfrak{F} \cap \mathcal{A}} \Delta \Pi(M_k).$$

There are several bargaining processes which could lead to joint profit maximization:

1. Multilateral "Coasian bargaining" under complete information amongst all firms would lead to an efficient (aggregate-profit maximizing) outcome.

2. Suppose the auctioneer (here, firm 0) conducts a "menu auction" in which each firm $i \geq 1$ submits a nonnegative bid $b_i(M_k) \geq 0$ for each feasible and approvable merger $M_k \in (\mathfrak{F} \cap \mathcal{A})$ with $k \geq 1$. Firm 0 then selects the merger that maximizes its profit, where the profit from selecting merger M_k is given by the sum of all bids for that merger, $\sum_{i \in \mathcal{N} \setminus \{0\}} b_i(M_k)$, and the profit from selecting the null merger M_0 is $\pi_0(M_0)$. Bernheim and Whinston (1996) show that there is an efficient equilibrium which, in this setting, implements the merger that maximizes aggregate profit.

3. Suppose the target (firm 0) can commit to any sales mechanism. Jehiel, Moldovanu and Stacchetti (1996) show that one such optimal mechanism has the following structure: As a compensation for implementing merger $M_k \in (\mathfrak{F} \cap \mathcal{A})$, the principal (here, the target) requires payment $\pi_i(M_k) - \pi_i(\underline{M}_i)$ from each firm $i \geq 1$, where $\underline{M}_i \in (\mathfrak{F} \cap \mathcal{A})$ is the merger that minimizes firm i's profit. If a firm i does not make its payment, then the principal commits to proposing merger $\underline{M}_i$ to the antitrust authority [who will then approve it since $\underline{M}_i \in (\mathfrak{F} \cap \mathcal{A})$].[1] Jehiel, Moldovanu and Stacchetti show that there exists an equilibrium in which all firms participate in the mechanism. Given the set of feasible and approvable mergers, $\mathfrak{F} \cap \mathcal{A}$, the resulting outcome maximizes aggregate profit; that is, merger $M^*(\mathfrak{F}, \mathcal{A})$ is proposed.[2]

In line with legal standards in the U.S. and many other countries, we assume that the antitrust authority acts in the consumers' interests. That is, the antitrust authority selects the approval set $\mathcal{A}$ that maximizes expected consumer surplus given that firms' proposal rule is $M^*(\cdot)$:

$$\max_{\mathcal{A}} E_{\mathfrak{F}} \left[\Delta CS\left(M^*(\mathfrak{F}, \mathcal{A})\right) \right],$$

where the expectation is taken with respect to the set of feasible mergers, $\mathfrak{F}$. (We discuss aggregate surplus maximization in Section 4.)

We are interested in studying how the optimal approval set depends on the pre-merger characteristics of the alternative mergers. For this reason, we assume that the potential acquirers differ in terms of their pre-merger marginal costs. Without loss of generality, let

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[1] That is, similar to Bernheim and Whinston's (1996) menu auction, firms make payments even for mergers that they are not a party to.

[2] To see this, note that the target's program can be written as:

$$\max_{M_k \in (\mathfrak{F} \cap \mathcal{A})} \Pi(M_k) - \sum_{i \in \mathcal{N}} \pi_i(\underline{M}_i).$$

But this is equivalent to $\max_{M_k \in (\mathfrak{F} \cap \mathcal{A})} \Pi(M_k)$.

$\mathcal{K} \equiv \{1, ..., K\}$ and re-label firms 1 through K in decreasing order of their pre-merger marginal costs: $c_1 > c_2 > ... > c_K$. Thus, in the pre-merger equilibrium, firm $k \in \mathcal{K}$ produces more than firm $j \in \mathcal{K}$, and has a larger market share, if $k > j$. We will say that merger M_k is *larger* than merger M_j if $k > j$ as the combined pre-merger market share of firms 0 and k is larger than that of firms 0 and j.

3 Optimal Merger Policy with Efficient Bargaining

We now investigate the form of the antitrust authority's optimal policy when firms bargain efficiently to determine the merger that is proposed. Given a set of feasible mergers $\mathfrak{F}$ and an approval set $\mathcal{A}$, this bargaining process results in the merger $M^*(\mathfrak{F}, \mathcal{A})$, as discussed in the previous section. We begin with some preliminary observations before turning to our main result.

3.1 Preliminaries

As firms produce a homogeneous good, a merger M_k raises [reduces] consumer surplus if and only if it raises [reduces] aggregate output Q. The following lemma summarizes some useful properties of a *CS-neutral* merger M_k, i.e., a merger that leaves consumer surplus unchanged, $\Delta CS(M_k) = 0$.

Lemma 1. *Suppose merger M_k is CS-neutral. Then*

1. *the merger causes no changes in the output of any nonmerging firm $i \notin \{0, k\}$ nor in the joint output of the merging firms 0 and k;*

2. *the merged firm's margin at the pre- and post-merger price $P(Q^\circ)$ equals the sum of the merging firms' pre-merger margins:*

$$P(Q^\circ) - \bar{c}_k = [P(Q^\circ) - c_0] + [P(Q^\circ) - c_k] ; \tag{1}$$

3. *the merger is profitable for the merging firms;*

4. *the merger increases the aggregate profit Π.*

Proof. See Nocke and Whinston (2008) for a proof of parts (1)-(3). For part (4), note that the merger raises the joint profit of the merging firms 0 and k by part (3) and it leaves the profit of any nonmerging firm unchanged (as neither price nor their output changes). $\square$

Rewriting equation (1), merger M_k is CS-neutral if the post-merger marginal cost satisfies

$$\bar{c}_k = \widehat{c}(Q^0) \equiv c_k - [P(Q^0) - c_0] . \tag{2}$$

A lower post-merger marginal cost induces a larger aggregate output, so that the merger is CS-increasing [i.e., $\Delta CS(M_k) > 0$] if $\bar{c}_k < \widehat{c}(Q^0)$ and CS-decreasing [i.e., $\Delta CS(M_k) < 0$] if $\bar{c}_k > \widehat{c}(Q^0)$.

An implication of (2), emphasized by Farrell and Shapiro (1990), is that a CS-neutral merger must involve a reduction in marginal cost below the marginal cost level of the more efficient merger partner: i.e., M_k can be CS-neutral only if $\bar{c}_k < \min\{c_0, c_k\}$.

We next introduce an assumption on how reductions in post-merger marginal cost affect the aggregate profit Π:[3]

Assumption 2 *If merger M_k for $k \geq 2$ is CS-nondecreasing [i.e., $\bar{c}_k \leq \widehat{c}(Q^0)$], then reducing its post-merger marginal cost $\bar{c}_k$ increases the aggregate profit Π.*

As we now show, this assumption must hold for merger M_k if whenever it is CS-nondecreasing we have $\bar{c}_k \leq \min_{l \neq k} c_l$; i.e., the merged firm has the lowest marginal cost. Since this would always be true were the firms in set $\mathcal{N}\backslash\{0\}$ to have identical initial marginal costs, it clearly holds provided their initial marginal costs are sufficiently close. To see why Assumption 2 holds in this case, note that summing up the post-merger first-order conditions for profit maximization yields

$$\Pi = \sum_{i \in \mathcal{N}\backslash\{0\}} [P(Q) - c_i]\, q_i = \left|Q^2 P'(Q)\right| H, \tag{3}$$

where $H \equiv \sum_{i \in \mathcal{N}\backslash\{0\}} (s_i)^2$ is the post-merger industry Herfindahl Index. Assumption 1 ensures that the first term, $\left|Q^2 P'(Q)\right|$, is increasing in Q. As reducing a firm's marginal cost leads to a larger Q, a sufficient condition for the claim to hold is that reducing the merged firm's marginal cost induces an increase in H. But this is indeed the case if the merged firm has lower costs, and hence a larger market share, than any of its (unmerged) rivals, since then a further reduction in its marginal cost increases its share and lowers the shares of all of its rivals, increasing H (see Lemma 4 in the Appendix).

To make the antitrust authority's problem interesting, and avoid certain degenerate cases we will henceforth assume the following:

Assumption 3 *For all $k \in \mathcal{K}$, the probability that the merger M_k is CS-increasing is positive but less than one: $\Delta CS(k, h_k) < 0 < \Delta CS(k, l)$.*

The following lemma gives a key result that indicates that there is a systematic bias in the proposal incentives of firms, relative to the interests of consumers, in favor of larger mergers:

Lemma 2. *Suppose two mergers, M_j and M_k, with $k > j \geq 1$, induce the same non-negative change in consumer surplus, $\Delta CS(M_j) = \Delta CS(M_k) \geq 0$. Then the larger merger M_k induces a greater increase in aggregate profit: $\Delta\Pi(M_k) > \Delta\Pi(M_j) > 0$.*

Proof. From the discussion above, the post-merger joint profit of the firms is given by (3). As both mergers induce the same level of consumer surplus (and thus the same Q), the first term on the right hand side of (3) is the same for both mergers. It thus suffices to show that the larger merger M_k induces a larger value of H than the smaller merger M_j.

Now, as both mergers induce the same Q, Assumption 1 implies that the output of any firm not involved in M_j or M_k is the same under both mergers. Hence,

$$s_k(M_k) + s_j(M_k) = s_k(M_j) + s_j(M_j). \tag{4}$$

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[3]Note that we do not require this assumption to hold for the smallest merger, M_1.

Next, recall that a CS-nondecreasing merger increases the share of the merging firms and reduces the share of all nonmerging firms. Thus, we have $s_k(M_k) \geq s_k + s_0 > s_k(M_j)$ and $s_j(M_j) \geq s_j + s_0 > s_j(M_k)$. In addition, since total output is the same after both mergers and $c_k < c_j$, we also have $s_j(M_k) < s_k(M_j)$. By (4), this in turn implies that $s_k(M_k) > s_j(M_j)$. Hence, the distribution of market shares after the larger merger M_k is a sum-preserving spread of those after the smaller merger M_j:

$$s_k(M_k) > \max\{s_j(M_j), s_k(M_j)\} \geq \min\{s_j(M_j), s_k(M_j)\} > s_j(M_k). \qquad (5)$$

By Lemma 4 in the Appendix, H is therefore larger after M_k than after M_j. $\qquad \square$

Assumptions 1-3 and Lemma 2 imply that the possible mergers can be represented as shown in Figure 1 (where the are four possible mergers; i.e., $K = 4$). In the figure, the change in the aggregate profit, $\Delta\Pi$, is measured on the horizontal axis and the change in consumer surplus, ΔCS, is measured on the vertical axis. The CS-increasing mergers therefore are those lying above the horizontal axis. The aggregate profit and consumer surplus changes induced by a merger between firms 0 and $k \geq 1$, $(\Pi(M_k), CS(M_k))$, fall somewhere on the curve labeled "M_k." (The figure shows only the parts of these curves foe which the aggregate profit change $\Delta\Pi$ is nonnegative.) Since by Lemma 1 a CS-neutral merger involves a strictly positive change in aggregate profit, each curve crosses the horizontal axis to the right of the vertical axis. By Assumption 2, the curve for each merger M_k with $k \geq 2$ is upward sloping everywhere above the horizontal axis. By Lemma 2, above the horizontal axis the curves for larger mergers lie everywhere to the right of those for smaller mergers.

A useful corollary of Lemma 2, which can easily be seen in Figure 1, is the following:

Corollary 1. *If two CS-nondecreasing mergers M_j and M_k with $k > j \geq 1$ have $\Delta\Pi(M_k) \leq \Delta\Pi(M_j)$, then $\Delta CS(M_k) < \Delta CS(M_j)$.*

Proof. Suppose instead that $\Delta CS(M_k) \geq \Delta CS(M_j)$. Then there exists a $\bar{c}'_k > \bar{c}_k$ such that $\Delta CS(k, \bar{c}'_k) = \Delta CS(M_j)$. But this implies (using Assumption 2 for the first inequality and Lemma 2 for the second) that $\Delta\Pi(M_k) > \Delta\Pi(k, \bar{c}'_k) > \Delta\Pi(M_j)$, a contradiction. $\qquad \square$

3.2 Optimal Merger Policy

We can now turn to the optimal policy of the antitrust authority. Recall that the antitrust authority can without loss restrict itself to approval sets in which the set of acceptable cost levels for a merger between firm 0 and each firm k, $\mathcal{A}_k \subseteq [l, h_k]$, is a union of closed intervals. Throughout we restrict attention to such policies.[4] Let $\bar{a}_k \equiv \max\{\bar{c}_k | \bar{c}_k \in \mathcal{A}_k\}$ denote the largest allowable post-merger cost level for a merger (i.e., the "marginal merger") between firms 0 and k. Also let $\underline{\Delta CS}_k \equiv \Delta CS(k, \bar{a}_k)$ and $\underline{\Delta\Pi}_k \equiv \Delta\Pi(k, \bar{a}_k)$ denote the changes in consumer surplus and aggregate profit levels, respectively, induced by that marginal merger. These are the lowest levels of consumer surplus and (for $k \geq 2$) aggregate profit in any allowable merger between firms 0 and k.

We now state our main result:

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[4]Thus, when we state that any optimal policy must have a particular form, we mean any optimal interval policy of this sort. There are other optimal policies that add or subtract in addition some measure zero sets of mergers, since these have no effect on expected consumer surplus.
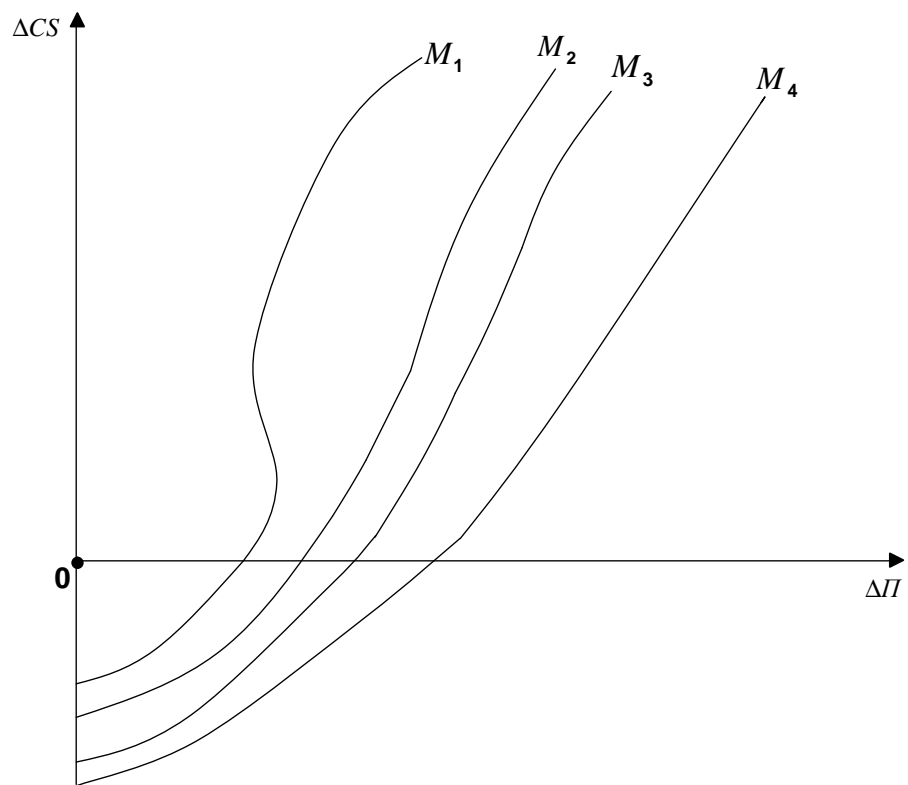
Proposition 1. *Any optimal approval policy $\mathcal{A}$ approves the smallest merger if and only if it is CS-nondecreasing, approves only mergers $k \in \mathcal{K}^+ \equiv \{1, ..., \widehat{K}\}$ with positive probability ($\widehat{K}$ may equal K) and satisfies $0 = \underline{\Delta CS}_1 < \underline{\Delta CS}_2 < ... < \underline{\Delta CS}_{\widehat{K}}$ for all $k \le \widehat{K}$. That is, the lowest level of consumer surplus change that is acceptable to the antitrust authority equals zero for the smallest merger M_1, is strictly positive for every other merger M_k with $k > 1$, and is monotonically increasing in the size of the merger, while the largest merger(s) may never be approved.*

Proof. The proof proceeds in a number of steps.

Step 1. We observe first that an optimal policy does not approve CS-decreasing mergers. To see this, suppose the approval set $\mathcal{A}$ includes CS-decreasing mergers, and consider the set $\mathcal{A}^+ \subseteq \mathcal{A}$ that removes any mergers in $\mathcal{A}$ that reduce consumer surplus. Figure 2 depicts such a pair of approval sets. Since this change only matters when the aggregate profit-maximizing merger $M^*(\mathfrak{F}, \mathcal{A})$ under set $\mathcal{A}$ is no longer approved under $\mathcal{A}^+$, the change in expected consumer surplus from this change in the approval policy equals $\Pr(M^*(\mathfrak{F}, \mathcal{A}) \in \mathcal{A} \backslash \mathcal{A}^+)$, the probability of this event happening, times the conditional expectation

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F}, \mathcal{A}^+)) - \Delta CS(M^*(\mathfrak{F}, \mathcal{A})) | M^*(\mathfrak{F}, \mathcal{A}) \in \mathcal{A} \backslash \mathcal{A}^+].$$

Since $\Delta CS(M^*(\mathfrak{F}, \mathcal{A}^+))$ is necessarily nonnegative by construction of $\mathcal{A}^+$, and $\Delta CS(M^*(\mathfrak{F}, \mathcal{A}))$ is strictly negative whenever $M^*(\mathfrak{F}, \mathcal{A}) \in \mathcal{A} \backslash \mathcal{A}^+$, this change is strictly positive.

Step 2. Next, any smallest merger M_1 that is CS-nondecreasing must be approved. To see this, suppose that the approval set is $\mathcal{A}$ but that $\mathcal{A} \subset \mathcal{A}' \equiv (\mathcal{A} \cup \{(1, \bar{c}_1) : \Delta CS(1, \bar{c}_1) \ge 0\})$. Figure 3 depicts two such sets, $\mathcal{A}$ and $\mathcal{A}'$. Because a change from $\mathcal{A}'$ to $\mathcal{A}$ matters only when the aggregate profit-maximizing merger $M^*(\mathfrak{F}, \mathcal{A}')$ under $\mathcal{A}'$ is no longer approved under $\mathcal{A}$, the change in expected consumer surplus by using $\mathcal{A}'$ rather than $\mathcal{A}$ equals $\Pr(M^*(\mathfrak{F}, \mathcal{A}') \in \mathcal{A}' \backslash \mathcal{A})$ times

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F}, \mathcal{A}')) - \Delta CS(M^*(\mathfrak{F}, \mathcal{A})) | M^*(\mathfrak{F}, \mathcal{A}') \in \mathcal{A}' \backslash \mathcal{A}]. \tag{6}$$

By Corollary 1 and the fact that $\mathcal{A}' \backslash \mathcal{A}$ contains only smallest mergers (between firms 0 and 1), whenever $M^*(\mathfrak{F}, \mathcal{A}') \in \mathcal{A}' \backslash \mathcal{A}$ [which implies $\Delta\Pi(M^*(\mathfrak{F}, \mathcal{A}')) > \Delta\Pi(M^*(\mathfrak{F}, \mathcal{A}))$] we have $\Delta CS(M^*(\mathfrak{F}, \mathcal{A}')) > \Delta CS(M^*(\mathfrak{F}, \mathcal{A}))$, so (6) is strictly positive. This can be seen in Figure 3. This implies in particular that $\underline{\Delta CS}_1 = 0$.

Step 3. Next, let $\mathcal{K}^+$ denote those acquirers with $k \ne 1$ for whom the probability of having a merger $M_k \in \mathcal{A}$ is strictly positive. We claim that in any optimal policy $\underline{\Delta CS}_k > 0$ for all $k \in \mathcal{K}^+$. To see this, consider switching from the policy $\mathcal{A}$ to $\mathcal{A}^\varepsilon \equiv \{M_k \in \mathcal{A} : k \in \mathcal{K}^+$ and $\Delta CS(M_k) > \varepsilon\}$ where $\varepsilon > 0$, as shown in Figure 4. The change in expected consumer surplus equals $\Pr(M^*(\mathfrak{F}, \mathcal{A}) \in \mathcal{A} \backslash \mathcal{A}^\varepsilon)$ times
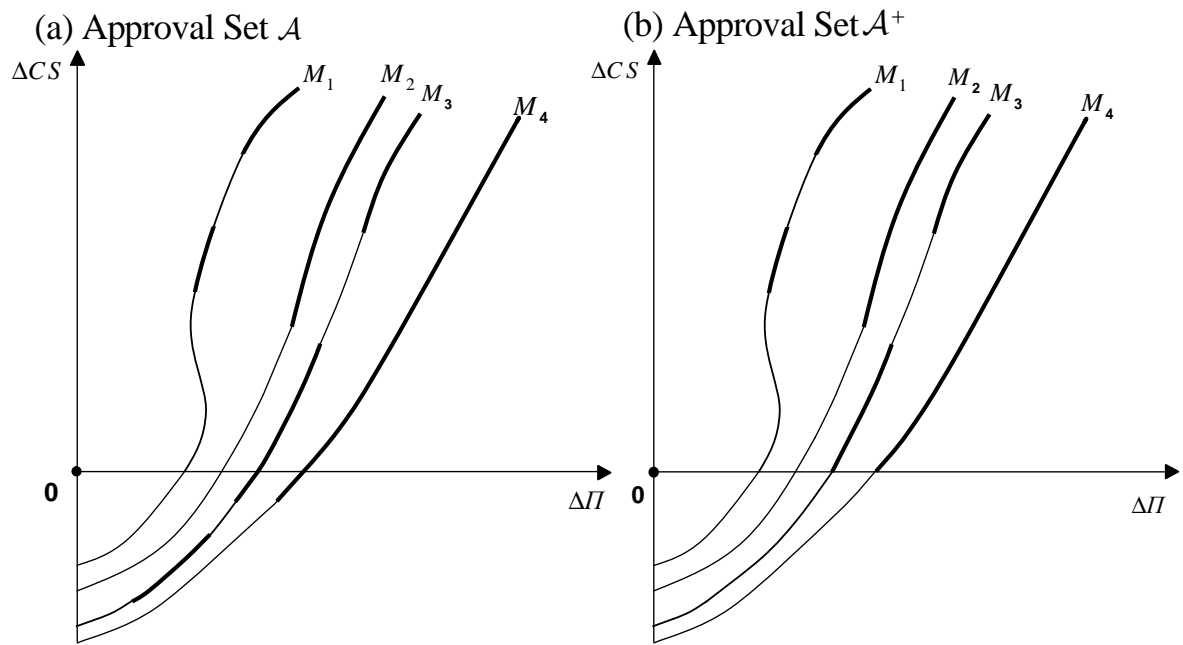
$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F}, \mathcal{A}^\varepsilon)) - \Delta CS(M^*(\mathfrak{F}, \mathcal{A})) | M^*(\mathfrak{F}, \mathcal{A}) \in \mathcal{A} \backslash \mathcal{A}^\varepsilon].$$

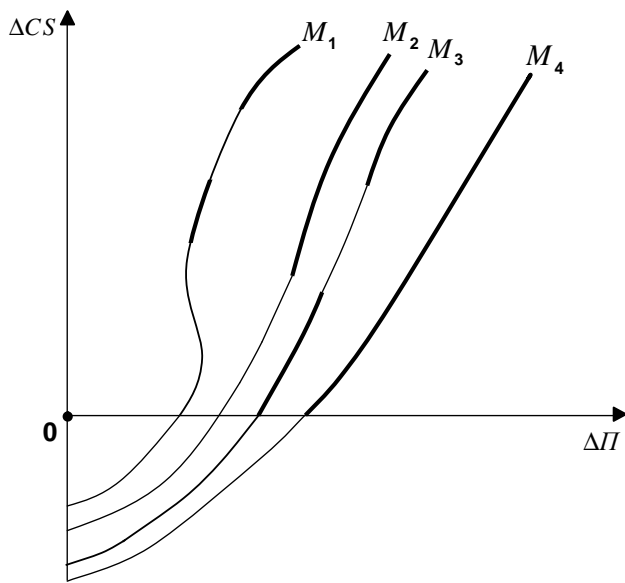Now, as $\varepsilon \to 0$, this conditional expectation approaches

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F}, \mathcal{A}^\varepsilon)) | M^*(\mathfrak{F}, \mathcal{A}) \in \mathcal{A} \backslash \mathcal{A}^\varepsilon],$$

which is strictly positive given steps 1 and 2.
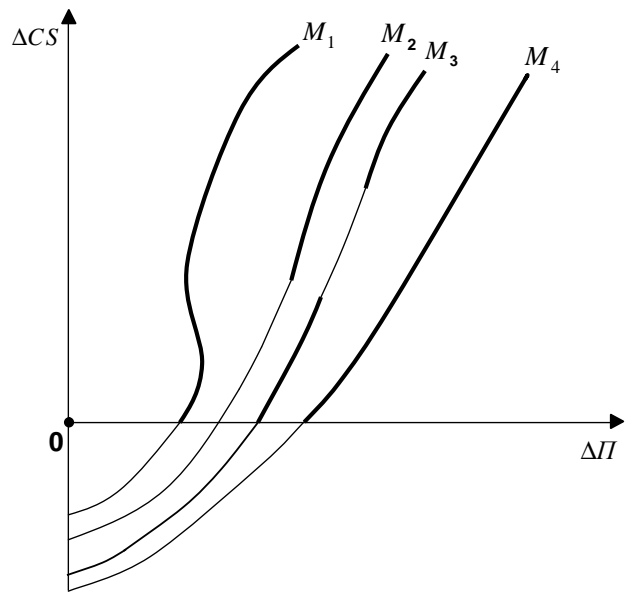
Step 4. Next, we claim that in any optimal policy, for all $k \in \mathcal{K}^+$, $\underline{\Delta CS}_k$ must equal the expected change in consumer surplus from the next-most-profitable merger $M^*(\mathfrak{F} \backslash (k, \bar{a}_k), \mathcal{A})$,

(a) Approval Set $\mathcal{A}$

(b) Approval Set $\mathcal{A}^{+}$

(a) Approval Set $\mathcal{A}$

(b) Approval Set $\mathcal{A}'$

(a) ApprovalSet $\mathcal{A}$

ΔCS

M_1 M_2 M_3 M_4

0

$\Delta \Pi$

(b) ApprovalSet $\mathcal{A}^{\varepsilon}$

ΔCS

M_1 M_2 M_3 M_4

ε

0

$\Delta \Pi$

conditional on merger $M_k = (k, \overline{a}_k)$ being the most profitable merger in $\mathfrak{F} \cap \mathcal{A}$. Defining the expected change in consumer surplus from the next-most-profit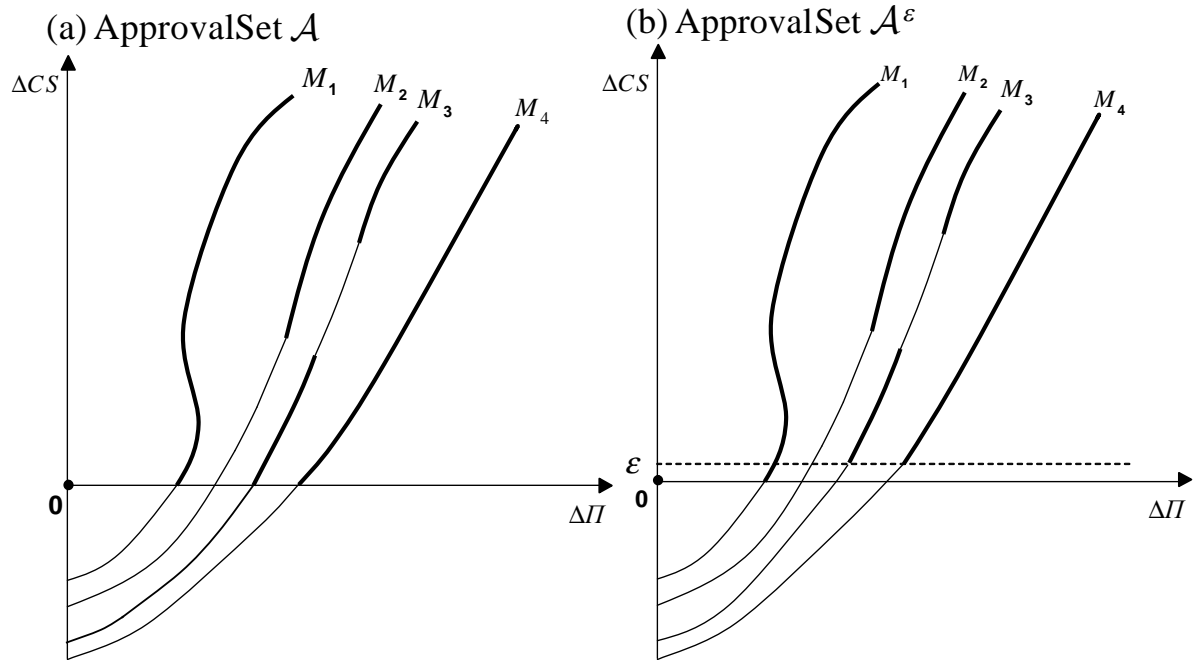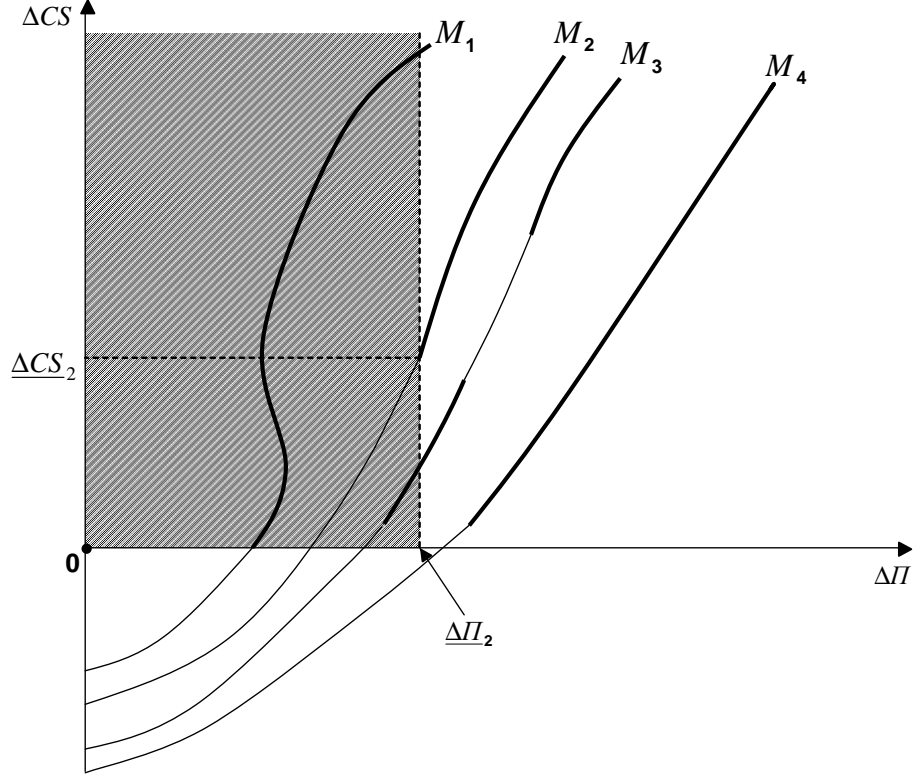able merger $M^*(\mathfrak{F} \backslash M_k, \mathcal{A})$, conditional on merger $M_k = (k, \overline{c}_k)$ being the most profitable merger in $\mathfrak{F} \cap \mathcal{A}$, to be

$$
\begin{aligned}
E_k^{\mathcal{A}}(\overline{c}_k) & \equiv E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F} \backslash M_k, \mathcal{A}))|M_k = (k, \overline{c}_k) \text{ and } M_k = M^*(\mathfrak{F}, \mathcal{A})] \qquad (7) \\
& = E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F} \backslash M_k, \mathcal{A}))|M_k = (k, \overline{c}_k) \text{ and } \Delta \Pi(M^*(\mathfrak{F} \backslash M_k, \mathcal{A})) \leq \Delta \Pi(M_k)], (8)
\end{aligned}
$$

this means that

$$
\underline{\Delta CS}_k = E_k^{\mathcal{A}}(\overline{a}_k). \qquad (9)
$$

In Figure 5 the possible locations of the next-most-profitable merger when the most profitable merger is $M_2 = (2, \overline{a}_2)$ are shown as a shaded set. The quantity $E_2^{\mathcal{A}}(\overline{a}_2)$ is the expectation of the change in consumer surplus for the merger that has the largest change in aggregate profit among mergers other than M_2, conditional on all of these other mergers lying in the shaded region of the figure.

To see that (9) must hold for all $k \in \mathcal{K}^+$, suppose first that $\underline{\Delta CS}_{k'} > E_{k'}^{\mathcal{A}}(\overline{a}_{k'})$ for some

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$k' \in \mathcal{K}^+$ and consider the alternative approval set $\mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon}$ where

$$\mathcal{A}_{k'}^{\varepsilon} \equiv \{M_k : M_k = (k', \overline{c}_{k'}) \text{ with } \overline{c}_{k'} \in (\overline{a}_{k'}, \overline{a}_{k'} + \epsilon)\}.$$

For any $\varepsilon > 0$, the change in expected consumer surplus from changing from $\mathcal{A}$ to $\mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon}$ equals $\Pr(M^*(\mathfrak{F}, \mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon}) \in \mathcal{A}_{k'}^{\varepsilon})$ times

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F}, \mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon})) - \Delta CS(M^*(\mathfrak{F}, \mathcal{A})) | M^*(\mathfrak{F}, \mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon}) \in \mathcal{A}_{k'}^{\varepsilon}]. \tag{10}$$

This conditional expectation can be rewritten as

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F}, \mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon})) - E_{k'}^{\mathcal{A}}(\overline{c}_{k'}) | M^*(\mathfrak{F}, \mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon}) \in \mathcal{A}_{k'}^{\varepsilon}], \tag{11}$$

where $\overline{c}_{k'}$ is the realized cost level in the aggregate profit-maximizing merger $M^*(\mathfrak{F}, \mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon})$, which is a merger of firms 0 and k' when the conditioning statement is satisfied. By continuity of $\Delta CS(k', \overline{c}_{k'})$ and $E_k^{\mathcal{A}}(\overline{c}_{k'})$ in $\overline{c}_{k'}$, there exists an $\overline{\varepsilon} > 0$ such that $\Delta CS(M_{k'}) > E_k^{\mathcal{A}}(\overline{c}_{k'})$ for all $M_{k'} \in \mathcal{A}_{k'}^{\varepsilon}$ provided $\varepsilon \in (0, \overline{\varepsilon}]$. For all such ε, the conditional expectation (11) is strictly positive so this change in the approval set would strictly increase expected consumer surplus. A similar argument applies if $\underline{\Delta CS}_{k'} < E_{k'}^{\mathcal{A}}(\overline{a}_{k'})$.

Step 5. Next, we argue that for all $j < k$ such that $j, k \in \mathcal{K}^+$ it must be that $\underline{\Delta \Pi}_j \leq \underline{\Delta \Pi}_k$; that is, the aggregate profit change in the marginal merger by acquirer j must be no greater than the aggregate profit change in the marginal merger by any larger acquirer k. Figure 6(a) shows a situation that violates this condition, where the marginal merger by acquirer 3 causes a smaller aggregate profit change $\underline{\Delta \Pi}_3$, than the marginal merger by the smaller acquirer 2, $\underline{\Delta \Pi}_2$.

For $j \in \mathcal{K}^+$, let $k' \equiv \arg\min_{k \in \mathcal{K}^+, k > j} \underline{\Delta \Pi}_k$ and suppose that $\underline{\Delta \Pi}_{k'} < \underline{\Delta \Pi}_j$. We know from the previous step that $\underline{\Delta CS}_{k'} = E_{k'}^{\mathcal{A}}(\overline{a}_{k'})$. Let $\overline{c}_j'$ be the post-merger cost level satisfying $\Delta \Pi(j, \overline{c}_j') = \underline{\Delta \Pi}_{k'}$ and consider a change in the approval set from $\mathcal{A}$ to $\mathcal{A} \cup \overline{\mathcal{A}}_j^{\varepsilon}$ where

$$\overline{\mathcal{A}}_j^{\varepsilon} \equiv \{M_j : M_j = (j, \overline{c}_j) \text{ with } \overline{c}_j \in (\overline{c}_j', \overline{c}_j' + \epsilon)\}.$$

The set $\overline{\mathcal{A}}_j^{\varepsilon}$ is shown in Figure 6(b). The change in expected consumer surplus from this change in the approval set equals $\Pr(M^*(\mathfrak{F}, \mathcal{A} \cup \overline{\mathcal{A}}_j^{\varepsilon}) \in \overline{\mathcal{A}}_j^{\varepsilon})$ times

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F}, \mathcal{A} \cup \overline{\mathcal{A}}_j^{\varepsilon})) - E_j^{\mathcal{A}}(\overline{c}_j) | M^*(\mathfrak{F}, \mathcal{A} \cup \overline{\mathcal{A}}_j^{\varepsilon}) \in \overline{\mathcal{A}}_j^{\varepsilon}], \tag{12}$$

where $\overline{c}_j$ is the realized cost level in the aggregate profit-maximizing merger $M^*(\mathfrak{F}, \mathcal{A} \cup \overline{\mathcal{A}}_j^{\varepsilon})$, which is a merger of firms 0 and k' when the conditioning statement is satisfied. As $\varepsilon \to 0$, the expected change in (12) converges to

$$
\begin{aligned}
\Delta CS(j, \overline{c}_j') - E_j^{\mathcal{A}}(\overline{c}_j') &= \Delta CS(j, \overline{c}_j') - E_{k'}^{\mathcal{A}}(\overline{a}_{k'}) \\
&> \underline{\Delta CS}_{k'} - E_{k'}^{\mathcal{A}}(\overline{a}_{k'}) \\
&= 0,
\end{aligned}
$$

where the inequality follows from Corollary 1 since $\Delta \Pi(j, \overline{c}_j') = \underline{\Delta \Pi}_{k'}$.

Step 6. We next argue that $\underline{\Delta CS}_j < \underline{\Delta CS}_k$ for all $j, k \in \mathcal{K}^+$ with $j < k$. Suppose otherwise; i.e., for some $j, h \in \mathcal{K}^+$ with $h > j$ we have $\underline{\Delta CS}_j \geq \underline{\Delta CS}_h$. Define $k = \arg\min_{h \in \mathcal{K}^+, h > j} \underline{\Delta \Pi}_h$ s.t. $\underline{\Delta CS}_j \geq \underline{\Delta CS}_h$. Figure 7 depicts such a situation where $j = 2$ and $k = 3$.

(a) Approval Set $\mathcal{A}^{\varepsilon}$

(b) Approval Set $\mathcal{A} \cup \overline{\mathcal{A}}_{j}^{\varepsilon}$
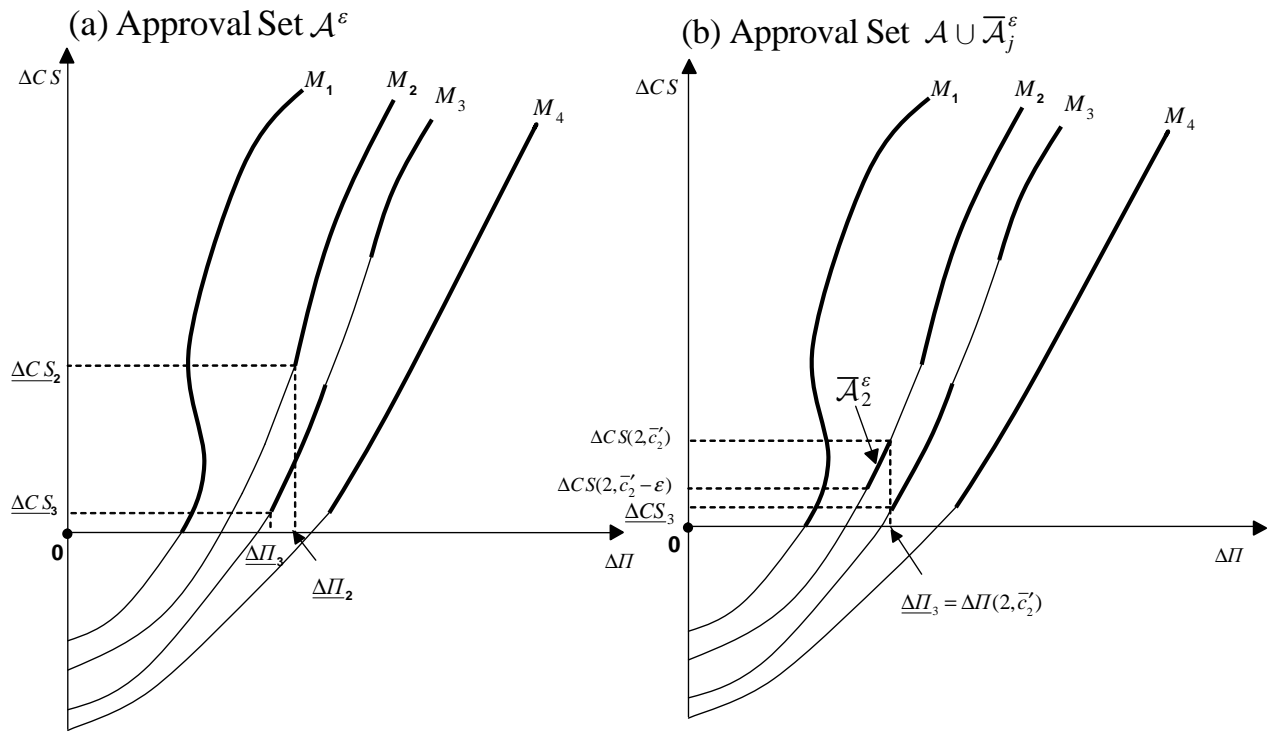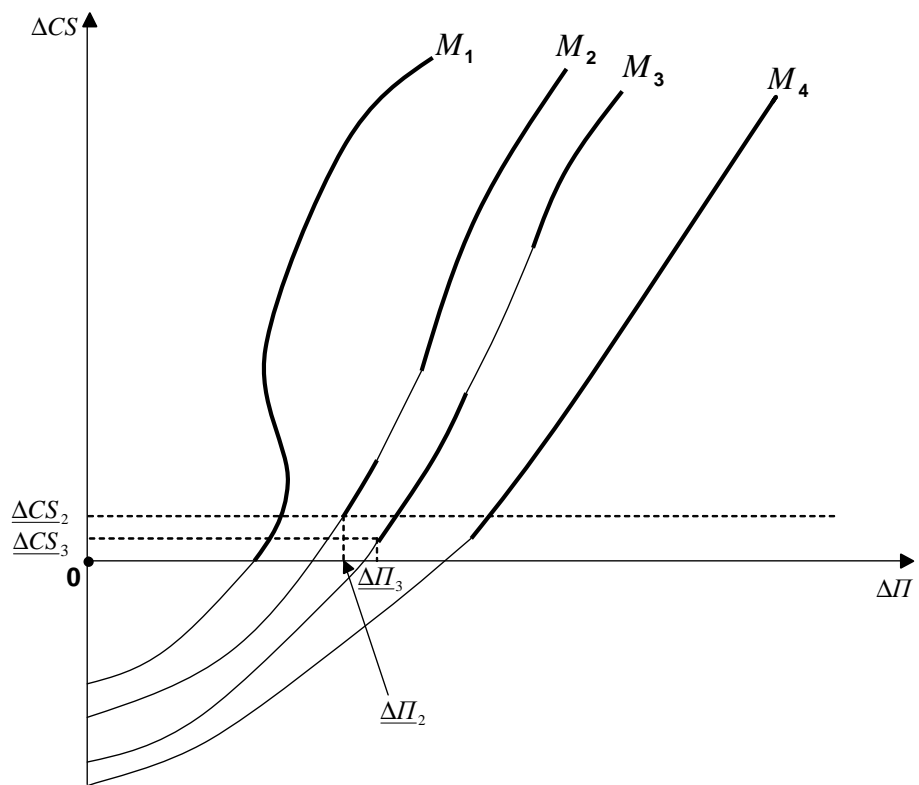
By Step 4, we must have $E_j^{\mathcal{A}}(\overline{a}_j) = \underline{\Delta CS}_j \geq \underline{\Delta CS}_k = E_k^{\mathcal{A}}(\overline{a}_k)$. But recalling (8), $E_k^{\mathcal{A}}(\overline{a}_k)$ can be written as a weighted average of two conditional expectations:

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F}\backslash M_k,\mathcal{A}))|M_k = (k,\overline{c}_k),\ M_k = M^*(\mathfrak{F},\mathcal{A}),\ \text{and}\ \Delta\Pi(M^*(\mathfrak{F}\backslash M_k,\mathcal{A})) <\underline{\Delta\Pi}_j] \quad (13)$$

and

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F}\backslash M_k,\mathcal{A}))|M_k = (k,\overline{c}_k),\ M_k = M^*(\mathfrak{F},\mathcal{A}),\ \text{and}\ \Delta\Pi(M^*(\mathfrak{F}\backslash M_k,\mathcal{A})) \in [\underline{\Delta\Pi}_j,\underline{\Delta\Pi}_k]]. \quad (14)$$

Expectation (13) conditions on the event that the next-most-profitable merger other than $(k,\overline{a}_k)$ has a profit change less than $\underline{\Delta\Pi}_j$, the profit change in merger $(j,\overline{a}_j)$. Since no merger in $\mathcal{A}$ by either acquirer k or j can have such a profit level (since $\underline{\Delta\Pi}_k > \underline{\Delta\Pi}_j$ by Step 5), the expectation (13) must exactly equal $E_j^{\mathcal{A}}(\overline{a}_j)$. Now consider the expectation (14). If $\Delta\Pi(M^*(\mathfrak{F}\backslash M_k,\mathcal{A})) \in [\underline{\Delta\Pi}_j,\underline{\Delta\Pi}_k)$, it could be that (i) $M^*(\mathfrak{F}\backslash M_k,\mathcal{A}) = (j,\overline{c}_j)$ for some $\overline{c}_j \leq \overline{a}_j$, or (ii) $M^*(\mathfrak{F}\backslash M_r,\mathcal{A}) = (r,\overline{c}_r)$ for some $r < j$, or (iii) $M^*(\mathfrak{F}\backslash M_r,\mathcal{A}) = (r,\overline{c}_r)$ for some $r > j$ and $r < k$. Now, in case (i) it is immediate that $\Delta CS(M^*(\mathfrak{F}\backslash M_k,\mathcal{A}) \geq \underline{CS}_j$, with strict inequality whenever $\overline{c}_j = \overline{a}_j$. In case (ii), the fact that $\Delta\Pi(r,\overline{c}_r) \geq \underline{\Delta\Pi}_j$ implies by Corollary 1 that

$$\Delta CS(M^*(\mathfrak{F}\backslash M_k,\mathcal{A}) = \Delta CS(r,\overline{c}_r) > \underline{CS}_j = E_j^{\mathcal{A}}(\overline{a}_j). \quad (15)$$

In case (iii), (15) follows from the definition of k. Thus, expectation (14) must strictly exceed $E_j^{\mathcal{A}}(\overline{a}_j)$, which leads to a contradiction.

Step 7. Finally, we argue that $\mathcal{K}^+ = \{1,...,\widehat{K}\}$ for some $\widehat{K}$. To establish this fact, we show that if $k \notin \mathcal{K}^+$, then $k+1 \notin \mathcal{K}^+$. We first observe that $\Delta CS(k,l) > \Delta CS(k+1,l)$, which follows because the profile of firms' costs following merger (k,l) are lower than following merger $(k+1,l)$ (the post-merger industry cost profile differs only for firms k and $k+1$, which have costs of l and c_{k+1} with the first merger and c_k and l with the second). Thus, if $k+1 \in \mathcal{K}^+$, then $\Delta CS(k+1,\overline{a}_{k+1}) < \Delta CS(k,l)$. But, an argument like that in Step 6 [using the fact that, by an argument like that in Step 4, $\Delta CS(k,l) \leq E_k^{\mathcal{A}}(\overline{a}_k)$] shows that $\Delta CS(k,l) < E_{k+1}^{\mathcal{A}}(\overline{a}_{k+1})$, so that $\Delta CS(k+1,\overline{a}_{k+1}) < E_{k+1}^{\mathcal{A}}(\overline{a}_{k+1})$, contradicting the conclusion of Step 4. $\square$

We have shown that there is a misalignment between firms' proposal incentives and the interests of the antitrust authority: firms tend to have an incentive to propose a merger that is larger (in terms of the pre-merger size of the merger partner) than the one that would maximize consumer surplus. To compensate for this intrinsic bias in firms' proposal incentives, the antitrust authority should optimally adopt a higher minimum CS-standard the larger is the proposed merger.

3.3 Cut-off Rules

In our analysis above, we have shown that the optimal approval policy has the property that the minimum CS-standard is zero for the smallest merger and increasing in the size of the proposed merger. Does this mean that any merger that generates a larger increase in consumer surplus than the minimum CS-standard would get approved? Put differently, does the optimal policy have a cut-off structure so that $\mathcal{A}_k = [l,\overline{a}_k]$? It turns out that the answer depends on whether or not the merger partners can under-report the efficiency gains induced by their merger.
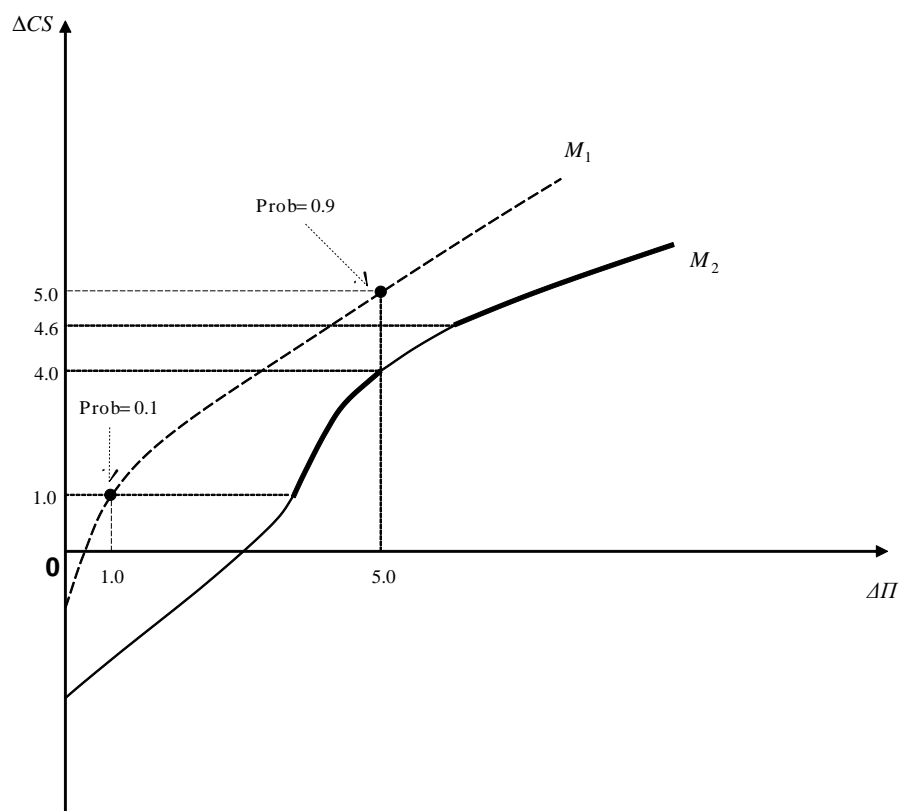
Consider first the case where the antitrust authority observes the efficiency gain of any proposed merger. In that case, the optimal approval policy may not have a cut-off structure, as the following example illustrates. (For simplicity, the example considers the case where, contrary to the assumption of the model, one of the mergers has a finite support of post-merger marginal costs. But the same insight would obtain if we perturbed the example and assumed that the support is continuous with no atoms.)

Example 1. Suppose that there are two possible mergers, M_1 and M_2. The smaller merger, M_1, is always feasible. Its post-merger marginal cost is either $\bar{c}_1 = l$ or $\bar{c}_1 = h_1$, where the probability on the latter is 0.9. The corresponding changes in consumer surplus and aggregate profit are given by $(\Delta CS(1,l), \Delta\Pi(1,l)) = (5,5)$ and $(\Delta CS(1,h_1), \Delta\Pi(1,h_1)) = (1,1)$. The unconditional expected increase in consumer surplus from approving M_1 is thus equal to 4.6. The post-merger marginal cost of the larger merger, M_2, has a continuous support $[l, h_2]$ with no atoms, satisfying $\Delta CS(2, h_2) < 1$ and $5 < \Delta CS(2, l)$. It is straightforward to verify that the optimal approval policy $\mathcal{A}^*$ is such that $\mathcal{A}_1 = \{l, h_1\}$ and $\mathcal{A}_2 = [l, c_2'] \cup [c_2'', \bar{a}_2]$, where c_2' and $c_2'' > c_2'$ are implicitly defined by $\Delta CS(2, c_2') = 4.6$ and $\Delta CS(2, c_2'') = 4$. This situation is illustrated in Figure 8. To see why the optimal approval policy for M_2 does not have a cut-off structure, note that for any post-merger marginal cost $\bar{c}_2 \in (c_2', c_2'')$, the induced change in consumer surplus is less than 5 (which is the induced change in consumer surplus of the best realization of M_1). But, if approved, the firms would propose the larger merger even if the realized M_1 is better for consumers as, for $\bar{c}_2 \in (c_2', c_2'')$, $\Delta\Pi(2, \bar{c}_2) > 5 = \Delta\Pi(1, l)$. The optimal policy corrects for this bias in firms' proposal policies by not approving M_2 whenever $\bar{c}_2 \in (c_2', c_2'')$.

Consider now the case where firms can under-report (but not over-report) any realizable efficiency gain. That is, suppose the merger partners can claim any level of post-merger marginal cost $\tilde{c}_k \geq \bar{c}_k$, where $\bar{c}_k$ is the true marginal cost level when the merger is implemented, and that the antitrust authority cannot or does not verify the actual marginal cost level ex post. (The idea might be that the antitrust authority cannot directly observe potential cost savings but requires documentation from the merger partners. While this documentation might be hard information, the firms may under-report potential efficiencies by providing documentation on only a subset of possible cost-saving measures. Ex post, the merged firm may always claim that the additional efficiencies were unexpected.) In this case, the optimal policy trivially has a cut-off structure: if the actual post-merger marginal cost level $\bar{c}_k$ is such that the merger would not get approved even though $\bar{c}_k < \bar{a}_k$, the merger partners would have an incentive to claim that the actual post-merger marginal cost level is higher, say $\bar{a}_k$, and get the merger approved.

If, for whatever reason, the approval policy does have a cut-off structure, then the optimal cut-offs can be constructed recursively. Let $\mathfrak{F}^{(t)} \equiv \{M_k, 1 \leq k \leq t : \phi_k = 1\}$ and $\mathcal{A}^{(t)} \equiv \{M_k, 1 \leq k \leq t : \bar{c}_k \in [l, \bar{a}_k]\}$ denote the sets of feasible and allowable mergers not larger than M_t. The optimal cut-offs $(\bar{a}_1, ..., \bar{a}_{\widehat{K}})$ are recursively defined as follows:

$$\Delta CS(1, \bar{a}_1) = 0,$$
$$\Delta CS(k, \bar{a}_k) = E_{\mathfrak{F}^{(k-1)}}\left[\Delta CS\left(M^*\left(\mathfrak{F}^{(k-1)}, \mathcal{A}^{(k-1)}\right)\right) \mid M_k = (k, \bar{a}_k)\right.$$
$$\left. \text{and } M_k = M^*\left(\mathfrak{F}^{(k)}, \mathcal{A}^{(k)}\right), \ 1 \leq k \leq \widehat{K}\right].$$

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4 Inefficient Bargaining Processes

In our analysis so far, we have focused on the benchmark case where the bargaining process between firms is efficient from the viewpoint of the industry. In this section, we explore two inefficient bargaining processes. First, we consider the case where there is (efficient) bargaining only between a subset of firms (including all of those firms that are involved in potential mergers). Second, we consider the case where the target (firm 0) can put itself up for sale by making a take-it-or-leave-it offer to a single acquirer of its choosing, which is the "offer game" of Segal (1999). We show that, in both cases, the main result continues to hold: the optimal approval policy has the property that the minimum CS-standard is increasing in the size of the proposed merger.

4.1 Bargaining Between a Subset of Firms

Suppose that the outcome of the bargaining process maximizes the joint profit of only a subset of firms, $\mathcal{L}$, that includes the target and all of the acquirers, i.e., $(\{0\} \cup \mathcal{K}) \subseteq \mathcal{L} \subset \mathcal{N}$. That is, we now assume that firms propose merger $M_{\mathcal{L}}^*(\mathfrak{F}, \mathcal{A})$, where

$$M_{\mathcal{L}}^*(\mathfrak{F}, \mathcal{A}) \equiv \arg \max_{M_k \in (\mathfrak{F} \cap \mathcal{A})} \Delta \Pi_{\mathcal{L}}(M_k),$$

and $\Delta \Pi_{\mathcal{L}}(M_k) \equiv \Pi_{\mathcal{L}}(M_k) - \Pi_{\mathcal{L}}(M_0)$ is the induced change in the joint profit of the firms in set $\mathcal{L}$ when merger M_k is implemented, where $\Pi_{\mathcal{L}}(M_0) \equiv \sum_{i \in \mathcal{L}} \pi_i^0$ and, for $k \in \mathcal{K}$, $\Pi_{\mathcal{L}}(M_k) = \sum_{i \in \mathcal{L} \setminus \{0\}} \pi_i(M_k)$.

We claim that Proposition 1 carries over to this bargaining process: the optimal approval policy $\mathcal{A}$ is such that the minimum CS-standard is zero for the smallest merger and increasing in the size of the proposed merger, $0 = \underline{\Delta CS}_1 < \underline{\Delta CS}_2 < \cdots < \underline{\Delta CS}_{\widehat{K}}$, where $\widehat{K}$ is the largest merger that is approved with positive probability. The key step in proving this claim is the following observation: If any CS-nondecreasing merger or any reduction in a merged firm's marginal cost induces an increase in aggregate profit, $\Delta \Pi > 0$, then it also induces an increase in the joint profit of the firms in set $\mathcal{L}$, $\Delta \Pi_{\mathcal{L}} > 0$. To see this, note that both a CS-nondecreasing merger and a reduction in a firm's post-merger marginal cost weakly reduce the profit of any other firm, including the firm(s) not in set $\mathcal{L}$, i.e., $\Delta \Pi_{\mathcal{N} \setminus \mathcal{L}} \leq 0$. Hence, $\Delta \Pi_{\mathcal{L}} = \Delta \Pi - \Delta \Pi_{\mathcal{N} \setminus \mathcal{L}} > 0$ if $\Delta \Pi > 0$. This observations has several implications. First, it means that Lemma 4, part 4, continues to hold if we replace aggregate profit Π by $\Pi_{\mathcal{L}}$. Second, it also means that Assumption 2 implies that a reduction in the post-merger marginal cost $\bar{c}_k$ raises the joint profit of the firms in set $\mathcal{L}$, $\Pi_{\mathcal{L}}$. Third, a similar type of argument implies that Lemma 2 continues to hold if we replace the induced change in industry profit, $\Delta \Pi(M_l)$, $l = j, k$, by the induced change in the joint profit of the firms in $\mathcal{L}$, $\Delta \Pi_{\mathcal{L}}(M_l)$. To see this, recall that both mergers in the statement of the lemma, M_j and M_k, induce (by assumption) the same change in consumer surplus. Hence, the profit of any firm $i \neq j, k$ is the same under both mergers, so that $\Delta \Pi(M_k) > \Delta \Pi(M_j)$ implies $\Delta \Pi_{\mathcal{L}}(M_k) > \Delta \Pi_{\mathcal{L}}(M_j)$. Finally, it is straightforward to see that all of the remaining steps in the proof of Proposition 1 carry over if we replace $\Pi(M_k)$ by $\Pi_{\mathcal{L}}(M_k)$.

4.2 The Offer Game

Suppose the bargaining process takes the form of an offer game, as in Segal (1999), where the target (firm 0) makes public take-it-or-leave-it offers. In Segal (1999), the principal's offers consist of a profile of trades $x = (x_1, ..., x_K)$ with x_k the trade of agent k. Here, $x_k \in \{0, 1\}$, where $x_k = 1$ if the target proposes a merger with firm k. Specifically, suppose firm 0 can make a take-it-or-leave-it offer t_k to a single firm k of its choosing, where k is such that $M_k \in (\mathfrak{F} \cap \mathcal{A})$. If the offer is accepted by firm k, then merger M_k is proposed to the antitrust authority, who will approve it since $M_k \in (\mathfrak{F} \cap \mathcal{A})$, and firm k acquires the target in return for the transfer payment t_k. If the offer is rejected, or if no offer is made, then no merger is proposed and no payments are made.

Let

$$\Delta\Pi_B(M_k) \equiv \pi_k(M_k) - \left[\pi_0^0 + \pi_k^0\right], \ k \geq 1,$$

denote the change in the bilateral profit to the merging parties, firms 0 and k, induced by merger M_k. Given the set of feasible and approvable mergers, the proposed merger in the equilibrium of the offer game is $M_B^*(\mathfrak{F}, \mathcal{A})$, where

$$M_B^*(\mathfrak{F}, \mathcal{A}) \equiv \begin{cases} \tilde{M}_B(\mathfrak{F}, \mathcal{A}) & \text{if } \Delta\Pi_B\left(\tilde{M}_B(\mathfrak{F}, \mathcal{A})\right) > 0 \\ M_0 & \text{otherwise,} \end{cases}$$

and

$$\tilde{M}_B(\mathfrak{F}, \mathcal{A}) \equiv \arg \max_{M_k \in (\mathfrak{F} \cap \mathcal{A})} \Delta\Pi_B(M_k).$$

That is, the proposed merger M_k is the one that maximizes the induced change in the bilateral profit to firms 0 and k, provided that change is positive; otherwise, no merger is proposed.

In the following, we show that Proposition 1 carries over to this bargaining process: the optimal approval policy $\mathcal{A}$ is such that the minimum CS-standard is zero for the smallest merger and increasing in the size of the proposed merger, $0 = \underline{\Delta CS}_1 < \underline{\Delta CS}_2 < \cdots < \underline{\Delta CS}_{\widehat{K}}$, where $\widehat{K}$ is the largest merger that is approved with positive probability. The key steps in proving this result are the following. First, from the proof of part 4 of Lemma 1, it follows that a CS-neutral merger M_k, $k \geq 1$, raises the joint profit of the merger partners 0 and k, i.e., $\Delta\Pi_B(M_k) > 0$. Second, it is straightforward to verify that our conditions on demand (Assumption 1) ensure that reducing the merged firm's marginal cost $\bar{c}_k$ not only increases consumer surplus but also the merged firm's profit. This means that we can dispense with Assumption 2 as its bilateral-profit analog is implied by Assumption 1. Third, we obtain the following analog of Lemma 2:

Lemma 3. *Suppose two mergers, M_j and M_k, with $j < k$, induce the same non-negative change in consumer surplus, $\Delta CS(M_j) = \Delta CS(M_k) \geq 0$. Then, the larger merger M_k induces a greater increase in the bilateral profit of the merger partners: $\Delta\Pi_B(M_k) > \Delta\Pi_B(M_j) > 0$.*

Proof. Suppose otherwise that $\Delta\Pi_B(M_k) \leq \Delta\Pi_B(M_j)$, i.e.,

$$\pi_k(M_k) - \pi_j(M_j) \leq \pi_k^0 - \pi_j^0. \tag{16}$$

Using the first-order conditions of profit maximization, the term on the r.h.s. of equation (16) can be re-written as

$$
\begin{aligned}
\pi_k^0 - \pi_j^0 &= \left[P(Q^0) - c_k\right] q_k^0 - \left[P(Q^0) - c_j\right] q_j^0 \\
&= \frac{\left[P(Q^0) - c_k\right]^2 - \left[P(Q^0) - c_j\right]^2}{-P'(Q^0)} \\
&= \left[\left(\frac{P(Q^0) - c_j}{-P'(Q^0)}\right) + \left(\frac{P(Q^0) - c_k}{-P'(Q^0)}\right)\right] [c_j - c_k] \\
&= \left[q_j^0 + q_k^0\right] [c_j - c_k].
\end{aligned}
$$

As both mergers induce the same aggregate output (i.e., $Q(M_j) = Q(M_k)$), the term on the l.h.s. of equation (16) can similarly be re-written as

$$
\pi_k(M_k) - \pi_j(M_j) = [q_j(M_j) + q_k(M_k)] \left[\bar{c}_j - \bar{c}_k\right].
$$

Next, we claim that

$$
\left[\bar{c}_j - \bar{c}_k\right] = [c_j - c_k].
$$

To see this, let $\overline{Q} \equiv Q(M_j) = Q(M_k)$ denote the level of aggregate output after either merger. Summing up the N first-order conditions of profit maximization after merger M_l, $l = j, k$, we obtain

$$
NP(\overline{Q}) - \left(\sum_{i \geq 1, i \neq l} c_i + \bar{c}_l\right) + \overline{Q}P'(\overline{Q}) = 0.
$$

It follows that $c_i + \bar{c}_l$, $i, l = j, k$, $i \neq l$, is the same under either merger, proving the claim.

Combining these observations, we can re-write equation (16) as

$$
[q_j(M_j) + q_k(M_k)] \leq \left[q_j^0 + q_k^0\right].
$$

Now, as merger M_l, $l = j, k$, is CS-nondecreasing by assumption, the merger induces a weak increase in the joint output of the merger partners and a weak decrease in the output of any other firm $i \neq 0, l$. That is,

$$
q_l(M_l) \geq q_0^0 + q_l^0 > q_l^0 \geq q_l(M_r), \; l, r = j, k, \; l \neq r,
$$

implying that

$$
[q_j(M_j) + q_k(M_k)] > \left[q_j^0 + q_k^0\right],
$$

and thus resulting in a contradiction. Hence, equation (16) cannot hold. □

The final step consists in noting that all of the remaining steps in the proof of Proposition 1 continue to hold if we replace the change in aggregate profit, $\Delta\Pi(M_k)$, by the change in the merging firms' bilateral profit, $\Delta\Pi_B(M_k)$.

5   Extensions

In this section, we consider three extensions of our baseline model. First, we study the optimal merger approval policy when the antitrust authority cares not only about consumer surplus but also about producer surplus. Second, we extend the model by allowing for synergies in fixed costs. Third, we consider a simple situation where there is no single "pivotal" firm that is part of every potential merger.

5.1 Alternative Welfare Standard

In our baseline model, we have assumed that the antitrust authority seeks to maximize consumer surplus. While this is in line with the legal standard in the U.S. and many other countries, it might seem unsatisfactory that the antitrust authority completely ignores any effect of its policy on producer surplus. We now show that our main result extends to the case where the antitrust antitrust authority seeks to maximize any convex combination of consumer surplus and aggregate surplus.

Specifically, suppose the antitrust authority's welfare criterion is $W \equiv CS + \lambda \Pi$, where $\lambda \in [0, 1]$. When $\lambda = 1$, welfare W thus amounts to aggregate surplus. Let

$$\Delta W(M_k) \equiv \Delta CS(M_k) + \lambda \Delta \Pi(M_k)$$

denote the change in welfare induced by approving merger M_k. We will say that merger M_k is W-increasing [W-decreasing] if $\Delta W(M_k) > 0$ [$\Delta W(M_k) < 0$], and W-nondecreasing [W-nonincreasing] if $\Delta W(M_k) \geq 0$ [$\Delta W(M_k) \leq 0$].

Since a W-increasing merger may be CS-decreasing, we require a slightly stronger version of Assumption 2:

Assumption 2' *If merger M_k for $k \geq 2$ is W-nondecreasing, then reducing its post-merger marginal cost $\bar{c}_k$ increases the aggregate profit Π. Moreover, for any W-nondecreasing merger M_k, $k \in K$, $\bar{c}_k < \min\{c_0, c_k\}$ [i.e., the merger involves synergies].*

To understand when Assumption 2' must hold, consider the extreme case where all firms have the same pre-merger marginal cost c. Then, for merger M_k to be W-nondecreasing, it must involve synergies in that $\bar{c}_k < c$.[5] Hence, if M_k is W-nondecreasing, the merged firm is the firm with the lowest marginal cost post merger. Reducing the merged firm's marginal cost $\bar{c}_k$ induces an increase in aggregate output Q, thereby raising $|Q^2 P'(Q)|$, and a further increase in the Herfindal index H. From equation (3), a lower level of post-merger marginal cost $\bar{c}_k$ thus results in a greater level of aggregate profit Π. By continuity of consumer and producer surplus in marginal costs, it follows that $\Delta W(M_k) \geq 0$ implies that $\bar{c}_k < \min\{c_0, c_k\}$, and that Π is decreasing in $\bar{c}_k$, if pre-merger marginal cost differences are sufficiently small.

We also impose the following analog of Assumption 3:

Assumption 3' *For all $k \in \mathcal{K}$, the probability that the merger M_k is W-increasing is positive but less than one: $\Delta W(k, h_k) < 0 < \Delta W(k, l)$.*

Assumption 2' allows us to obtain a slightly stronger version of Lemma 2:

Lemma 2' *Suppose two W-nondecreasing mergers, M_j and M_k, with $k > j \geq 1$, induce the same change in consumer surplus, $\Delta CS(M_j) = \Delta CS(M_k)$. Then the larger merger M_k induces a greater increase in aggregate profit: $\Delta \Pi(M_k) > \Delta \Pi(M_j) > 0$.*

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[5]To see this, suppose otherwise that $\bar{c}_k \geq c$. We can decompose the induced change in market structure into two steps: (i) a move from N to $N-1$ firms, each with marginal cost c, and (ii) an increase in the marginal cost of one firm from c to $\bar{c}_k \geq c$. Step (i) induces a reduction in aggregate output but does not affect average production costs, and so reduces W. Step (ii) weakly reduces aggregate output and weakly increases average costs in the industry, and so weakly reduces W.
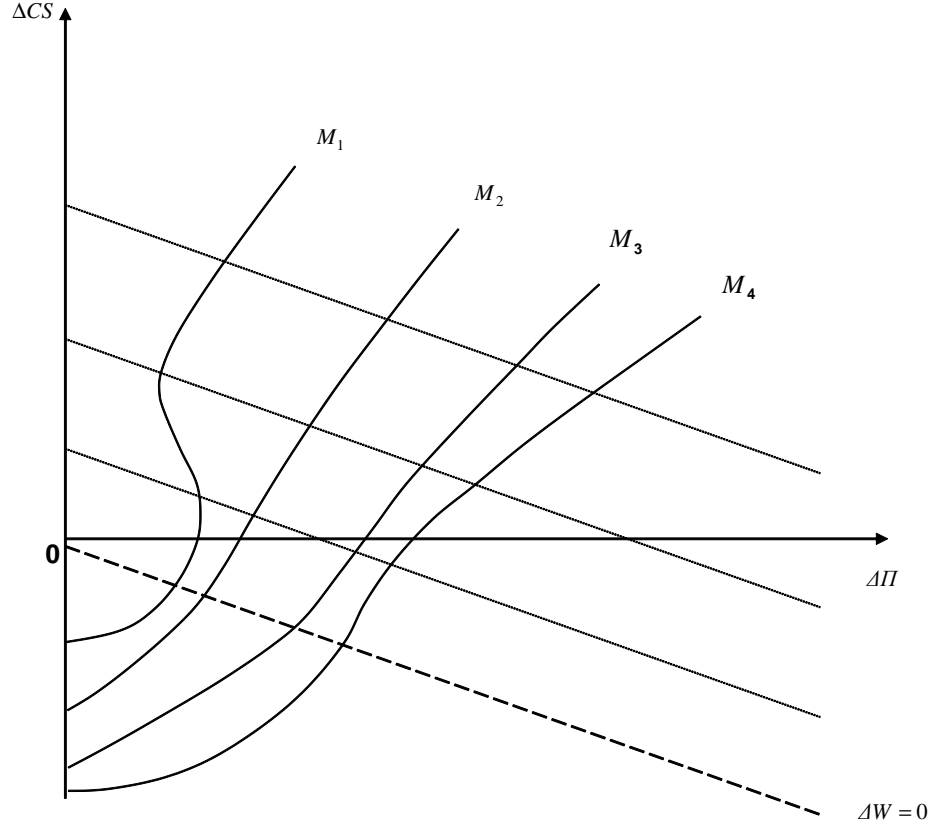
Proof. The proof proceeds exactly as that of Lemma 2, except that the inequalities $s_k(M_k) > s_k(M_j)$ and $s_j(M_j) > s_j(M_k)$ in equation (5) now hold since any W-nondecreasing merger involves synergies, $\bar{c}_k < c_k$ and $\bar{c}_j < c_j$, by Assumption 2' (and since $Q(M_k) = Q(M_j)$ as both mergers induce the same CS-level by assumption). $\qquad\square$

Figure 9 depicts the "merger curves" in $(\Delta\Pi, \Delta CS)$-space. The dotted lines are isowelfare curves, each with slope $-\lambda$; the hatched line is the isowelfare curve corresponding to no welfare change, $\Delta W = 0$. Lemma 2' states that, above the line $\Delta W = 0$, the curve corresponding to a larger merger lies everywhere to the right of that corresponding to a smaller merger. The figure also illustrates another result. That result is the analog of Corollary 1 and shows that there is a systematic misalignment between the proposal incentives of firms and the objectives of the antitrust authority:

Corollary 1' *If two W-nondecreasing mergers M_j and M_k with $k > j \geq 1$ have $\Delta\Pi(M_k) \leq \Delta\Pi(M_j)$, then $\Delta W(M_k) < \Delta W(M_j)$.*

Proof. Suppose instead that $\Delta W(M_k) \geq \Delta W(M_j)$. As $\Delta\Pi(M_k) \leq \Delta\Pi(M_j)$ by assumption,
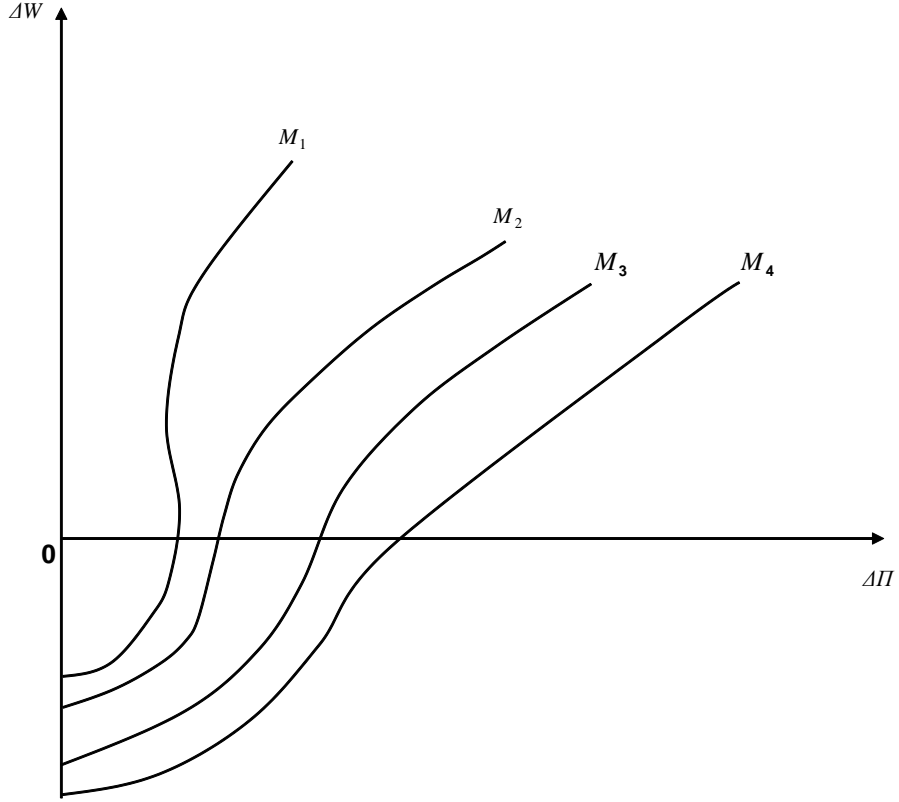
this implies that $\Delta CS(M_k) \geq \Delta CS(M_j)$. Then there exists a $\bar{c}'_k > \bar{c}_k$ such that $\Delta CS(k, \bar{c}'_k) = \Delta CS(M_j)$. But this implies (using Assumption 2' for the first inequality and Lemma 2' for the second) that $\Delta\Pi(M_k) > \Delta\Pi(k, \bar{c}'_k) > \Delta\Pi(M_j)$, a contradiction. $\qquad\square$

Figure 10 depicts the merger curves in $(\Delta\Pi, \Delta W)$-space. Note that each merger curve has a positive horizontal intercept: since a CS-nondecreasing merger raises aggregate profit, a W-neutral merger must be CS-decreasing and therefore increase aggregate profit. Moreover, each curve is upward-sloping in the positive orthant (except possibly for the curve corresponding to M_1). Finally, in the positive orthant, the curve of a larger merger lies everywhere to the right of that of a smaller merger.

Let $\underline{\Delta W}_k \equiv \Delta W(k, \bar{a}_k)$ denote the welfare level of the "marginal merger," i.e., the lowest welfare level in any allowable merger between firms 0 and k. The following proposition shows that our main result (Proposition 1) extends to the case where the antitrust authority maximizes an arbitrary convex combination of consumer surplus and aggregate surplus:

Proposition 1' *Any optimal approval policy A approves the smallest merger if and only if it is*

W-nondecreasing, and satisfies $0 = \underline{\Delta W}_1 < \underline{\Delta W}_j < \underline{\Delta W}_k$ for all $j, k \in \mathcal{K}^+$, $1 < j < k$, where $\mathcal{K}^+ \subseteq \mathcal{K}$ is the set of mergers that is approved with positive probability. Moreover, if $j \notin \mathcal{K}^+$ and $k \in \mathcal{K}^+$, $j < k$, then $\Delta W(j, l) < \underline{\Delta W}_k$. That is, the lowest level of welfare change that is acceptable to the antitrust authority equals zero for the smallest merger M_1, is strictly positive for every other merger M_k with $k > 1$, and is monotonically increasing in the size of the merger.

Proof. The proof proceeds in seven steps. Steps 1 through 6 are as in the proof of Proposition 1 but with the welfare criterion replacing the consumer surplus criterion. Step 7 does not carry over as we cannot guarantee that $\Delta W(k, l) > \Delta W(k+1, l)$. But the same type of argument can be used to show that if $j \notin \mathcal{K}^+$ and $k \in \mathcal{K}^+$, $j < k$, then $\Delta W(j, l) < \underline{\Delta W}_k$. $\qquad\square$

5.2 Synergies in Fixed Costs

tbw.

5.3 No Single Pivotal Firm

So far, we have assumed that there is a single target (and therefore a single 'pivotal player'), firm 0, that is part of every potential merger. We now show that our main result continues to hold in the simplest possible setting where there is no single target but, as before, all mergers are mutually exclusive. Specifically, we assume that there are three potential mergers, a merger between firms 1 and 2, a merger between firms 1 and 3, and a merger between firms 2 and 3. The merger between firms i and $j > i$ is denoted $M_{ij} \equiv (\{i, j\}, \bar{c}_{ij})$, where $\bar{c}_{ij}$ is the corresponding post-merger marginal cost, which (conditional on the merger being feasible, $\phi_{ij} = 1$) is drawn from distribution G_{ij} with support $[l, h_{ij}]$.

Note that any two of these three potential mergers have in common exactly one merger partner. As $c_1 > c_2 > c_3$, this implies that we can order the three mergers by the combined pre-merger market shares of their merger partners: M_{23} is larger than M_{13}, which in turn is larger than M_{12}. With this ordering of merger size, our previous analysis carries over to this setting. In particular, any optimal policy approves the smallest merger M_{12} if and only if it is CS-nondecreasing, satisfies $\underline{CS}(M_{ij}) > 0$ if merger M_{ij} is approved with positive probability, and $\underline{CS}(M_{13}) < \underline{CS}(M_{23})$ if both M_{13} and M_{23} are approved with positive probability. Moreover, if the largest merger M_{23} is approved with positive probability, then so is M_{13}.

6 Appendix

Lemma 4. *Consider the function $H(s_1, ..., s_N) = \sum_n (s_n)^2$ and two vectors $s' = (s'_1, ..., s'_N)$ and $s'' = (s''_1, ..., s''_N)$ having $\sum_{n=1}^{N} s'_n = \sum_{n=1}^{N} s''_n$. If for some r, (i) $s'_r \geq s'_j$ for all $j \neq r$, (ii) $s''_r > s'_r$, and (iii) $s''_j \leq s'_j$ for all $j \neq r$, then $H(s'') > H(s')$.*

Proof. Without loss of generality, take $r = 1$ and define $\Delta_n \equiv s'_n - s''_n$ for $n > 1$. Observe that $\Delta_n \geq 0$ for all $n > 1$ and $\Delta_n > 0$ for some $n > 1$. Define as well the vectors $s^n \equiv (s'_1 + \sum_{t=2}^{n} \Delta_t, s'_2 - \Delta_2, ..., s'_n - \Delta_n, s'_{n+1}, ..., s'_N)$ for $n > 1$ and $s^1 \equiv s'$. Note that $s^N = s''$.

Then

$$H(s'') - H(s') = \sum_{n=1}^{N-1} [H(s^{n+1}) - H(s^n)].$$

Now letting $\bar{s}_1^1 \equiv s_1'$ and $\bar{s}_1^n \equiv s_1' + \sum_{t=2}^{n} \Delta_t \geq s_1'$ for all $n > 1$, each term in this sum is nonnegative,

$$\begin{aligned}
H(s^{n+1}) - H(s^n) &= (\bar{s}_1^n + \Delta_{n+1})^2 + (s_n' - \Delta_{n+1})^2 - (\bar{s}_1^n)^2 - (s_n')^2 \\
&= 2\Delta_{n+1}(\bar{s}_1^n - s_n') + 2(\Delta_{n+1})^2 \geq 0,
\end{aligned}$$

and strictly positive if $\Delta_{n+1} > 0$. Since $\Delta_{n+1} > 0$ for some $n \geq 1$, the result follows. $\qquad\square$

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