Procurement Contracts with Hidden Investments and Endogenous Contracting Time

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Abstract

This paper analyses a procurement setting where the supplier can make a cost-reducing investment prior to trade and the parties have to agree on signing the contract before or after the investment decision. There exist three types of equilibria which all lead to inefficiently low investment and inefficiently low trade when the supplier does not invest. In the first type the parties always contract before the investment decision, in the second always after the investment decision. In the third type they mix between the two but allocations are identical. The types of equilibria can be ranked by pareto dominance.

1 Introduction

This paper analyses a procurement setting with one buyer, one supplier and the following features: The supplier can make a cost-reducing, private, and relation-specific investment at privately known cost prior to trade and the parties have to agree on signing the contract either before or after the investment decision.

The investment is cost-reducing as it improves the distribution of production costs that are learned after the investment decision. The investment decision is private as the buyer observes the investment decision neither directly, nor indirectly through the production costs which are privately observed by the agent. The investment is relation-specific as it does not influence the value of the outside option. In contrast to the literature on the hold-up problem there is an equivalence of observability and contractibility of actions in this model.[1]

The parties have to agree on whether the contract should be signed before or after the investment decision. When they meet the buyer has all bargaining power and offers a contract to the supplier in a take-it-or-leave-it manner. Before the parties have met, they cannot commit to any action. The point in time at which the contract is signed is therefore endogenous. The contracting problems before and after the investment decision are quite different. If the parties sign a contract before the investment decision, then the contract has to elicit investment costs,

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[1] see Hart and Moore (1988)

give incentives for a certain investment decisions and finally to elicit the production costs. If the parties sign a contract after the investment decision, then the contract has to elicit the investment decision and the production costs. There is no moral hazard anymore, rather the principal faces a problem of sequential screening as in Courty and Li (2000).

There exists three types of equilibria which all lead qualitatively to the same allocations: Inefficiently low investment and inefficiently low trade when the supplier has high investment costs and does not invest. In the first type of equilibrium, the parties contract before the investment decision independently of the investment cost of the supplier. The contracting time is therefore uninformative about investment costs. This type of equilibrium is called *Uninformative Equilibrium with Early Contracting.* The second type of equilibrium features contracting after investment independently of the investment cost of the agent. This type is called *Uninformative Equilibrium with Late Contracting.* Finally there is the third type of equilibrium where the parties sometimes contract before and sometimes after the investment decision for all investment costs of the supplier. The resulting investment and production decisions are independent of whether the contract was signed before or after the investment decision. However conditional on having invested, the supplier is more likely to have invested before the investment decision. This type is therefore called *Informative Equilibrium with Identical Allocations.*

From a normative point of view it can be shown that uninformative equilibria with late contracting are pareto dominated by both other types of equilibria as they lead to the largest distortions in investment and production. Furthermore it holds that informative equilibria with identical allocations are pareto dominated by uninformative equilibria with early contracting. This results from a hold-up problem in late contracting as the buyer cannot commit to terms of trade that sustain higher levels of investment for the supplier.

The relevance of pre-trade investments in contractual relationships is widely recognized in economics. This paper looks at a setting where neither the investment decision nor its consequences are observable. Such an assumption is justified when procurement projects are complex, lie in the supplier's area of expertise and are distant from the buyer's area of expertise[2], and require largely intangible inputs. The procurement of nonstandard IT goods is an example that satisfies these three points.

Related Literature This paper is related to the literature on sequential screening that originated from Baron and Besanko (1984) and Courty and Li (2000).[3] This paper contributes to this literature by showing that if the agent can favorably influence the ex-ante type, then there is a mutual benefit from signing contracts early. Furthermore the optimal early contract is an example for a sequential screening problem with moral hazard where the agent receives an information rent for the private knowledge of the production costs due to moral hazard, even though production cost and investment cost are uncorrelated. A paper from the sequential screening literature which comes to different conclusions is Krähmer and Strausz (2011b). In their framework, moral hazard about an information acquisition decision after trade does

[2]This might motivate the buyer to buy and not to make.

[3]see also Eső and Szentes (2007a) and Pavan et al. (2013)

not lead to additional agency costs for a large number of cases. The key difference lies in the fact that in the present paper, the agent has private information about the cost of investment while in Krähmer and Strausz (2011b) the agent has initially some noisy information about production costs while the cost of the investment is publicly known.

In this model the buyer cannot commit to offer certain contracts before she has met the supplier. This is a form of limited commitment. A paper with dynamic information and limited commitment is Deb and Said (2013). In their paper, the agent can reject a contract in the first period and accept another contract later on. This is in contrast to this paper where the agent has a private action which increases the value of the interaction but cannot reject one contract and accept another contract later on.

In the next section the model is introduced. Section 3 analyses equilibrium play when the parties contract before the investment decision. Section 4 analyses equilibrium play after the investment decision. In Section 5 the meeting decisions of the players are analysed. Section 6 concludes.

2 The Model

An organisation (the principal) seeks to procure a single indivisible good from a supplier (the agent). The value of the good to the principal is commonly known to be v. The agent can supply the good at some cost c. This cost is initially unknown and depends on the technology that the agent uses.

In the beginning of the game, the agent is endowed with the technology T_0. Alternatively he can install a different technology T_1. This would lead to investment costs of θ. θ is the realization of a random variable $\tilde{\theta}$ drawn from $[\underline{\theta}, \overline{\theta}] = \Theta \subseteq \mathbb{R}_+$ according to the distribution function F with density f. The agent first makes the investment decision and then learns the production costs c for the good. The use of T_0 will lead to production costs that are a realization of the random variable $\tilde{c}_0$, drawn from $C = [\underline{c}, \overline{c}] \subseteq \mathbb{R}$ according to the distribution function G_0.

If T_1 is installed, then the costs of supply will be a realization of $\tilde{c}_1$ where $\tilde{c}_1 \in C$ following the distribution function G_1. Both distribution functions admit a density g_τ which is strictly positive on the common support: $g_r(c) > 0$ for all $c \in [\underline{c}, \overline{c}]$ and all $r \in \{0, 1\}$. To capture the cost reducing aspect of the investment I assume G_1 to be first order stochastically dominated by G_0: $G_1(c) \geq G_0(c)$ with a strict inequality for a positive mass of points. Let $v \in (\underline{c}, \overline{c})$, so trade is sometimes but not always socially desirable. Similarly I assume that under efficient production decisions the investment is profitable for some types but not for all. I denote by θ^* the gross benefit of investment under efficient production decisions.

$$\theta^* \equiv \int_{\underline{c}}^{v} (v - c) dG_1(c) - \int_{\underline{c}}^{v} (v - c) dG_0(c) \in (\underline{\theta}, \overline{\theta}) \tag{1}$$

Production costs and investments costs are independently distributed. Both players are risk neutral and have quasilinear preferences. If no contract is signed, then the agent receives an outside option of zero.

Information The agent privately observes the investment costs θ in the beginning of the game. He then takes the investment decision which is neither observable nor verifiable. After the investment decision is made the agent learns the production cost c privately.

Actions, Strategies, and Beliefs After the agent has learned the investment cost, the principal and the agent simultaneously choose an action $d_i \in \{E_i, L_i\}$, $i = A, P$. If both players play E_i, then they meet early. Otherwise they meet late. I refer to this action as the *meeting decision.* When the players meet, the principal offers a contract to the agent in a take-it-or-leave-it manner. By the revelation principle due to Myerson (1986), there is no loss of generality in focusing on direct, truthful, and obedient contracts. The agent reports new information as soon as he learns it. The contract recommends certain actions to the agent and the contract satisfies obedience if the agent finds it optimal to follow these recommendations.

If the players meet early, the principal offers a contract M_E from the set of direct, truthful, and obedient contracts $\mathcal{M}_E$. A contract M_E is the tuple

$$\left\{\alpha(\theta), \{q_r^e(\theta, c), t_r^e(\theta, c)\}_{r=0,1}\right\}$$

After the agent has reported θ, the contract recommends an investment decision. The investment ($r = 1$) is recommended with probability $\alpha(\theta)$. No investment ($r = 0$) is recommended with complementary probability. Depending on the report about θ and the recommendation the agent faces a production schedule $q_r^e(\theta, \cdot)$ and a transfer schedule $t_r^e(\theta, \cdot)$. These represent the probability of trade and the associated transfer for any report about the production cost c.

If the players meet late, the principal chooses a contract M_L from the set of direct, truthful, and obedient contracts $\mathcal{M}_L$. A contract from this set is a tuple

$$\left\{\left\{q_\rho^l(c), t_\rho^l(c)\right\}_{\rho=0,1}\right\}$$

The agent reports whether he has invested ($\rho = 1$) or not ($\rho = 0$). Depending on this report he faces a pair of quantity and transfer schedules where $q_\rho^l(c)$ is the probability of trade after a report c and $t_\rho^l(c)$ is the associated transfer.

After the agent has received a contract offer he decides where to accept or reject the offer.

A strategy for the principal is $\sigma_P = (x_P, M_E, M_L)$. x_P is a randomization over $\{E_P, L_P\}$, M_E is a contract offer if the players meet early and M_L is a contract offer if the players meet late. Note that for any randomization over $\mathcal{M}_E$ ($\mathcal{M}_L$) there exists an element in $\mathcal{M}_E$ ($\mathcal{M}_L$) that is outcome equivalent to the randomization. It is therefore not restrictive to focus on strategies where some contract is chosen with certainty.

A strategy of the agent is $\sigma_A = (x_A(\theta), \beta(\theta), n_E(M_E, \theta), n_L(M_P, \theta))$. $x_A(\theta)$ is a randomization over $\{E_A, L_A\}$ depending on the investment cost. $\beta(\theta)$ is the probability that the type θ invests if the principal and the agent meet late. n_E and n_L are mappings from the set of mechanisms and investment cost types into the randomizations over $\{accept, reject\}$.

If the players meet early the principal has the belief F_E over Θ. If they meet late, the belief is F_L on Θ.

The Timing

t=0 Agent privately learns $\theta \in \Theta$

t=1 Agent and Principal choose d_i

t=2 If $(d_A, d_P) = (E_A, E_P)$, then P chooses M_E and A accepts or rejects. If $(d_A, d_P) \neq (E_A, E_P)$, nothing happens.

t=3 Agent takes investment decision

t=4 If $(d_A, d_P) \neq (E_A, E_P)$, then P chooses M_L and A accepts or rejects. If $(d_A, d_P) = (E_A, E_P)$ nothing happens.

t=5 Agent learns $c \in C$

t=6 Allocations realize

Equilibrium Concept Denote the payoff to the principal from some contract M_E by Π_E and from some contract M_L by Π_L. The expected utility for the agent of type θ from the contract M_E by $U^E(\theta)$ and from the contract M_L by $U^L(\theta)$. The equilibrium concept used is Sequential Equilibrium.[4]

Definition 1. $(\sigma_A, \sigma_P, F_E, F_L)$ *constitutes a Sequential Equilibrium of the game if*

i) $M_m \in \arg\max_{\mathcal{M}_m} \Pi_m$ *for* $m = E, L$

ii) $x_P \in \arg\max_{x\in[0,1]} \int_\Theta x x_A(\theta)\Pi_E(M_E) + (1 - x x_A(\theta))\Pi_L(M_L)$

iii) $x_A(\theta) \in \arg\max_{x\in[0,1]} x x_P U^E(\theta) + (1 - x x_P)U^F(\theta)$, $\forall\theta$

iv) $\beta(\theta) \in \arg\max_{\beta\in[0,1]} U^L(\theta)$, $\forall\theta$

v) $n_m(M_m, \theta) \in \arg\max_{n\in[0,1]} nU_m(\theta)$, $m = E, L$, $\forall\theta$

vi) $F_E(\theta) = \int_{\underline{\theta}}^{\theta} x_A(z)f(z)dz / \int_{\underline{\theta}}^{\overline{\theta}} x_A(z)f(z)dz$ *and* $F_L(\theta) = \int_{\underline{\theta}}^{\theta}(1 - x_A(z))f(z)dz / \int_{\underline{\theta}}^{\overline{\theta}}(1 - x_A(z))f(z)dz$ *if well defined.*

First Best Benchmark The expected social surplus that is generated from the interaction of the principal and the agent with investment cost θ is

$$S(\theta) \equiv \alpha(\theta)\int_C q_1(\theta, c)(v - c)dG_1(c) - \alpha(\theta)\theta + (1 - \alpha)\int_C (v - c)q_0(\theta, c)dG_0(c) \qquad (2)$$

The social surplus is maximal for the production rules $q_0^*(c) = q_1^*(c) = \mathbb{1}_{c\leq v}$ and for the investment rule $\alpha^*(\theta) = \mathbb{1}_{\theta\leq\theta^*}$ where θ^* is defined in (1).

[4]As defined in Osborne and Rubinstein (1994) on p.225

3 Early Contracting

In this section I analyse equilibrium play after the two parties have agreed upon meeting early. The principal offers a contract to the agent. This contract requires the agent to report his investment cost and to make an investment decision following a recommendation. I refer to this part of the game as period 1 of the early contracting game. After the investment decision is made, the agent observes his production costs c and reports them. I will refer to this part of the game as period 2 of the early contracting game.

The principal is committed to not offer another contract to the agent if the agent rejects. If the agent is offered a contract which gives him a strictly positive payoff, then sequential rationality requires the agent to accept this contract with certainty. Similarly the agent rejects any contract that gives him a strictly negative payoff. If the contract gives an expected payoff equal to zero, the agent is willing to randomize. But this can never be an equilibrium. The principal would be willing to increase the agent's rent slightly above zero to ensure certain participation. The utility that the agent derives from a given contract is given by

$$U^E(\theta) \equiv \alpha(\theta) \int_C t_1^e(\theta,c) - cq_1^e(\theta,c)dG_1(c) - \alpha(\theta)\theta + (1-\alpha(\theta)) \int_C t_0^e(\theta,c) - cq_0^e(\theta,c)dG_0(c)$$

Any equilibrium strategy for the principal will include a contract M_E that satisfies a participation constraint

$$U^E(\theta) \geq 0, \forall \theta \in \Theta \qquad (PC_E)$$

The principal's payoff from some contract is

$$\Pi_E = \int_\Theta \left\{ \alpha(\theta) \int_C vq_1^e(\theta,c) - t_1^e(\theta,c)dG_1(c) + (1-\alpha(\theta)) \int_C vq_0^e(\theta,c) - t_0^e(\theta,c)dG_0(c) \right\} dF_E(\theta)$$

The principal's problem consists in choosing a contract M_E from the set $\mathcal{M}_E$ in order to maximize Π_E, subject to the participation constraint. A contract which is in $\mathcal{M}_E$ is an *incentive compatible* contract.

3.1 When is a contract incentive compatible?

On the equilibrium path with early contracting the agent with investment cost θ, investment decision r, and production cost c has the following ex-post utility: $u_r^e(\theta,c) \equiv t_r^e(\theta,c) - cq_r^e(\theta,c)$. If this agent reported investment costs $\hat{\theta}$ and production cost $\hat{c}$, his ex-post utility were $v_r^e(\hat{\theta},\hat{c};\theta,c) = t_r^e(\hat{\theta},\hat{c}) - cq_r^e(\hat{\theta},\hat{c})$. A feasible contract induces the agent to report the production costs truthfully on the equilibrium path. It must therefore satisfy

$$u_r^e(\theta,c) \geq v_r^e(\theta,\hat{c};\theta,c) = u_r^e(\theta,\hat{c}) + (\hat{c}-c)q_r^e(\theta,\hat{c}) \quad \forall \hat{c}, c \in C, \forall \theta \in \Theta, r = 0,1 \qquad (IC_E^2)$$

Note that this constraint ensures that the agent reports production costs truthfully even if he has lied about investment costs. The costs of investment are sunk at this stage. The fact that the agent with true types $(\hat{\theta}, c)$ has no incentive to lie about c implies that the true type (θ, c) who has reported $\hat{\theta}$ in the first stage has no incentive to lie about c.

Directly after the contract has been signed, the agent can choose from a large set of deviations from truthtelling. He can make a false report about investment costs, disobey the recommendation given by the contract, and lie about production costs. The previous observation allows to restrict attention to deviation strategies where the agent truthfully reports production costs as long as (IC_E^2) holds. For a feasible contract these deviations are not profitable. It must hold for all θ and $\hat{\theta}$ in Θ that

$$\begin{aligned} U^E(\theta) \geq \alpha(\hat{\theta}) \max \left\{ \int_C u_1^e(\hat{\theta}, c) dG_1(c) - \theta, \int_C u_1^e(\hat{\theta}, c) dG_0(c) \right\} \\ +(1-\alpha(\hat{\theta})) \max \left\{ \int_C u_0^e(\hat{\theta}, c) dG_0(c), \int_C u_0^e(\hat{\theta}, c) dG_1(c) - \theta \right\} \end{aligned} \tag{IC_E^1}$$

(IC_E^1) reflects the combined moral hazard and adverse selection problem that the principal faces. These two agency problems are interrelated and cannot simply be treated in two different constraints. Nevertheless it is helpful to first consider deviation strategies where the agent misreports investment costs but follows obediently the recommendation. The utility from such a deviation strategy is

$$V^E(\hat{\theta}, \theta) \equiv \alpha(\hat{\theta}) \int_C u_1^e(\hat{\theta}, c) dG_1(c) - \alpha(\hat{\theta})\theta + (1-\alpha(\hat{\theta})) \int_C u_0^e(\hat{\theta}, c) dG_0(c)$$

The set of constraints that ensure that this is unprofitable is a subset of (IC_E^1) and is given by

$$U^E(\theta) \geq V^E(\hat{\theta}, \theta) = U^E(\hat{\theta}) + \alpha(\hat{\theta})(\hat{\theta} - \theta) \quad \forall \hat{\theta}, \theta \in \Theta \tag{AS^1}$$

3.2 Simplifying the Incentive Constraints

(IC_E^2) and (AS^1) are two adverse selection constraints, that can be treated by the standard first-order approach of mechanism design. I state the following two well-known equivalence results without proof.

Lemma 1. *A contract satisfies (IC_E^2) if and only if it satisfies for all $r \in \{0, 1\}$*

$$u_r^e(\theta, c) = u_r^e(\theta, \overline{c}) + \int_c^{\overline{c}} q_r^e(\theta, y) dy \tag{RE_E^2}$$

$$q_r^e(\theta, c) \text{ is nonincreasing in } c \tag{MON_E^2}$$

Lemma 2. *A contract satisfies (AS^1) if and only if it satisfies*

$$U^E(\theta) = U^E(\overline{\theta}) + \int_\theta^{\overline{\theta}} \alpha(y) dy \tag{RE^1}$$

$$\alpha(\theta) \text{ is nonincreasing in } \theta \tag{MON^1}$$

A final step is needed to achieve a tractable characterisation of incentive compatibility. In order to exclude that some type of the agent has a profitable deviation strategy that includes disobedient behavior, the following two constraints are helpful.

$$(1-\alpha(\theta))\int_C q_0^e(\theta,c)(G_1(c)-G_0(c))dc \leq (1-\alpha(\theta))\theta - \int_{\underline{\theta}}^{\theta}(1-\alpha(y))dy \qquad (OB_l)$$

$$\alpha(\theta)\int_C q_1^e(\theta,c)(G_1(c)-G_0(c))dc \geq \alpha(\theta)\theta + \int_{\theta}^{\overline{\theta}}\alpha(y)dy \qquad (OB_h)$$

(OB_l) ensures that the agent with lowest investment costs $\underline{\theta}$ has no profitable deviation in reporting investment cost θ and investing independently of the recommendation. In contrast, (OB_h) guarantees that the agent with the highest investment costs $\overline{\theta}$ does not find it profitable to make some report θ and to produce with the inefficient technology independently of the recommendation that he receives. The mathematical expressions are derived from using the revenue equivalence conditions (RE^1) and (RE_E^2) in (IC_E^1). I can state the following result.

Lemma 3. *Some mechanism satisfies (IC_E^1) and (IC_E^2) if and only if it satisfies (RE^1), (RE_E^2), (MON^1), (MON_E^2), (OB_l), and (OB_h).*

The proof of this lemma can be found in the appendix. Lemmata 2 and 1 state that all deviation strategies that include false reports but obedient investment behavior are unprofitable. But what about strategies that include disobedient behavior?

Consider a deviation strategy for the agent with type θ in which he does not invest independently of the recommendation from the contract. With this deviation strategy the agent will never invest. An agent with higher investment costs $\theta' > \theta$ will therefore receive the same utility from this strategy. However the utility on the equilibrium path is lower for the type θ' then for θ by the revenue equivalence condition in period 1. It follows that the ex-ante type θ' finds it also profitable to deviate to this strategy. Conversely it holds that if an agent will not deviate to the disobedient strategy if he has ex-ante type θ', then he will also not deviate to this strategy if he has ex-ante type θ.

Assume now that the agent has an ex-ante type θ and a profitable deviation from the truthful and obedient strategy by investing independently of the recommendation. With this strategy the agent will always invest. An agent with lower investment costs $\theta'' < \theta$ has a strictly higher utility from this deviation strategy. His payoff is the same as for the type θ plus the saved investment costs $\theta - \theta''$. The type θ'' will also receive a higher utility than θ under the truthful and obedient strategy. However the difference in the utilities on the equilibrium path is at most as high as the difference in utilities for the deviation strategy. Revenue equivalence in the first period implies $U^E(\theta'') - U^E(\theta) = \int_{\theta''}^{\theta}\alpha(z)dz$ which is smaller than $\theta - \theta''$: If θ'' finds it unprofitable to use the strategy with disobedience after a recommendation to not invest, then θ will not deviate to this strategy neither.

From this argument it follows that we can focus on disobedient deviation strategies for the most extreme investment cost types. If the agent finds it optimal to be obedient for the highest and the lowest investment cost, then he will also be obedient for intermediate types.

3.3 The optimal Contract with Early Contracting

We are now equipped with a characterization of incentive compatibility that will allow us to find the optimal contract. Using (RE^1) and standard arguments one can express the principal's expected payoff from the contract as virtual surplus.

$$\tilde{\Pi}_E \equiv \int_\Theta S(\theta) - \alpha(\theta)\left(\theta - \frac{F_E(\theta)}{f_E(\theta)}\right) dF_E(\theta) - U^E(\overline{\theta})$$

There are two distinct cases to be considered. In the first, the meeting decision of the agent is uninformative about his type, i.e. $F_E = F$. In the second case, the meeting decision is informative about the agent's type: $F_E \neq F$. For the first case one can give standard assumptions on F that allow to solve for the optimal contract without restrictions. In the second case, those assumptions cannot be made as F_E depends on equilibrium strategies. I characterise the optimal contract in the class of contract with deterministic investment recommendations. This turns out to be not restrictive: In section 5 I show that in any equilibrium where the meeting decision is informative, the optimal contract for early contracting has to have a deterministic investment recommendation.

Consider first the class of contracts where also the production decisions are deterministic. The monotonicity conditions (MON^1) and (MON^2_E) imply that one can focus on cutoff rules: $\alpha(\theta) = \mathbb{1}_{\theta \leq \eta}$, $q_1(\theta, c) = \mathbb{1}_{c \leq p_1(\theta)}$, and $q_0(\theta, c) = \mathbb{1}_{c \leq p_0(\theta)}$. ($OB_l$) simplifies to $\int_{\underline{c}}^{p_0(\theta)} G_1(c) - G_0(c)dc \leq \eta$. The production rules q_0 and q_1 do not influence the information rent that the principal has to give to the agent. She should therefore choose them such that the social surplus is maximal. If one neglects (OB_h), the principal chooses the efficient production rule after a recommendation to invest: $q_1(\theta, c) = q^*(c)$. Furthermore (OB_l) is strictly binding as long as $\eta < \theta^*$. The principal's problem is then

$$\max_\eta \int_{\underline{\theta}}^{\eta} \left\{ \int_{\underline{c}}^{v} (v - c)dG_1(c) - \theta - \frac{F_E(\theta)}{f_E(\theta)} \right\} dF_E(\eta) + \int_\eta^{\overline{\theta}} \int_{\underline{c}}^{p(\eta)} (v - c)dG_0(c)dF_E(\eta)$$

where $p(\eta)$ is defined by

$$\int_{\underline{c}}^{p(\eta)} G_1(c) - G_0(c)dc = \eta \tag{3}$$

for $\eta \leq \theta^*$ and $p(\eta) = v$ for $\eta > v$. The first order condition of this problem is

$$\int_{\underline{c}}^{v} G_1(c)dc - \eta - \frac{F_E(\eta)}{f_E(\eta)} - \int_{\underline{c}}^{p(\eta)} (v - c)dG_0(c) + \frac{(v - p(\eta))g_0(p(\eta))}{G_1(p(\eta)) - G_0(p(\eta))} \frac{1 - F_E(\eta)}{f_E(\eta)} = 0 \tag{4}$$

As one cannot restrict the shape of F_E, there might be multiple solutions to (4). However the global maximum must be one of them. To see this note that if $\eta \geq \theta^*$, then the left hand of (4) side is strictly smaller than zero. For $\eta = \underline{\theta}$, the left hand side is strictly greater than zero. As the left hand side of (4) is continuous, there exists a global maximizer that lies in $(\underline{\theta}, \theta^*)$. Note that this also implies that (OB_h) is satisfied: $\int_{\underline{c}}^{v} G_1(c) - G_0(c)dc = \theta^* > \eta$.

The following assumption ensures that it is indeed optimal to choose a deterministic production rule q_0 after the recommendation not to invest.

Assumption 1. $(G_1(c) - G_0(c))/g_0(c)$ *is concave.*

For the case where the meeting decision is uninformative, the following assumption ensures that the solution to (4) is unique and defines together with (3) the optimal contract from the unrestricted class of contracts.

Assumption 2. $(z - F(\theta))/f(\theta)$ *is not increasing in* θ *for all* $z \in [0, 1]$.

Note, that these assumptions are frequently used in the literature on dynamic mechanism design.[5]

Proposition 1. *Suppose Assumptions 1 and 2 are satisfied. In any equilibrium where the principal offers a contract* M_E *with a deterministic investment decision,* M_E *includes investment and production rules of the following form:*

$$\alpha_E(\theta) = \begin{cases} 1 & \text{if } \theta \leq \theta_E \\ 0 & \text{otherwise} \end{cases}; \quad q_{0E}(\theta, c) = \begin{cases} 1 & \text{if } c \leq p_E \\ 0 & \text{otherwise} \end{cases}; \quad q_{1E}(\theta, c) = q^*(c)$$

where θ_E *satisfies* (4) *and* $p_E = p(\theta_E)$*. It holds that* $\theta_E \in (\underline{\theta}, \theta^*)$ *and* $p_E < v$*. In any equilibrium with* $F_E = F$*,* θ_E *is uniquely defined and the principal offers a contract that includes* α_E*,* q_{0E}*, and* q_{1E}*.*

The contract specified in Proposition 1 leads to two kinds of distortions compared to first best. The first distortion concerns production: If the agent does not invest, then his production decision will be inefficient. For realisations of the production costs lying in (p_E, v) the agent will not deliver the good although it were socially efficient to do so. The second inefficiency lies in the investment decision: Under efficient production rules the social value of the investment by some type θ is $\theta^* - \theta$. But as the production decision for noninvesting types is distorted, the social value of investment is now even higher. $\theta_E < \theta^*$ implies that there are socially efficient investment opportunities that are not realized. The two inefficiencies are connected by the fact that the investment decision is private, i.e. not observable and leading to privately known costs. If the principal were implementing the efficient production decision for those types of the agent, that do not invest, then she would create efficient investment incentives: The agent would find it profitable to invest if $\theta \leq \theta^*$. However the whole surplus that is generated through the investment would go to the agent. The principal can extract surplus created from the investment only by distorting the production decisions of noninvesting types.

4 Late Contracting

In this section I will analyse equilibrium play after one of the players has refused to meet early. The principal faces a standard sequential screening problem. She offers a contract to the agent.

[5] For Assumption 1 see the discussion following Proposition 2.2 in Courty and Li (2000) or Krähmer and Strausz (2011a) on p. 16. Assumption 2 is made by Eső and Szentes (2007b). It is equivalent to assuming F to be logconave and to have a decreasing hazard rate.

In any equilibrium of the game, the agent accepts the contract if and only if the utility U^L from participating in the contract is at least zero. This follows from the same arguments as made in the beginning of the analysis of early contracting. As the investment decision has already been made, U^L only depends on the production technology that the agent uses and not on the sunk investment cost. Given some contract M_L, the agent receives the utility

$$U_\rho^L = \int_C t_\rho^l(c) - cq_\rho^l(c)dG_\rho(c)$$

if he uses T_ρ where $\rho = 0, 1$. The principal chooses a contract M_L from the set $\mathcal{M}_L$ in order to maximize Π_L under the participation constraint

$$U_\rho^L \geq 0, \quad \rho = 0, 1 \tag{PC_L}$$

Suppose the principal believes that a fraction γ of the agents that she meets for late contracting has made the investment. The principal's payoff from some accepted contract M_L is then given by

$$\Pi_L = \gamma \int_C vq_1^l(c) - t_1^l(c)dG_1(c) + (1-\gamma) \int_C vq_0^l(c) - t_0^l(c)dG_0(c)$$

For any contract in $\mathcal{M}_L$ the agent finds it optimal to report the truth about the installed technology in the beginning of the contract and to truthfully communicate his production costs when he has learned them. Lets call the phase when the agent reports his technology period 1, and period 2 shall refer to the time after the agent has learned his production costs and has to report them. On the equilibrium path, the utility that the agent receives if he has installed the technology T_ρ and has production costs c is $u_\rho^l(c) \equiv t_\rho^l(c) - cq_\rho^l(c)$. For some report $\hat{\rho}$ about technology and $\hat{c}$ about costs, the agent with production technology T_ρ and production costs c receives ex-post utility $v_{\rho,\hat{\rho}}^l(\hat{c}, c) \equiv t_{\hat{\rho}}^l(\hat{c}) - cq_{\hat{\rho}}^l(\hat{c})$. The agent is willing to truthfully reveal his production costs on the equilibrium path

$$u_\rho^l(c) \geq v_{\rho,\rho}^l(\hat{c}, c) \quad \forall c, \hat{c} \in C, \rho = 0, 1 \tag{IC_L^2}$$

Note that this constraint implies that the agent also finds it profitable to truthfully reveal his production costs after some false report about his production technology. One therefore only needs to consider period 1 deviation strategies where the agent reports his production cost in period 2 truthfully, given that (IC_L^2) is satisfied. Truthtelling in period 1 is optimal if

$$U_\rho^L \geq \int_C u_{\rho'}^l(c)dG_\rho(c) \quad \rho, \rho' = 0, 1 \tag{IC_L^1}$$

The Optimal Late Contract Offer

The optimal contract for the principal, given his belief γ can be found by the standard method of sequential screening[6]

First I state a standard equivalence result analogue to Lemma 2.

[6] e.g. Courty and Li (2000)

Lemma 4. *A contract satisfies* (IC_L^2) *if and only if it satisfies for all* $\rho \in \{0,1\}$

$$u_\rho^l(\theta, c) = u_\rho^l(\bar{c}) + \int_c^{\bar{c}} q_\rho^l(y)dy \qquad (RE_L^2)$$

$$q_\rho^l(\theta, c) \text{ is nonincreasing in } c \qquad (MON_L^2)$$

I neglect the first period incentive constraint for the noninvesting types. For an investing type, (IC_L^1) can be written as

$$U_1^L \geq U_0^L + \int_C q_0^l(c)(G_1(c) - G_0(c))dc$$

This constraint binds by standard arguments. The principal's payoff expressed as virtual surplus is

$$\tilde{\Pi}_L = \gamma \int_C (v - c)q_1^l(c)dG_1(c) + (1-\gamma)\int_C \left(v - c - \frac{\gamma}{1-\gamma}\frac{G_1(c) - G_c(0)}{g_0(c)}\right) q_0^l(c)dG_0(c)$$

$\tilde{\Pi}_L$ can be maximized pointwise and we can state the following result.

Lemma 5. *Suppose Assumption 1 is satisfied. Given a belief* γ *about the fraction of types who invest, the principal offers a contract* M_L *that satisfies*

$$q_0^l(c) = \begin{cases} 1 & \text{if } c \leq p_L \\ 0 & \text{otherwise} \end{cases}; \quad q_1^l(c) = q^*(c)$$

where p_L *is uniquely determined by*

$$v = p_L + \frac{\gamma}{1-\gamma}\frac{G_1(p_L) - G_c(p_L)}{g_0(p_L)}. \tag{5}$$

Investment Behaviour with Late Contracting

Given the principal's belief γ, the value of the investment for the agent lies in receiving an information rent. As a function of the belief, this rent equals

$$U_1^L(\gamma) = \int_{\underline{c}}^{p_L(\gamma)} G_1(c) - G_0(c)dc$$

where $p_L(\gamma)$ is defined as the solution to (5) for belief γ. An agent with investment cost θ optimally invests if and only if $U_1^L(\gamma) \geq \theta$. The fraction of investing agents is constituted by all agents with investment costs below a certain threshold θ_L. In equilibrium it must hold that the principal's belief about this fraction is correct and the agent invests if and only if his investment cost lies below this threshold:

$$U_1^L(F_L(\theta_L)) = \theta_L \tag{6}$$

This equation has a unique interior solution. As $p_L(\gamma)$ is decreasing in γ, $U_1^L(\gamma)$ is also decreasing in γ. F_L is increasing in θ_L. The left hand side of (6) is therefore decreasing in θ_L while the right hand side is increasing. For $\theta_L = \underline{\theta}$, the rent given to investing types equals $\int_{\underline{c}}^{v} G_1(c) - G_0(c)dc = \theta^*$ which is greater than $\theta_L = \underline{\theta}$. For $\theta_L = \overline{\theta}$, the rent given to high types becomes zero. The right hand side of (6) equals $\overline{\theta}$, which is greater.

Proposition 2. *Suppose Assumption 1 is satisfied. In any equilibrium the principal offers a contract M_L and the agent plays an investment strategy $\beta(\theta)$ which satisfy*

$$\beta_L(\theta) = \begin{cases} 1 & \text{if } \theta \leq \theta_L \\ 0 & \text{otherwise} \end{cases}; \quad q_{0L}(\theta, c) = \begin{cases} 1 & \text{if } c \leq p_L \\ 0 & \text{otherwise} \end{cases}; \quad q_{1L}(\theta, c) = q^*(c)$$

where θ_L is uniquely defined by (6) *and $p_L = p_L(F_L(\theta_L))$. Furthermore $\theta_L \in (\underline{\theta}, \theta^*)$ and $p_L < v$.*

Equilibrium behavior after late contracting leads to similar inefficiencies as in the case of early contracting. An agent with technology T_0 might not deliver the good although his production costs lie below the principal's value. Furthermore there is too little investment since agents with investment cost types between θ_L and θ^* do not invest although the social value from the investment exceeds the costs. The intuition for the uniqueness of the equilibrium behaviour in the case of late contracting is the following: If the principal believes the fraction of investing agents to be small, then she is willing to give a high rent to them in order to distort the production for noninvesting types less. But then there is a high value for the agent from the investment and so the fraction of investing types will be large. This does not confirm the original expectations of the principal and is therefore not an equilibrium.

If however the principal believes the fraction of investing types to be high, then she will distort production of noninvesting types farer away from first best in order to give less information rent to an agent who has invested. The value of the investment for the agent is small in such a case and only agents with very small investment costs can be expected to invest.

In equilibrium these two forces are exactly balanced.

5 Meeting Decision

In this section I analyse the player's meeting decisions d_A and d_P in equilibrium.

The Principal's meeting Decision

The meeting decision is essentially made by the agent. The principal always weekly prefers to meet early over meeting later. If the parties meet early the principal can implement the same investment and production rules that she would optimally implement, if the parties would meet later. The principal could therefore always play early meeting $x_P = 1$ and this is outcome equivalent to all equilibria in which the agent plays another strategy.

Categorizing Equilibria

Before I turn to the agent's meeting decision, it is helpful to categorize potential equilibria of the game.

Definition 2. *An equilibrium $(\sigma_A, \sigma_P, F_E, F_L)$ is called a revealing equilibrium if there exists a positive mass of investment cost types $\Theta_{rev} \subset \Theta$ such that $x_A(\theta) = 1 (= 0)$ if and only if $\theta \in \Theta_{rev}$.*

In a revealing equilibrium the principal can infer either for early or for late contracting that he does *not* face any of the investment cost types in the set Θ_{rev}.

Definition 3. *An equilibrium* $(\sigma_A, \sigma_P, F_E, F_L)$ *is called an informative equilibrium if* $x_A(\theta) \in (0, 1)$ *for all* $\theta \in \Theta$ *and* $F_E \neq F_L$.

In an informative equilibrium, the principal cannot exclude that she is facing any of the investment cost types. However she holds different beliefs about the investment costs of the agent for early and late contracting: Different investment cost types mix differently about meeting early or meeting late.

Definition 4. *An equilibrium* $(\sigma_A, \sigma_P, F_E, F_L)$ *is called an uninformative equilibrium if* $x_A(\theta) = x \in [0, 1]$ *for all* $\theta \in \Theta$.

In an uninformative equilibrium, the agent does not contingent his meeting decision on his investment costs. The principal therefore does not learn new information from the meeting time.

The Agent's Meeting Decision

For given beliefs F_E and F_L, the agent with investment cost type θ receives an expected utility of $U^E(\theta)$ from meeting the principal early and $U^L(\theta)$ from meeting late.

$$U^E(\theta) = \int_{\theta}^{\overline{\theta}} \alpha(y)dy \quad \text{and} \quad U^L(\theta) = [\theta_L - \theta]_+$$

An agent with investment costs θ mixes between $d_A = E_A$ and $d_A = L_A$ only if $U^E(\theta) = U^L(\theta)$. Note that $\partial U^E(\theta)/\partial\theta \geq \partial U^L(\theta)/\partial\theta$. This observation is helpful to derive the following result.

Lemma 6. *Suppose Assumption 1 is satisfied. The game has no revealing equilibrium. If an equilibrium is informative, then the principal offers contracts* M_E *and* M_L *such that* $\alpha(\theta) = \beta(\theta)$ *for all* θ *and* $p_L = p_E$.

Proof. Consider first an equilibrium with $U^E(\underline{\theta}) > U^L(\underline{\theta})$: This implies that $U^E(\theta) > U^L(\theta)$ for all θ with $\alpha(\theta) > 0$. So only types θ who do not invest under any of the two contracts are willing to mix between meeting early and late. If they come late with a certain positive probability, then the principal optimally offers a contract which does not distort the production of noninvesting types. An agent with a lower investment cost type has then the possibility to meet late and earn the maximal rent $\int_{\underline{c}}^{v} G_1(c) - G_0(c)dc = \theta^*$. This is a profitable deviation.

Next assume some equilibrium with $U^E(\underline{\theta}) < U^L(\underline{\theta})$: There are two possible cases to be considered. Either there exists some θ' such that $U^E(\theta') = U^L(\theta')$ or not. Suppose θ' exists. Denote by θ'' the lowest type with $\alpha(\theta'') = 0$.[7] All types $\theta \in (\theta', \theta'')$ strictly prefer to contract early. All types $\theta < \theta'$ strictly prefer to contract late. Furthermore $\theta_L \in (\theta', \theta'')$. When meeting the agent late, the principal infers that the agent's investment type does not lie in (θ', θ''). She can then profitably deviate by setting θ_L down to θ'.

[7] respectively the infimum

Suppose next that θ' does not exist. It follows that $\theta'' < \theta_L$. This implies $\theta'' < \theta^*$ and $p_E < v$. When contracting early the principal can infer that the agent does not invest, as all types smaller than θ'' prefer the late contract. She therefore has a profitable deviation by setting p_E up to v.

Finally consider some equilibrium with $U^E(\underline{\theta}) = U^L(\underline{\theta})$. Suppose that there exists some θ such that $U^E(\theta) > U^L(\theta)$. Denote the infimum of the set of these investment cost types by θ'. Denote by θ'' the lowest type with $\alpha(\theta'') = 0$. When the principal and the agent are contracting late, the principal can infer that the investment cost of the agent does not lie in (θ', θ''). As above she can then profitably deviate by setting θ_L down to θ'.

We are left with three possible equilibria: $U^E(\theta) \geq U^L(\theta)$ for all investment cost types θ and the agent always contracts early. $U^E(\theta) \leq U^L(\theta)$ for all investment cost types θ and the agent always contracts late. $U^E(\theta) = U^L(\theta)$ for all θ and $x_A(\theta) \in (0, 1)$. This concludes the proof. □

Lemma 6 reduces the number of possible equilibria and it justifies the restriction to contracts with a deterministic investment decision made in section 3. It shows furthermore that if an equilibrium is informative, then the early and the late contract are equivalent in terms of allocations.

I state the main result of this paper.

Proposition 3. *Suppose Assumptions 1 and 2 are satisfied. Any equilibrium of the game belongs to one of the following groups:*

1. *Uninformative Equilibrium with Early Contracting:* $x_A(\theta) = 1$ *for all* $\theta \in \Theta$, $F_E = F$, M_E *includes* α_E, q_{0E}, *and* $q_1^*(c)$
2. *Uninformative Equilibrium with Late Contracting:* $x_A(\theta) = 0$ *for all* $\theta \in \Theta$, $F_L = F$, M_F *includes* β_L, q_{0L}, *and* $q_1^*(c)$
3. *Informative Equilibrium with Identical Allocations:* $x_A(\theta) \in (0, 1)$ *for all* $\theta \in \Theta$, $p_E = p_L = \hat{p}$, $\theta_E = \theta_L = \hat{\theta}$, *and* $F_E(\hat{\theta}) > F_L(\hat{\theta})$

Any uninformative equilibrium with late contracting and any informative equilibrium with identical allocations is strictly Pareto dominated by any uninformative equilibrium with early contracting.

The complete proof is relegated to the appendix. Here I show why an uninformative equilibrium with late contracting is strictly Pareto dominated by an uninformative equilibrium with early contracting. Plug $\eta = \theta_L$ into the first order condition (4) of the principal under early contracting. By (5) and Proposition 2, we know that

$$\frac{(v - p(\theta_L))g_0(p(\theta_L))}{G_1(p(\theta_L)) - G_0(p(\theta_L))} \frac{1 - F(\theta_L)}{f(\theta_L)} = \frac{F(\theta_L)}{f(\theta_L)}$$

Using this in the first order condition:

$$\int_{\underline{c}}^{v} G_1(c)dc - \theta_L - \int_{\underline{c}}^{p(\theta_L)} (v - c)dG_0(c) > 0$$

It follows that the principal prefers $\theta_E > \theta_L$ to θ_L. The agent also prefers θ_E to θ_L as his rent is given by $U^Z(\theta) = [\theta_Z - \theta]_+$ where $Z = E, L$. The uninformative equilibrium with late contracting is Pareto dominated by the uninformative equilibrium with early contracting.

The intuition behind this result is the following: If principal and agent contract after the investment decision, then the principal takes the distribution of technologies in the population of agents as given. Consequently she has an incentive to distort the production decision for noninvesting types in order to save information rent payments to the investing types. If principal and agent contract before the investment decision is made, then there is still an incentive for the principal to save rent by lowering the production for the noninvesting types. But now there is an opposing effect: Lowering the production of noninvesting types lowers the fraction of investing types, i.e. the distribution of types in the population depreciates. Due to this opposing effect the principal prefers less distortions in the production rule for the noninvesting types. With late contracting, the agent correctly anticipates that the principal has after the investment decision no incentive to influence the agent's investment decision. She offers a contract with higher distortions in the production for noninvesting types. He will therefore reduce the investments compared to the case with early contracting. In the late contracting case, we have a hold up problem that can be mitigated by contracting early. Furthermore the optimal early contract offered by the principal has to share the surplus from early contracting with the agent.

6 Conclusion

This paper analyses a procurement setting with an endogenous choice of the contracting date. The set of equilibria was fully characterised. The allocations for all equilibria have inefficiently low investment and inefficiently low production if the agent has high investment cost and does therefore not invest. There exists one type of equilibrium in which the parties contract before the investment decision independently of the investment cost of the supplier. Another type leads to contracting after the investment decision independently of the investment cost of the supplier. In a third type of equilibrium the parties sometimes contract before and sometimes after the investment for all investment costs of the supplier. The allocations in both contracts are identical but conditional on having invested, a supplier is more likely to have contracted before the investment decision. The first type of equilibrium pareto dominates the second and the third type.

Appendix

A Omitted Proofs

A.1 Proof of Lemma 3

Proof. Lets begin with the "only if"-part. By Lemma 2 we have that (IC_E^1) implies (RE^1) and (MON^1). By Lemma 1, (IC_E^2) implies (RE_E^2) and (MON_E^2). Consider now the constraint

(IC_E^1). This constraint can be written as $U^E(\theta) \geq V^E(\hat{\theta},\theta) + W^E(\hat{\theta},\theta)$ where

$$W^E(\hat{\theta},\theta) \equiv \alpha(\hat{\theta}) \max\left\{0, \theta - \int_C u_1^e(\hat{\theta},c)d(G_1(c)-G_0(c))\right\}$$
$$+(1-\alpha(\hat{\theta}))\max\left\{0, \int_C u_0^e(\hat{\theta},c)d(G_1(c)-G_0(c)) - \theta\right\}$$

From (RE^1) it follows that

$$U^E(\theta') - V^E(\theta,\theta') = \int_{\theta'}^{\theta} \alpha(y) - \alpha(\theta')dy \tag{7}$$

Using (RE_E^2) we have furthermore that $W^E(\theta,\theta')$ equals

$$\alpha(\theta)\left[\theta' - \int_C q_1^e(\theta,c)(G_1(c)-G_0(c))dc\right]_+ + (1-\alpha(\theta))\left[\int_C q_0^e(\theta,c)(G_1(c)-G_0(c))dc - \theta'\right]_+ \tag{8}$$

where $[x]_+ = \max\{0,x\}$.

As (IC_E^1) ensures that obedient investment behavior is optimal also after a truthful report, it must hold that $W^E(\theta',\theta') = 0$ as $U^E(\theta') = V^E(\theta',\theta')$. So we have that $\theta' - \int_C q_1^e(\theta',c)(G_1(c)-G_0(c))dc \leq 0$ and $\int_C q_0^e(\theta',c)(G_1(c)-G_0(c))dc - \theta' \leq 0$. From this it follows that $\int_C q_0^e(\theta',c)(G_1(c)-G_0(c))dc \leq \int_C q_1^e(\theta',c)(G_1(c)-G_0(c))dc$. In words: there is no type θ' who is disobedient after both types of recommendations. It follows that at least one of the "max"-expressions in $W^E(\theta',\theta)$ must be zero. If the first is zero, then one can write (IC_E^1) as

$$(1-\alpha(\theta))\int_C q_0^e(\theta,c)(G_1(c)-G_0(c))dc \leq (1-\alpha(\theta))\theta' + \int_{\theta'}^{\theta}(\alpha(y)-\alpha(\theta))dy \tag{9}$$
$$\forall\theta \in \{\theta \in \Theta | \alpha(\theta) < 1\}$$

Set $\theta' = 0$ and note that this conditions is trivially satisfies for some $\alpha(\theta) = 1$. This gives (OB_l).

If the second is zero, then one can write (IC_E^1) as

$$\alpha(\theta)\int_C q_1^e(\theta,c)(G_1(c)-G_0(c))dc \geq \alpha(\theta)\theta + \int_{\theta}^{\theta'} \alpha(y)dy \tag{10}$$
$$\forall\theta \in \{\theta \in \Theta | \alpha(\theta) > 0\}$$

Set $\theta' = \overline{\theta}$ and note that the condition is trivially satisfied for $\alpha(\theta) = 0$. So we have (OB_h). This concludes the "only if"-part.

Lets show the "if"-part. By Lemma 2 (MON^1) and (RE^1) imply (AS^1) and by Lemma 1 (MON_E^2) and (RE_E^2) imply (IC_E^2). It remains to show that constraints which are in (IC_E^1) but not in (AS^1) are satisfied by adding (OB_l) and (OB_h). Note that the left hand side of (9) is increasing in θ'. (9) is therefore strictest for $\theta' = 0$. This means that (OB_l) implies (9). The left hand side of (10) is increasing in θ'. (10) is therefore strictest for $\theta' = \overline{\theta}$. (OB_h) implies (10). Using (7) and (8), we have that (9) and (10) together with (RE^1) and (RE_E^2) ensure that all constraints in (IC_E^1) that concern deviation strategies with disobedience after at most one recommendation hold. Consider some deviation strategy where the agent reports

θ and is disobedient after all recommendations. Such a strategy is only of interest if both recommendations are given with positive probability, i.e. $\alpha(\theta) \in (0,1)$. $\int_C q_0^e(\theta,c)(G_1(c) - G_0(c))dc$ captures the gain from using T_1 instead of T_0 after some recommendation r for a given θ. From (9) and (10) it follows that $\int_C q_1^e(\theta,c)(G_1(c) - G_0(c))dc \geq \int_C q_0^e(\theta,c)(G_1(c) - G_0(c))dc$ for all θ. This means that there does not exist some θ' such that $\int_C q_1^e(\theta,c)(G_1(c) - G_0(c))dc < \theta'$ and $\int_C q_0^e(\theta,c)(G_1(c)-G_0(c))dc > \theta'$. (OB_l) and (OB_h) ensure that any deviation strategy with disobedience after both recommendations is dominated by a deviation strategy with disobedience after only one of the recommendations. This completes the proof. □

A.2 Proof of Proposition 1

A relaxed version of the principal's problem is

$$\max_{\alpha,q,U_E(\bar{\theta})} \tilde{\Pi}_E \quad \text{s.t.} \quad (MON^1), (OB_l), U^E(\bar{\theta}) \geq 0$$

Consider a Lagrangian which includes (OB_l)

$$\mathcal{L} \equiv$$

$$\tilde{\Pi}_E - \int_\Theta \mu(\theta) \left\{ (1-\alpha(\theta)) \left(\int_C q_0^e(\theta,c)(G_1(c) - G_0(c))dc - \theta \right) - \int_{\underline{\theta}}^{\theta} (1-\alpha(y))dy \right\} d\theta$$

$\mu(\theta)$ is the Lagrange-Multiplier. As (OB_l) is trivially binding for $\alpha(\theta) = 1$, $\mu(\theta) = 0$ for some θ with $\alpha(\theta) = 1$.

Lemma 7. *Suppose Assumption 1 is satisfied. $\mathcal{L}$ is maximized with respect to q_0^e by $q_0^e(\theta,c) = \mathbb{1}_{c \leq p(\theta)}$ for some $p(\theta)$.*

Proof. For fixed α and q_1^e, $\mathcal{L}$ can be written as

$$\mathcal{L} = \int_\Theta (1-\alpha(\theta)) \int_C q_0^e(\theta,c) \left(v - c - \frac{\mu(\theta)}{f(\theta)} \frac{G_1(c) - G_0(c)}{g_0(c)} \right) dG_0(c) dF_E(\theta) + Rest$$

This is linear in $q_0^e(\theta,c)$ and under Assumption 1 and $v \in (\underline{c}, \bar{c})$, $v - c - \frac{\mu(\theta)}{f(\theta)} \frac{G_1(c)-G_0(c)}{g_0(c)}$ has a unique root where the sign changes from positive to negative. □

Together with the arguments made in the main text, this proves the statement for the case $F_E \neq F$. For $F_E = F$ we can go a step further.

The Lagrangian cannot be maximized with respect to α in a point wise fashion. Therefore I introduce a function $\tilde{\mathcal{L}}$

$$\tilde{\mathcal{L}} \equiv \int_\Theta \alpha(\theta) \left(\int_C (v-c) q_1^e(\theta,c) dG_1(c) - \theta + \frac{\int_\Theta \mu(\theta)d\theta - F(\theta)}{f(\theta)} \right)$$
$$+ (1-\alpha(\theta)) \int_C q_0^e(\theta,c) \left(v - c - \frac{\mu(\theta)}{f(\theta)} \frac{G_1(c) - G_0(c)}{g_0(c)} \right) dG_0(c) dF(\theta)$$

$\tilde{\mathcal{L}}$ has the following properties:

Lemma 8. *$\mathcal{L}$ and $\tilde{\mathcal{L}}$ satisfy*

1. $\mathcal{L} \leq \tilde{\mathcal{L}}$

2. If α *is deterministic and satisfies* (MON^1)*, then* $\mathcal{L} = \tilde{\mathcal{L}}$*.*

Proof. Straightforward calculations show that

$$\mathcal{L} = \tilde{\mathcal{L}} - \int_\Theta \alpha(\theta) \int_0^\theta \mu(y)dyd\theta - \int_\Theta \mu(\theta)\theta\alpha(\theta)d\theta$$

Point 1. is then immediate. For 2. note that if (MON^1) is satisfied and $\alpha(\theta)$ is deterministic, then either $\alpha(\theta) = 0$, or $\alpha(\theta) = 1$ implying $\mu(\theta) = 0$ and by (MON^1) $\mu(\theta') = 0$ for all $\theta' < \theta$. □

Lemma 9. *Suppose Assumptions 1 and 2 are satisfied. There exists a maximizer of* $\tilde{\mathcal{L}}$ *such that* $\alpha(\theta)$ *is deterministic and satisfies* (MON^1)*.*

Proof. $\tilde{\mathcal{L}}$ is linear in $\alpha(\theta)$. It remains to check whether the derivative of $\tilde{\mathcal{L}}$ with respect to $\alpha(\theta)$ is decreasing in θ. $\theta - \frac{\int_\Theta \mu(\theta)d\theta - F(\theta)}{f(\theta)}$ is therefore increasing under Assumption 1 if $\int_\theta \mu(\theta)d\theta \in [0, 1]$. To show that $\int_\theta \mu(\theta)d\theta \in [0, 1]$ for any optimum of $\tilde{\mathcal{L}}$, note that some α cannot be optimal if $\alpha(\theta) = 1$ for all $\theta \leq \theta^*$. For this α, the constraint would have no bite, implying that $\mu(\theta) = 0$ for all θ. One can then increase $\tilde{\mathcal{L}}$ by decreasing α for all $\theta \geq \theta^*$. We therefore know that for any optimal α, there must exist some θ' with $\theta' < \theta^*$ such that $\alpha(\theta') = 0$. It follows that at $\theta = \theta'$ we have

$$\begin{aligned} &\int_C (v - c)q_1^e(\theta', c)dG_1(c) - \theta' + \frac{\int_\Theta \mu(\theta)d\theta - F(\theta')}{f(\theta')} \\ &- \int_C q_0^e(\theta', c)(v - c - \frac{\mu(\theta')}{f(\theta')}\frac{G_1(c) - G_0(c)}{g_0(c)})dG_0(c) \leq 0 \end{aligned}$$

This can be rewritten as

$$\begin{aligned} \int_\Theta \mu(\theta)d\theta &\leq f(\theta')\theta' + F(\theta') - f(\theta')\Big(\int_C (v - c)q_1^e(\theta', c)dG_1(c) \\ &- \int_C q_0^e(\theta', c)(v - c - \frac{\mu(\theta')}{f(\theta')}\frac{G_1(c) - G_0(c)}{g_0(c)})dG_0(c)\Big) \\ &\leq f(\theta')\theta' + F(\theta') - f(\theta')\theta^* < F(\theta') \leq 1 \end{aligned}$$

where the first inequality in the last line follows from the fact that q_1^e is the efficient production plan in any maximizer of $\tilde{\mathcal{L}}$.

Finally note that for any investment rule α that satisfies (MON^1), (OB_l) is relaxed as θ rises. From this it follows that $\mu(\theta)/f(\theta)$ has to be decreasing in θ. The result follows. □

Lemmata 8 and 9 imply that the investment rule and the production rules given in Proposition 1 have to be part of any optimal contract offered if $F = F_E$.

A.3 Proof of Proposition 3

Lemma 10. *In any equilibrium with* $U^E(\theta) = U^L(\theta)$ *for all* θ $F_E(\theta_L) = F_E(\theta_E) > F_L(\theta_E) = F_L(\theta_L)$*.*

Proof. Using (5) in (4):

$$\int_{\underline{c}}^{v} G_1(c)dc - \hat{\theta} - \frac{F_E(\hat{\theta})}{f_E(\hat{\theta})} - \int_{\underline{c}}^{p(\hat{\theta})} (v-c)dG_0(c) + \frac{F_L(\hat{\theta})}{1-F_L(\hat{\theta})}\frac{1-F_E(\hat{\theta})}{f_E(\hat{\theta})} = 0 \tag{11}$$

As $\hat{\theta} < \theta^*$ this implies that

$$\frac{F_L(\hat{\theta})}{1-F_L(\hat{\theta})}\frac{1-F_E(\hat{\theta})}{f_E(\hat{\theta})} > \frac{F_E(\hat{\theta})}{f_E(\hat{\theta})}$$

which implies $F_E(\hat{\theta}) > F_L(\hat{\theta})$. □

From this it also follows that there is no uninformative equilibrium in which $x_A(\theta) = x \in (0,1)$.

The uninformative equilibria in which the agent either always chooses to meet early or always to meet late exist under appropriate specification of out-off-equilibrium beliefs. In particular the principal's belief off-the-equilibrium path need to put sufficient mass on low investment costs. This results in contracts which have high distortions in the production of noninvesting types and small rents for investing types.

It remains to show that

Lemma 11. *Uninformative equilibria with early contracting Pareto dominate informative equilibria with identical allocations.*

Proof. From Lemma 10 we know that $F_E(\hat{\theta}) > F_L(\hat{\theta})$ at any $\hat{\theta} = \theta_L = \theta_E$ in an informative equilibrium with identical allocations. It follows that $F(\hat{\theta}) \in (F_L(\hat{\theta}), F_E(\hat{\theta}))$ by the fact that beliefs are derived by Bayes' rule. $\hat{\theta}$ satisfies the first order condition in (11) Note that by Bayes' rule $F_E(\hat{\theta}) = \int_{\underline{\theta}}^{\hat{\theta}} x_A(z)f(z)dz / \int_{\underline{\theta}}^{\overline{\theta}} x_A(z)f(z)dz$ and $F_L(\hat{\theta}) = \int_{\underline{\theta}}^{\hat{\theta}}(1-x_A(z))f(z)dz / \int_{\underline{\theta}}^{\overline{\theta}}(1-x_A(z))f(z)dz$. One can decrease the difference between $F_E(\hat{\theta})$ and $F_L(\hat{\theta})$ by either decreasing $x_A(\theta)$ slightly for some appropriate $\theta \in [\underline{\theta}, \hat{\theta})$ or by increasing $x_A(\theta)$ slightly for appropriate $\theta \in (\hat{\theta}, \overline{\theta}]$. As $\hat{\theta}$ is a maximizer the first derivative changes its sign from plus to minus at $\hat{\theta}$. It follows that the slight variation in $x_A(\theta)$ increase the optimal choice of $\hat{\theta}$. This increases payoffs for both players. □

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