GOVERNANCE AND THE EFFICIENCY
OF ECONOMIC SYSTEMS
GESY

Discussion Paper No. 399

Subjective Evaluation versus Public Information

Helmut Bester*
Johannes Münster**

* Free University Berlin
** University of Cologne

May 2013

Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

Sonderforschungsbereich/Transregio 15 · www.sfbtr15.de
Universität Mannheim · Freie Universität Berlin · Humboldt-Universität zu Berlin · Ludwig-Maximilians-Universität München
Rheinische Friedrich-Wilhelms-Universität Bonn · Zentrum für Europäische Wirtschaftsforschung Mannheim

Speaker: Prof. Dr. Klaus M. Schmidt · Department of Economics · University of Munich · D-80539 Munich,
Phone: +49(89)2180 2250 · Fax: +49(89)2180 3510

Subjective Evaluation versus Public Information*

Helmut Bester† and Johannes Münster‡

May 24, 2013

Abstract

This paper studies a principal–agent relation in which the principal's private information about the agent's effort choice is more accurate than a noisy public performance measure. For some contingencies the optimal contract has to specify ex post inefficiencies in the form of inefficient termination (firing the agent) or third–party payments (money burning). We show that money burning is the less efficient incentive device: it is used at most in addition to firing and only if the loss from termination is small. Under an optimal contract the agent's wage may depend only on the principal's report and not on the public signal. Nonetheless, public information is valuable as it facilitates truthful subjective evaluation by the principal.

Keywords: Subjective evaluation, moral hazard, termination clauses, third–party payments

JEL Classification No.: D23, D82, D86, J41, M12

*We wish to thank Daniel Krähmer, Roland Strausz, and seminar participants at Freie Universität Berlin, Humboldt Universität Berlin, University of Cologne, University of Copenhagen, and WHU Vallendar for comments and interesting discussions. Support by the German Science Foundation (DFG) through SFB/TR 15 is gratefully acknowledged.

†Free University Berlin, Dept. of Economics, Boltzmannstr. 20, D-14195 Berlin (Germany); email: *hbester@wiwiss.fu-berlin.de*

‡University of Cologne, Dept. of Economics, Albertus-Magnus-Platz, D-50923 Cologne (Germany); email: *johannes.muenster@uni-koeln.de*

1 Introduction

In the textbook moral hazard problem, the agent chooses some unobservable effort, and the only information about his success is some noisy but objective performance measure which is verifiable by outsiders. As Prendergast (1999) has pointed out, however, most people do not work in jobs like these. Rather, many firms use subjective performance evaluations.

This paper studies subjective performance evaluation in a contracting problem between a risk-neutral principal and one risk-neutral agent with limited liability. The principal may use a publicly verifiable but noisy objective performance signal to provide effort incentives for the agent. But, he privately receives more accurate information about the output produced by the agent. This information is not observable by outsiders and in this sense 'subjective'. We show that the optimal contract always relies not only on the public performance measure but also on subjective evaluation by the principal. Therefore, it has to address two incentive problems. On the one hand, the agent must be given incentives to exert effort. On the other hand, the principal has to be incentivized to report his information truthfully: giving a bad performance evaluation must be costly for the principal if the performance is in fact good; otherwise, the principal would be tempted to report bad performance to save on wage costs. Thus some ex-post inefficiencies are unavoidable.

The literature has studied two different solutions to the incentive problem of truthful subjective evaluation. First, Kahn and Huberman (1988) study up-or-out contracts in a dynamic setting where in an initial period the agent should acquire some firm specific human capital. The agent chooses an effort to learn, and then the principal privately receives information about the agent's success. In the optimal contract, the principal commits ex ante either to promote the agent and pay a high wage, or else to end the relationship by *firing* the agent. The principal is prevented from giving a bad performance evaluation when the agent was successful by his commitment to fire upon a bad performance evaluation. There is an ex post inefficiency, however, since the agent is fired after a bad evaluation, even if it would be ex post optimal to keep him.

Second, MacLeod (2003) allows for payments to third parties ("*money burning*"). Here the principal commits to pay out the same amount of money, irrespective of the performance evaluation, but the agent receives only a part of this payment when the evaluation is bad, while the remaining part is paid to a third party. The principal thus has no incentive to give bad evaluations to safe costs. Again, this involves an ex post inefficiency, which here takes the form of money burning.

While terminations of economic relationships are frequently observed in practice, examples for third–party payments are rather hard to come by.[1] Our paper provides an eco-

[1]Besides the obvious examples of layoffs and dismissals, option contracts where one party keeps the authority to terminate the relationship are a case in point. See Lerner and Malmendier (2010) on the use of

nomic explanation of why firing is more frequently observed than money burning. One might think that money burning and firing share essentially the same properties; indeed MacLeod (2003) motivates money burning as a shortcut for protracted conflict within an organization or the termination of a valuable work relationship. We argue, however, that money burning and firing have subtly different implications for the incentives of the principal: The principal's cost of burning one dollar does not depend on the agent's success or failure, but the principal's cost from firing the agent often depends on how successful the agent was. This is the case for up-or-our contracts as in Kahn and Huberman (1988), where a success of the agent means that the agent has acquired human capital valuable for the firm. Similarly, Schmitz (2002) studies a buyer–seller relationship where the seller (agent) produces a good, the quality of which depends on the seller's effort, but only the buyer (principal) knows his true willingness to pay for the good. Here 'firing' corresponds to the buyer not buying the good after it has been produced, and the principal's loss from not buying the good depends on the realized quality of the good.[2]

To capture the dependence of the principal's cost of firing on the agent's success in straightforward manner, we assume that upon firing the principal loses a fraction α of the output produced by the agent. We show that, under this assumption, firing is the more cost-effective instrument, and the principal will prefer firing over money burning. Since the costs of firing are high when the agent was in fact successful, a commitment to fire the agent after a bad performance evaluation gives the principal strong incentives to report successes truthfully. Moreover, the costs of this commitment are relatively low, since on the equilibrium path the principal will give a bad report only if the agent was, after all, not successful, and hence firing him is not that costly for the principal.[3] Only a bounded amount of incentives, however, can be generated with firing. When α is small, the principal cannot be given strong incentives for truthful revelation of his information, since he will not lose much after firing the agent. Thus, money burning can occur under the optimal contract, but only as a secondary instrument in addition to firing the agent.

We also provide several insights into the interaction between subjective and objective performance evaluation. If the principal receives his private information *before* the less informative public signal becomes available, the agent's wage schedule is not uniquely

option contracts in biotechnology research. An example of third party payments is given by Fuchs (2007): some baseball teams can fine their players, and the fines are not paid to the club, but rather to a charity.

[2]Bester and Krähmer (2012) consider a buyer–seller relation where the buyer observes the seller's quality choice, but his observation is not verifiable. They show that 'exit option' contracts, corresponding to the option of 'firing' in the present context, can implement the first–best. Here and in Schmitz (2002) this is not possible because the agent's (the seller's) effort is not directly observable.

[3]Obviously, firing might inflict a cost on the agent, and the threat of firing may be used in to motivate the agent. The use of non-monetary fines to overcome limited liability has been studied in Chwe (1990) and Sherstyuk (2000). To focus on the implications of firing versus money burning for the *principal*'s incentives, we assume that firing imposes no costs at all on the agent.

determined. It can be chosen in such a way that wages are contingent exclusively on subjective evaluation and do not depend on the public performance measure. This does not mean, however, that public information is useless. Wile it is not directly used to incentivize the agent, it facilitates providing incentives for truthful subjective evaluation by the principal. We show that for this reason the principal's payoff is increasing in the precision of public information.

In contrast, if the principal's information arrives *after* the public signal, the agent's wage schedule is uniquely determined by the optimal contract and it necessarily depends on both types of performance measures. This is so because the principal in this case faces an *ex post* truthtelling constraint for each single realization of the public signal rather than an *ex ante* constraint in expectation of the public signal. Perhaps surprisingly, however, it turns out that for the principal's payoff it does not matter whether he receives his private information earlier or later than the realization of public information.

By the latter observation, the principal has no incentive to acquire information early on. But, we also discuss a slight extension of our model where the fraction of output lost due to project termination is increasing over time. Here the principal is strictly better off by delaying his report. When the timing of subjective evaluation can be selected by the principal, he will report when the fraction of output lost due to project termination is high enough such that no payments to third parties are necessary to solve the incentive problem of truthful subjective evaluation. This reinforces our argument that money burning is a less attractive instrument than firing the agent in contracting problems with subjective evaluations.

Related Literature

As described above, our paper contributes to the literature on optimal contracting with subjective evaluation by comparing the use of project termination with payments to third parties. From this literature, Schmitz (2002) and Khalil, Lawaree and Scott (2012) are most closely related to our paper. Schmitz (2001) allows the use of both project termination and money burning and assumes, as is natural in his buyer–seller setting, that the complete output produced by the agent (seller) is lost when the principal (buyer) terminates the relation. In his setting, the optimal contract never involves any money burning. Khalil, Lawaree and Scott (2012) study a related issue in an adverse selection model. In their model, the agent knows the productivity of his effort, and the principal receives some subjective information about the agent's type. If the principal receives his information before the agent chooses his effort, the optimal contract specifies an effort that depends on the agent's report about his type and on the principal's report. In particular, there is a rescaling of the project to a lower level of effort and wage if the agent reports a low productivity but the principal's signal indicates a high productivity. Khalil, Lawaree and Scott (2012) find that this rescaling is superior to money burning. There are several differences between their result and our comparison of firing versus money burning. First, rescaling as in Khalil, Lawaree and Scott (2012) presupposes that the principal receives his private information *before* the agent chooses his effort. Therefore the principal strictly prefers to receive his information early. In our moral hazard setting, the principal's private information is a signal about the effort chosen by the agent, and thus necessarily becomes available only *after* the effort has been chosen. Moreover, in our setting the principal has no incentive to acquire information early; in contrast, he will strictly prefer to acquire information late if the fraction of output lost upon firing is increasing over time. Second, rescaling works in Khalil, Lawaree and Scott (2012) since different types of the *agent* trade off producing output and receiving wages at different rates. In contrast, firing works in our model since the *principal*'s expected cost from firing depends on his private signal. This has implications concerning the set of implementable contracts. In Khalil, Lawaree and Scott (2012) the principal's incentive constraints jointly imply that they hold with equality in every possible state, such that the principal will always be indifferent between his reports. As in MacLeod (2003), this indifference of the principal is an implication of the principal's incentive constraints. In contrast, in our setting the principal's incentive constraints can all be fulfilled without the principal ever being indifferent between sending different reports. As in a standard adverse selection model, in the optimal contract the principal's incentive constraint after having received bad news is slack, while the principal's incentive constraint after having received favorable information is binding. The latter, however, is an implication of optimality of the contract, and not of implementability alone.

Two recent contributions on optimal contracting with subjective evaluation include Lang (2013) and Sonne and Sebald (2012). In Lang (2013) the principal can justify subjective evaluation by sending a costly message. Sonne and Sebald (2012) consider a behavioral economics model in which unfair subjective evaluation by the principal induces a costly conflict with the agent. Similarly to money burning this may help the principal to truthfully commit to a higher wage.

Subjective evaluations have also been studied in models of repeated interactions, where intertemporal incentives for truthful revelation play a key role (e.g. Levin (2003) and Fuchs (2007)). Baker, Gibbons and Murphy (1994) and Pearce and Stacchetti (1998) study the combination of subjective and objective performance measures in infinitely repeated interactions; the focus of these papers differ from ours since thy impose exogenous assumptions on the set of admissible contracts that imply that, in the stage game, the private information of the principal cannot be used. While their focus is on the provision of intertemporal incentives to solve the principal's incentive constraints, we study the optimal contract in a one-shot relation without any exogenous restrictions on the set of admissible contracts.

The paper is organized as follows. Section 2 introduces the model. Section 3 uses the revelation principle to formulate the contract design problem. As a benchmark, we show in Section 4 that under unlimited liability, the principal can implement the first–best effort and extract the full surplus. Section 5 introduces limited liability of the agent and contains the core results of the paper, which are illustrated by an example in Section 6. Whereas the main part of the paper assumes that the principal reports his information before the public signal is realized, in Section 7 we show that the principal realizes the same payoff if he reports ex post, which implies that the principal has no incentive to acquire information early. Moreover, Section 7 also contains the extension of the model where the fraction of output lost upon firing is growing over time. Under this assumption, the principal will always report late enough such that no money burning is needed in the optimal contract. We summarize our results and discuss extensions in Section 8. Formal proofs are collected in an appendix.

2 The Model

There is one principal and one agent, who are both are risk–neutral. At some initial date the principal offers the agent an employment contract for a joint project. The agent's outside option payoff at the contracting stage is normalized to zero. After being employed, the agent chooses some effort $e \in E \equiv [0,1]$. From this effort choice the principal receives at some future date the (*expected*) output or benefit $x = x_H$ with probability e and $x = x_L$ with probability $1 - e$, where $0 < x_L < x_H$. The agent's monetary equivalent of his disutility of effort is $c(e)$. His choice of effort is not observable, neither to outsiders nor to the principal. The principal pays the agent the (expected) wage w at the end of their contractual relationship.

After the agent has chosen e, the principal privately observes whether the output will be x_L or x_H. The principal's information and the realization of output are not publicly observable. The output or benefit received by the principal may, for example, represent the quality of a good or service whose private value is difficult to determine.[4] The output may also represent the cash–flow from a project, which may not be be verifiable. For instance, if the principal operates in several businesses it may be impossible to ascribe money–streams to a particular project.[5] But we assume that there is a imprecise public signal $s \in \{s_L, s_H\}$, which is observable by outsiders and therefore verifiable. The public signal is correct with probability $\sigma \in (1/2, 1)$: if the output is x_i the public signal is s_i with probability $\sigma > 1/2$ and $s_j \neq s_i$ with probability $1 - \sigma < 1/2$. In the limit $\sigma \to 1$ our

[4]Cf. MacLeod (2003) and Schmitz (2002).

[5]Indeed, it is common in the literature to assume that cash–flow is non–observable (see e.g. Baker (1992), Bolton and Scharfstein (1996), or Lewis and Sappington (1997)).

setup becomes equivalent to the standard principal–agent setting, where output is publicly observed and not only by the principal.[6]

The principal can terminate cooperation with the agent after observing the expected output. If he dismisses the agent before the project is completed, he loses a fraction $\alpha \in (0,1]$ of output. The parameter α indicates the extent to which the project is already completed at this stage. In a buyer–seller relation, for example, where the principal refuses to trade after the agent has finished production of a good, $\alpha = 1$ as in Schmitz (2002). The agent's gross payoff from being dismissed is equal to zero. The termination decision is observable and contractible. We allow for stochastic contracts and denote by $\theta \in [0,1]$ the probability that the agent is fired.

The inefficiency of premature project termination may be used to provide incentives for information revelation and effort choice (cf. Kahn and Huberman (1988)). A similar incentive device are fines paid to a third party (cf. MacLeod (2003)). Indeed, we permit non–negative payments to a third party as part of the contract and refer to such payments as 'money-burning', because they reduce the available surplus. Without loss of generality, we assume that only the principal makes payments to a third party and denote by $b \geq 0$ the amount of 'money-burning'.[7]

The agent's effort cost $c(\cdot)$ satisfies $c(0) = 0$ and $c'(e) > 0$, $c''(e) > 0$ for all $e > 0$. Further

$$c'(0) = 0, \quad c'(1) > x_H - x_L. \tag{1}$$

Assumption (1) is sufficient to eliminate corner solutions for the agent's effort when the agent's enumeration is not restricted to be non–negative. For the analysis of the limited liability case, in which wages have to be non–negative, we assume in addition that

$$c'''(e) \geq 0, \quad c''(0) < \frac{\sigma}{1-\sigma}\left(x_H - x_L\right). \tag{2}$$

These conditions avoid corner solutions with zero effort under limited liability. In addition, the first condition in (2) guarantees that the second–order conditions for the principal's optimization problem are satisfied.[8]

The agent's utility is $w - c(e)$, and the principal's utility is $x(1-\alpha\theta) - w - b$. If the agent's effort were contractible and in the absence of limited liability restrictions, it would be chosen to maximize the expected joint surplus

$$S(e) \equiv e\,x_H + (1-e)\,x_L - c(e), \tag{3}$$

[6] See e.g. Holmstrom (1979), Grossmann and Hart (1983), and Sappington (1983).

[7] Whether the principal or the agent pays b plays no role because the wage payment can be adjusted accordingly.

[8] Note that (1) and (2) hold for the specification $c(e) = ke^a/2$ with $k > x_H - x_L$ for all $a > 2$. If $a = 2$, the public signal has to be sufficiently precise so that $\sigma(x_H - x_L)/(1-\sigma) > k$.

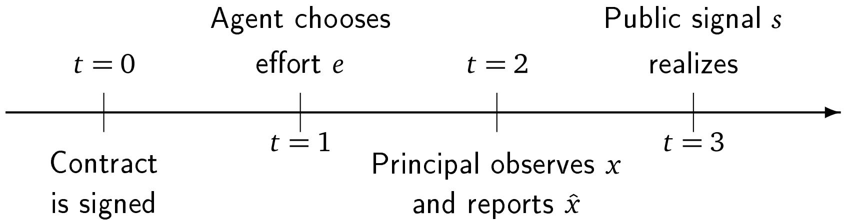

Figure 1: THE SEQUENCE OF EVENTS

which is obtained by setting $\theta = b = 0$. The first order condition

$$x_H - x_L = c'(\tilde{e}), \tag{4}$$

thus determines the first–best effort level $\tilde{e}$, and the first–best surplus is $S(\tilde{e})$.

3 Contract Design

Whenever contracting parties observe new information during the course of their relation, optimal contract design stipulates that they publicly report their information (see Myerson (1986)). Therefore, we consider contracts that require the principal to choose some verifiable message after observing the realization of output. By the Revelation Principle (Myerson (1979)), it is sufficient to consider messages that enable the principal to report simply some output $\hat{x} \in \{\hat{x}_L, \hat{x}_H\}$. Since the terms of the contract can be conditioned on the report, the principal's subjective evaluation of performance complements the objective performance measure provided by the public signal.

We first consider the case where the principal observes output and chooses a report *before* the public signal realizes. We discuss an alternative timing in Section 7. In Sections 4–6 the sequence of events is as follows: After a contract has been signed in stage $t = 0$, the agent chooses his effort e in stage $t = 1$. Then in stage $t = 2$ the principal observes the realization of output $x \in \{x_L, x_H\}$ and reports $\hat{x} \in \{\hat{x}_L, \hat{x}_H\}$. In stage $t = 3$ the contract is executed after the public signal s is observed. Figure 1 summarizes the sequence of events.

A contract specifies the wage, the probability of firing the agent before project completion, and the amount of money-burning contingent on the public signal s and the principal's report $\hat{x}$. Let θ_{ij} denote the probability of firing when the public signal is s_i and the principal's report is $\hat{x}_j$. Similarly, w_{ij} is the wage and b_{ij} represents money burning if the public signal is s_i and the report is $\hat{x}_j$. Let

$$\begin{aligned} w &= (w_{HH}, w_{HL}, w_{LH}, w_{LL}), \quad \theta = (\theta_{HH}, \theta_{HL}, \theta_{LH}, \theta_{LL}), \\ \text{and} \quad b &= (b_{HH}, b_{HL}, b_{LH}, b_{LL}). \end{aligned} \tag{5}$$

A contract $\gamma = (w, \theta, b)$ then has to satisfy $\gamma \in \Gamma \equiv \{(w, \theta, b) \in \mathbf{R}^{12} | \, b \geq 0, \, \theta \in [0,1]^4\}$.

If the principal observes the output x_L and reports $\hat{x}_j$ in stage 2, he receives the expected payoff

$$\begin{aligned} V_L(\gamma, \hat{x}_j) \;\; &\equiv \;\; \sigma \left[(1-\alpha\,\theta_{Lj})x_L - w_{Lj} - b_{Lj}\right] \\ &+ \;\; (1-\sigma)\left[(1-\alpha\,\theta_{Hj})x_L - w_{Hj} - b_{Hj}\right], \end{aligned} \tag{6}$$

because the public signal in stage 3 is s_L with probability σ and s_H with probability $1-\sigma$. Analogously, if the output realization is x_H, the principal's payoff is equal to

$$\begin{aligned} V_H(\gamma, \hat{x}_j) \;\; &\equiv \;\; \sigma \left[(1-\alpha\,\theta_{Hj})x_H - w_{Hj} - b_{Hj}\right] \\ &+ \;\; (1-\sigma)\left[(1-\alpha\,\theta_{Lj})x_H - w_{Lj} - b_{Lj}\right] \end{aligned} \tag{7}$$

when he reports $\hat{x}_j$.

By the Revelation Principle, we can restrict ourselves to contracts that satisfy the incentive compatibility constraints

$$V_L(\gamma, \hat{x}_L) \geq V_L(\gamma, \hat{x}_H), \quad V_H(\gamma, \hat{x}_H) \geq V_H(\gamma, \hat{x}_L). \tag{8}$$

These constraints ensure that reporting truthfully is optimal for the principal. In what follows, we refer to the principal's incentive compatibility constraints in (8) as the *ICP* constraints. Since the principal reports truthfully in stage 2, his ex ante expected payoff at the contracting stage is

$$V(\gamma, e) \equiv e\, V_H(\gamma, \hat{x}_H) + (1-e)\, V_L(\gamma, \hat{x}_L). \tag{9}$$

Truthful reporting by the principal also implies that the agent's expected wage is

$$U_L(\gamma) \equiv \sigma\, w_{LL} + (1-\sigma) w_{HL} \tag{10}$$

if the principal observes x_L, and

$$U_H(\gamma) \equiv \sigma\, w_{HH} + (1-\sigma) w_{LH} \tag{11}$$

otherwise. Therefore, the agent's ex ante payoff is

$$U(\gamma, e) \equiv e\, U_H(\gamma) + (1-e)\, U_L(\gamma) - c(e) \tag{12}$$

at the contracting stage.

Since effort is not observable, the agent chooses e in stage 1 to maximize his expected payoff in (12). This implies that e is determined by the first order condition[9]

$$U_H(\gamma) - U_L(\gamma) = c'(e). \tag{13}$$

[9]Our assumptions (1) and (2) ensure that $0 < e < 1$.

This condition ensures that e maximizes $U(\gamma, e)$ because $U(\gamma, e)$ is strictly concave in e. In what follows, we refer to the incentive compatibility condition for the agent's effort in (13) as the *ICA* constraint.

At the contracting stage, the principal proposes a contract γ that the agent can either accept or reject. As the agent's outside option payoff is zero, he accepts the contract if it satisfies his individual rationality constraint

$$U(\gamma, e) \geq 0. \tag{14}$$

In the following we refer to (14) as the *IRA* constraint.

4 Unlimited Liability Contracts

In this section, we briefly consider as a benchmark the optimal contract in the absence of non–negativity restrictions on the wage schedule w. Thus the agent is not protected by limited liability and he may face a penalty $w_{ij} < 0$ for some realization (s_i, x_j) of the public signal and output. In this situation the principal's problem is

$$\max_{(\gamma,e)\in\Gamma\times E} V(\gamma, e) \quad \text{subject to (8), (13), and (14)} \tag{15}$$

because he has to satisfy the *ICP*, *ICA*, and *IRA* constraints.

As is well–known (see e.g. Harris and Raviv (1979)), with a risk-neutral agent and without limited liability restrictions the principal is able to appropriate the first–best surplus by making the agent the residual claimant in the relationship. This can be done by ignoring the principal's information and conditioning the agent's wage exclusively on the public signal s. This explains the following observation:

Proposition 1 *Let (γ, e) solve problem (15). Then $\theta = b = 0$ and the wages can be chosen such that γ ignores the principal's information:*

$$w_{HH} = w_{HL}, \quad w_{LL} = w_{LH}.$$

Moreover, e is equal to the first–best effort $\tilde{e}$ and the principal's payoff $V(\gamma, \tilde{e})$ is equal to the first–best surplus $S(\tilde{e})$.

Under unlimited liability, subjective evaluation by the principal plays no role, independently of the precision of the public signal.[10] Therefore, the principal actually has no

[10] There are contracts that achieve the first–best, where payments depend on the principal's report, but reporting is not truthful. Formally all four wage parameters could be different, but only two different wages will be paid with positive probability.

incentive to supervise the agent to acquire information about the future realization of output. It is important for this result that negative wage payments are feasible, because the wage $w_{LL} = w_{LH}$ in Proposition 1 is negative. Indeed, it tends to minus infinity in the limit $\sigma \to 1/2$ where the public signal becomes uninformative.[11]

5 Limited Liability Contracts

We now turn to the more interesting case where the agent is protected by limited liability. Thus the principal has to obey the additional constraint that the agent's wage cannot be negative and so his problem becomes

$$\max_{(\gamma,e)\in\Gamma\times E} V(\gamma, e) \quad \text{subject to (8), (13), (14) and } w \geq 0. \tag{16}$$

In what follows, we analyze how the principal's subjective information affects the terms of the contract γ and the agent's effort e. Since the principal's information is more accurate than the public signal, the Informativeness Principle of Holmstrom (1979) suggests that his information should be used in determining the agent's pay. This principle, however, is not directly applicable in the present context because the principal's observation of performance is not publicly verifiable. Nonetheless, even though subjective evaluation is constrained by the *ICP* conditions, we will show that it will always be used in an optimal contract.

We begin with several lemmas that identify the binding constraints in problem (16).

Lemma 1 *Let* (γ, e) *solve problem (16). Then*

(a) the IRA *constraint (14) is not binding;*

(b) γ *satisfies*

$$w_{HL} = w_{LL} = 0 \quad \text{and} \quad \theta_{HH} = b_{HH} = \theta_{LH} = b_{LH} = b_{LL} = 0; \tag{17}$$

(c) $b_{HL} > 0$ *implies* $\theta_{HL} = 1$, *and* $\theta_{LL} > 0$ *implies* $\theta_{HL} = 1$;

(d) in the ICP *constraints (8), only the inequality* $V_H(\gamma, \hat{x}_H) \geq V_H(\gamma, \hat{x}_L)$ *is binding;*

(e) (γ, e) *satisfies*

$$\sigma\left(\alpha\,\theta_{HL}\,x_H + b_{HL}\right) + (1-\sigma)\,\alpha\theta_{LL}x_H = c'(e). \tag{18}$$

[11]This follows from equation (33) in the proof of Proposition 1 in the Appendix.

Part (a) of Lemma 1 shows that agent's individual rationality constraint (14) is automatically satisfied. Because the agent could choose zero effort at zero cost and wages are non–negative by limited liability, his utility cannot become negative.

By part (b), the agent's wage payment can be positive only if the principal reports that output is high. The reason is that, if w_{HL} or w_{LL} were positive, the principal could decrease these payments while increasing b_{HL} or b_{LL} by the same amount. Thereby the agent's incentive constraint could be relaxed while all other constraints remain unaffected. Moreover, part (b) also implies that firing and money burning can occur only if output is low. This result is driven by the observation under part (d): the relevant principal's incentive constraint in (8) is that he should have no incentive to *underreport* output, i.e. to claim that output is low while it is in fact high. Lowering any of θ_{HH}, b_{HH}, θ_{LH} or b_{LH} makes underreporting less tempting for the principal, leaves the agent's incentive constraint unaffected, and increases the principal's payoff; therefore all these variables must be zero.

The argument for why b_{LL} must be zero is a bit more involved since a positive b_{LL} could in principle be used to deter the principal from underreporting. However, if b_{LL} were positive, one could decrease it while simultaneously increasing b_{HL} and thereby increase the principal's payoff. To see why, note that the effect of b_{LL} on the principal's incentive to underreport is proportional to $1-\sigma$, i.e. the probability that the public signal is low, given that the true output is high. In contrast, the effect of b_{HL} is proportional to the probability that the public signal is high, given that the true output is high, which is $\sigma > 1-\sigma$. Therefore, b_{HL} has a stronger deterrence effect for the principal than b_{LL}. Moreover, b_{HL} affects the principal's payoff less adversely than b_{LL}, since b_{HL} has to be paid only when output is low but the public signal is high (which occurs with probability $(1-e)(1-\sigma)$), whereas b_{LL} has to be paid in the more likely event that output is low and the public signal is low as well (which occurs with probability $(1-e)\sigma$).

The second statement in part (c), follows from a similar comparison of the effects of θ_{LL} and θ_{HL}, the only additional complication being that, since θ_{HL} is a probability, it cannot be greater than one. Roughly speaking, the result means that one should first use θ_{HL} before using θ_{LL}.

The first statement in part (c) concerns the case where the principal reports low output, but the public signal is high: there will be money burning only if the agent is also fired with probability one. Compared with firing, burning money is a less attractive way to deter the principal from underreporting. The reason is that, while burning one dollar always costs one dollar, firing is more costly if output is high. Using θ_{HL} to deter underreporting has the advantage that firing occurs only when it is less costly since output is low, but the principal is deterred from underreporting in case of high output.

As in standard adverse selection problems, statement (d) shows that only the downward incentive constraint is binding for truthful reporting. Finally, by part (e) of Lemma

1 the agent's effort choice is related to the principal's cost of firing and money burning. Indeed, the principal can incentivise the agent by a positive wage for high output only if he is committed not to underreport. Therefore, it must be costly for him to claim low output. The following lemma gives more details on the structure of wages.

Lemma 2 *Let (γ, e) solve problem (16). Then the wages $w_{HH} \geq 0$ and $w_{LH} \geq 0$ are (not uniquely) determined by*

$$\sigma w_{HH} + (1-\sigma) w_{LH} = c'(e) \tag{19}$$

The lemma allows the principal to set $w_{HH} = w_{LH} = c'(e)$ in an optimal contract. Since $w_{HL} = w_{LL}$ by (17), this means that the agent's enumeration w can be chosen such that it depends only on the principal's report and not at all on the public signal.

Lemmas 1 and 2 substantially simplify the principal's problem. Only four of the principal's choice variables remain to be determined: effort e, firing probabilities θ_{HL} and θ_{LL}, and money burning b_{HL}. Moreover, to give the principal incentives to reveal information truthfully, the instrument θ_{HL} should be used first, and only if it is exhausted in the sense that $\theta_{HL} = 1$, the instruments θ_{LL} or b_{HL} should be used. Lemmas 1 and 2 do not, however, help to compare the latter two instruments. As we show in our next Lemma, their relative attractiveness turns out to depend on the precision of the public signal. Define the critical value

$$\bar{\sigma} \equiv \frac{\sqrt{x_H}}{\sqrt{x_H} + \sqrt{x_L}}. \tag{20}$$

Note that $1/2 < \bar{\sigma} < 1$.

Lemma 3 *Let (γ, e) solve problem (16). Then*

(a) $\theta_{LL} = 0$ if $\sigma > \bar{\sigma}$;

(b) if $\sigma < \bar{\sigma}$, $b_{HL} > 0$ implies $\theta_{LL} = 1$.

Lemma 3 further simplifies our analysis of problem (16). By part (a), there will be no project termination if both output and the public signal are low and the public signal is sufficiently informative: θ_{LL} is zero. It is then cheaper to deter the principal from underreporting by making him burn money, that is, by using the instrument b_{HL}. In fact, if the public signal is sufficiently informative, using b_{HL} is attractive for two reasons. First, there is only a small chance that the public signal is high when output is low; therefore also the likelihood that the principal actually has to pay b_{HL} is low. Second, b_{HL} is quite effective in deterring the principal from underreporting if the public signal is sufficiently informative: given that output is high, the public signal is likely to be high as well; thus if the principal underreports, he has to pay b_{HL} with high probability. As long as $\sigma > \bar{\sigma}$, these considerations outweigh the countervailing consideration (related to those mentioned in the discussion of Lemma 1) that burning money is less effective than firing since the deterrence effect of firing is proportional to x_H while the actual costs are proportional to x_L.

If $\sigma < \bar{\sigma}$, however, these countervailing considerations make θ_{LL} a more attractive instrument than b_{HL}. Therefore, by part (b) of Lemma 3, if the public signal is not very informative, there is no money burning unless the agent is fired with probability one if the public signal correctly indicates low output.

Together with our previous findings, the following proposition characterizes the optimal contract for the case where the public signal is sufficiently precise.

Proposition 2 *Let* (γ, e) *solve problem (16). Suppose that* $\sigma > \bar{\sigma}$. *Then there exists a critical* $\bar{\alpha} \in (0,1)$ *such that*

(a) $b_{HL} > 0$ *and* $\theta_{HL} = 1$ *if* $\alpha < \bar{\alpha}$*;*

(b) and $b_{HL} = 0$ *and* $\theta_{HL} \in (0,1)$ *if* $\alpha \geq \bar{\alpha}$.

In combination with Lemmas 1 and 2, Proposition 2 shows that as long as the public signal is sufficiently accurate, project termination and money burning occur if and only if the public signal conflicts with the principal's report that output is low. When this happens, the public signal of high output is actually incorrect because the principal always reports truthfully. But, to credibly overrule the public signal, the principal has to be committed to some action that reduces his payoff.

Proposition 2 also shows that project termination and money burning are clearly ranked as incentive devices for truthful reporting: Money burning occurs only as a secondary instrument when the probability of firing the agent cannot be further increased because it is already equal to one. Indeed, money burning is not used at all in an optimal contract if $\alpha \geq \bar{\alpha}$, which means that the loss from terminating the project is relatively high.

Our next result shows that the properties of the optimal contract are similar, albeit slightly more complicated, in the case where the public signal is rather imprecise:

Proposition 3 *Let* (γ, e) *solve problem (16). Suppose that* $\sigma < \bar{\sigma}$. *There exists critical* $\bar{\alpha}_1$ *and* $\bar{\alpha}_2$, *with* $0 < \bar{\alpha}_1 < \bar{\alpha}_2 < 1$, *such that*

(a) $b_{HL} > 0$ *and* $\theta_{HL} = \theta_{LL} = 1$ *if* $\alpha < \bar{\alpha}_1$*;*

(b) $b_{HL} = 0, \theta_{LL} \in (0,1)$ *and* $\theta_{HL} = 1$ *if* $\alpha \in (\bar{\alpha}_1, \bar{\alpha}_2)$*;*

(c) and $b_{HL} = \theta_{LL} = 0$ *and* $\theta_{HL} \in (0,1)$ *if* $\alpha > \bar{\alpha}_2$.

The main difference with Proposition 2 is that now the principal may have to fire the agent even if the public signal corroborates his report of low output. The reason is that the principal must be given additional incentives not to underreport if the public signal is relatively imprecise. But note that money burning never occurs if the public signal agrees with the principal's report of low output, because $b_{LL} = 0$ by Lemma 1.

Again, the incentive devices for truthful evaluation are hierarchically ordered. After the principal states low output, money burning is optimal only if at the same time the project is terminated with certainty. This is the case if the loss of output from firing the agent is rather low as $\alpha < \bar{\alpha}_1$. For higher values of α the loss from project termination is sufficient to keep the principal from underreporting and so money burning is suboptimal. But also the termination probabilities θ_{LL} and θ_{HL} are ranked as θ_{LL} can be positive only if $\theta_{HL} = 1$. Indeed, this happens for intermediate values of α in the interval $(\bar{\alpha}_1, \bar{\alpha}_2)$. In contrast, if $\alpha > \bar{\alpha}_2$ the principal has to fire the agent with positive probability only if the public signal $s = s_H$ provides no support for his evaluation $\hat{x} = \hat{x}_L$.

Our final result in this section shows that the principal benefits from increases in the parameters σ and α.

Proposition 4 *Let (γ, e) solve problem (16). Then the principal's payoff $V(\gamma, e)$ is strictly increasing in σ. Moreover, $\partial V(\gamma, e)/\partial \alpha > 0$ over the range where $\theta_{HL} = 1$ in Propositions 2 and 3, and $\partial V(\gamma, e)/\partial \alpha = 0$ if $\theta_{HL} < 1$.*

The direct effect of a more precise public signal is not that it allows providing stronger incentives for the agent's effort choice. Indeed, our conclusion from Lemma 2 shows that under an optimal contract the agent's enumeration can be chosen to be independent of the public signal. The reason that the principal gains from an increase in σ is that it relaxes his *ICP* constraints for truthful subjective evaluation. If the public signal becomes more accurate, it becomes easier to punish the principal for underreporting. As a consequence, the expected loss from money burning or project termination is reduced. For example, if $\sigma > \bar{\sigma}$ such losses occur by Proposition 2 only if the public signal s_H is incorrect because the true output is x_L. As σ increases, the likelihood of an incorrect signal decreases and therefore expected losses are reduced. In fact, in the limit $\sigma \to 1$ the expected loss from money burning or project termination tends to zero.

At first sight it may look paradoxical that the principal gains if firing the agent generates a higher loss of output. But again the intuition is that this relaxes the *ICP* conditions. Whenever $\theta_{HL} = 1$, an increase in α makes the principal better off because this allows him to reduce the less effective incentive instruments b_{HL} or θ_{LL}. This argument no longer holds if for high values of α it becomes optimal to set $\theta_{HL} < 1$ and $b_{HL} = \theta_{LL} = 0$. Then the principal simply keeps $\alpha\theta_{HL}$ constant and so the expected loss from firing the agent does not depend on α.

6 An Example

In this section we illustrate the solution of the principal's problem (16) under limited liability by a numerical example for the case $\sigma > \bar{\sigma}$. Let

$$c(e) = 5e^2/2, \quad x_L = 6, \quad x_H = 10, \quad \sigma = 3/4. \tag{21}$$

Notice that $\sigma > \bar{\sigma}$ because $\bar{\sigma} \approx 0.5645$. Further, the first–best effort is $\tilde{e} = 4/5$.

By Lemma 1 we can ignore the *IRA* constraint (14) and the first of the two *ICP* constraints in (8). Since the optimal contract γ satisfies (17) and $\theta_{LL} = 0$ by Lemma 3 (a), the principal's ex ante payoff $V(\gamma, e)$ simplifies for the specification in (21) to

$$6 + 4e - \frac{e}{4}\left(3w_{HH} + w_{LH}\right) - \frac{1-e}{4}\left(6\alpha\,\theta_{HL} + b_{HL}\right). \tag{22}$$

Similarly, the second *ICP* constraint in (8) becomes

$$30\alpha\,\theta_{HL} + 3b_{HL} \geq 3w_{HH} + w_{LH}, \tag{23}$$

and the *ICA* constraint (13) reduces to

$$3w_{HH} + w_{LH} = 20e. \tag{24}$$

The principal's problem is therefore to choose e and $(w_{HH}, w_{LH}, \theta_{HL}, b_{HL}) \geq 0$ to maximize his payoff in (22) subject to (23), (24), and $\theta_{HL} \leq 1$.[12]

It is a bit tedious but straightforward to derive the solution of this optimization problem from the Kuhn–Tucker conditions: The critical value $\bar{\alpha}$ mentioned Proposition 2 is given by $\bar{\alpha} = 7/33$, and the solution for (e, θ_{HL}, b_{HL}) is

$$e = \frac{7 - 3\alpha}{20}, \quad \theta_{HL} = 1, \quad b_{HL} = \frac{7 - 33\alpha}{3}, \quad \text{if} \quad \alpha \leq \bar{\alpha}, \tag{25}$$

and

$$e = \min\left[\frac{3\alpha}{2}, \frac{3}{8}\right], \quad \theta_{HL} = \min\left[1, \frac{1}{4\alpha}\right], \quad b_{HL} = 0, \quad \text{if} \quad \alpha \geq \bar{\alpha}. \tag{26}$$

The wages $w_{HH} \geq 0$ and $w_{LH} \geq 0$ are determined by (24) together with the solution for the agent's effort e in (25) and (26), respectively.

As Figure 2 illustrates, the solution variables (e, θ_{HL}, b_{HL}) are continuous functions of the parameter α. But these functions may have a kink at $\alpha = \bar{\alpha}$ and at $\alpha = 1/4 > \bar{\alpha}$. The kinks can occur at those values of α where the constraints $b_{HL} \geq 0$ and $\theta_{HL} \leq 1$ become binding. Indeed, for $\alpha \in (\bar{\alpha}, 1/4)$ these constraints are both binding so that b_{HL} and θ_{HL} remain constant within this interval. For $\alpha < \bar{\alpha}$ only the constraint $\theta_{HL} \leq 1$ is binding and

[12] The constraint $0 \leq e \leq 1$ can be ignored because it is not binding.

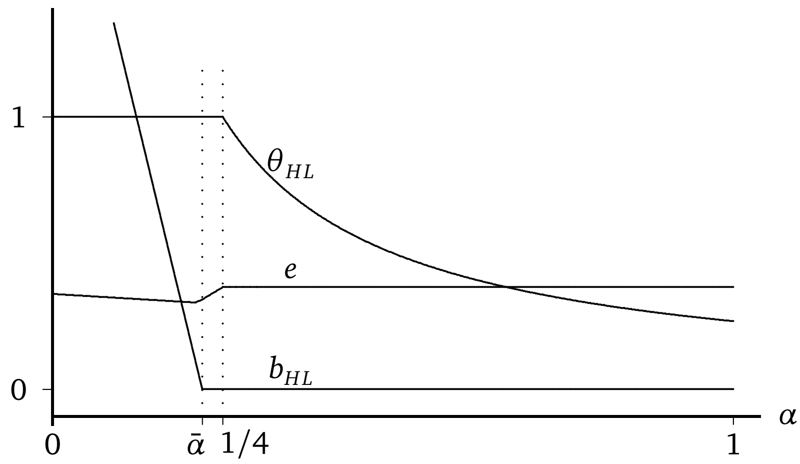

Figure 2: SOLUTION VARIABLES (e, θ_{HL}, b_{HL})

b_{HL} is strictly decreasing in α. Similarly, θ_{HL} is strictly decreasing when for $\alpha > 1/4$ only the constraint $b_{HL} \geq 0$ is binding .

Interestingly, the agent's effort e is not a monotone function of α. It is decreasing over the interval $[0, \bar{\alpha})$, increasing over the interval $[\bar{\alpha}, 1/4)$, and constant for $\alpha \geq 1/4$. This is so because, as stated in Lemma 1 (e), the agent's effort incentive is positively related to the principal's willingness to incur an efficiency loss after reporting low output. As long as $b_{HL} > 0$, an increase in the cost of project termination makes it optimal for the principal to reduce the amount of money burning at a rate that requires also reducing the agent's effort. In contrast, over the range where we have a corner solution with $b_{HL} = 0$ and $\theta_{HL} = 1$, the principal's cost of reporting low output necessarily increases with α and so he can provide stronger incentives for the agent. Finally, if $\theta_{HL} < 1$, the principal optimally adjusts to a higher value of the parameter α by keeping $\alpha\theta_{HL}$ constant. Thus the expected cost of project termination and, therefore, also the agent's effort are not changed.

7 The Timing of Evaluation

We now consider the alternative timing of events where the principal becomes informed about the output realization after the public signal is observed. This means the sequence of events in Figure 1 is reversed in stages $t = 2$ and $t = 3$. Whereas this does not affect the *ICA* and *IRA* constraints for the agent, the principal's *ICP* constraints have to be reformulated because at the reporting stage he already knows the public signal.

If the principal observes the output x_L and reports $\hat{x}_j$ in stage 3, his payoff depends on whether in stage 2 the public signal has been s_L or s_H according to

$$\begin{aligned} V_L(\gamma,\hat{x}_j|s_L) &\equiv (1-\alpha\,\theta_{Lj})x_L - w_{Lj} - b_{Lj}, \\ V_L(\gamma,\hat{x}_j|s_H) &\equiv (1-\alpha\,\theta_{Hj})x_L - w_{Hj} - b_{Hj}. \end{aligned} \tag{27}$$

Similarly, his payoffs after observing x_H depend on the public signal and are equal to

$$\begin{aligned} V_H(\gamma,\hat{x}_j|s_H) &\equiv (1-\alpha\,\theta_{Hj})x_H - w_{Hj} - b_{Hj}, \\ V_H(\gamma,\hat{x}_j|s_L) &\equiv (1-\alpha\,\theta_{Lj})x_H - w_{Lj} - b_{Lj}. \end{aligned} \tag{28}$$

The *ICP* constraints for truthful reporting in the four possible (x,s)–constellations therefore are

$$\begin{aligned} V_L(\gamma,\hat{x}_L|s_L) \geq V_L(\gamma,\hat{x}_H|s_L), &\qquad V_L(\gamma,\hat{x}_L|s_H) \geq V_L(\gamma,\hat{x}_H|s_H), \\ V_H(\gamma,\hat{x}_H|s_H) \geq V_H(\gamma,\hat{x}_L|s_H), &\qquad V_H(\gamma,\hat{x}_H|s_L) \geq V_H(\gamma,\hat{x}_L|s_L). \end{aligned} \tag{29}$$

Obviously, in comparison with the previous *ICP* conditions in (8) these constraints are more restrictive: The principal now has to report truthfully *ex post* for each realization of the public signal, while under (8) this is required only *ex ante* in expectation. Therefore, whenever γ satisfies the *ICP* conditions in (29) it also satisfies these conditions in (8).

When the principal observes output after the realization of the public signal, his contracting problem becomes

$$\max_{(\gamma,e)\in\Gamma\times E} V(\gamma,e) \quad \text{subject to (29), (13), (14) and } w \geq 0. \tag{30}$$

The only difference between this problem and problem (16) in Section 5 is that the ex ante *ICP* constraints (8) are replaced by the ex post constraints (29).

It is easy to see that in the case of unlimited liability contracts, which we studied in Section 4, it does not matter for the principal whether he reports his evaluation before or after the realization of the public signal. This is so because by Proposition 1 he can appropriate the first–best surplus by setting $b = \theta = 0$ and using a wage schedule that is independent of his evaluation. The same contract thus trivially satisfies also the *ICP* constraints for ex post reporting.[13] Perhaps more surprising is the following observation that also with limited liability the time at which the principal observes and reports output is irrelevant for his payoff.

Proposition 5 *Let (γ,e) solve problem (16). Then γ satisfies the* ICP *constraints in (29), and therefore (γ,e) also solves problem (30), if and only if $w_{LH} = \alpha\theta_{LL}x_H$ in (19). Thus for the principal's payoff it does not matter whether he observes the realization of output before or after the public signal.*

[13] Indeed, if the principal reports after having observed the public signal, any contract that solves the unlimited liability problem must ignore the principal's information. This immediately follows from (29) because $b = \theta = 0$.

As Lemma 2 shows, the agent's enumeration is not uniquely determined by the solution of problem (16). This degree of freedom turns out to be sufficient for meeting also the more restrictive requirements for ex post truthful reporting. By (18) and (19), Proposition 5 implies that the agent's wages in the solution of problem (30) satisfy

$$w_{HH} = \alpha\,\theta_{HL}\,x_H + b_{HL}, \quad w_{LH} = \alpha\theta_{LL}x_H. \tag{31}$$

The payments w_{HL} and w_{LL} are zero by (17). Thus the agent is never rewarded by a positive wage if the principal submits an unfavorable evaluation $\hat{x}_L$. If, however, he reports $\hat{x}_H$ the public signal becomes decisive because $w_{HH} > w_{LH}$ by Propositions 2 and 3. In contrast with our findings for ex ante reporting, the agent's wage schedule now necessarily depends not only on the principal's report but also on the public signal.

In our analysis the timing of subjective evaluation by the principal is exogenous. But from Proposition 5 we can draw some immediate conclusions for environments in which the principal can decide at which stage he evaluates the agent. Since the timing is irrelevant for his payoff, the principal has no incentive to acquire information at an early stage. Indeed, a slight modification of our model leads to the conclusion that delaying his report can even increase his payoff. Suppose that the parameter α, which presents the degree of project completion, increases over time. Then we can conclude from Propositions 2–4 that the principal gains from postponing the agent's evaluation as long as α lies in the range where $\theta_{HL} = 1$. The optimal time of reporting occurs when α is sufficiently large so that $\theta_{HL} < 1$. Interestingly, then money burning is no longer needed to prevent underreporting by the principal. Thus, if the timing of evaluation can be freely selected, reporting low output requires the principal to terminate the project and dismiss the agent with a positive probability, but he is not forced to burn money in addition.

8 Conclusions

We have studied a principal–agent relation where the principal possesses more accurate information about the outcome of the agent's effort than a publicly verifiable performance measure. Despite being noisier than the principal's information, public information is helpful to reduce the ex post inefficiencies that are unavoidably associated with subjective evaluation. As long as the public performance measure is not too imprecise, such inefficiencies occur only if the principal's subjective evaluation is contradictory to the public signal. In general, the presence of public information relaxes the principal's incentive compatibility constraints for truthful subjective evaluation.

Our analysis further shows that there is a clear pecking–order of the instruments that can be used to support truthful subjective evaluation. We show that 'firing' the agent, thereby destroying some part of the output, is more efficient than 'burning money' in the form of payments to a passive third party. When the efficiency loss from firing is large enough, an optimal contract makes no use of money burning. Also, money burning is not optimal as long as there is a positive probability that the agent is not fired.

The problem of subjective performance evaluation consists of creating effort incentives for the agent and, at the same time, incentives for truthful reporting by the principal. This double incentive problem can be extended to a setting with more than one agent where the principal's private information is about some aggregate measure such as the sum or the mean of the efforts. As is standard in the literature on subjective evaluation, in our model the principal does not have to invest in information acquisition. An additional moral hazard problem occurs, however, if the principal's information acquisition is costly and not observable. How this problem interacts with the other two incentive problems of subjective evaluation may be an interesting subject of further research.

9 Appendix

Proof of Proposition 1 Suppose that $\theta = b = 0$, $w_{HH} = w_{HL}$, and $w_{LH} = w_{LL}$. Then the principal's incentive constraints (8) are obviously satisfied. Let the difference of the wages satisfy

$$w_{HH} - w_{LL} = \frac{c'(\tilde{e})}{2\sigma - 1}. \tag{32}$$

Then by (13) the agent will choose the first–best effort $\tilde{e}$. In addition, by unlimited liability one can choose the wage w_{LL} such that the agent's individual rationality constraint holds with equality:

$$w_{LL} = c(\tilde{e}) - \left[\frac{1-\sigma}{2\sigma - 1} + \tilde{e}\right] c'(\tilde{e}). \tag{33}$$

This contract implements the first–best effort $\tilde{e}$. Moreover, the principal's payoff is equal to the first–best surplus $S(\tilde{e})$ because the agent receives his outside option payoff. Obviously, the payoff of the principal cannot be higher; thus the contract considered here is optimal. Moreover, any optimal contract must implement the first–best effort $\tilde{e}$, for otherwise the principal's payoff must be lower than the first–best surplus $S(\tilde{e})$.

It remains to show that $\theta = b = 0$ in any optimal contract. By assumption (1), $\tilde{e} \in (0,1)$. Since $\sigma < 1$, this implies that all four possible combinations of output and the public signal occur with positive possibility. Therefore, whenever $\theta \neq 0$ or $b \neq 0$, total surplus is below the first–best surplus $S(\tilde{e})$, and hence the principal's payoff is below $S(\tilde{e})$ as well. Q.E.D.

Proof of Lemma 1 (a) The agent's utility is

$$U(\gamma, e) = \max_{e'} U(\gamma, e') \geq U(\gamma, 0). \tag{34}$$

Since $w \geq 0$ and $c(0) = 0$, $U(\gamma, 0) \geq 0$. Thus (14) is automatically satisfied.

(b) If (γ, e) solves problem (16), then obviously γ must maximize $V(\gamma, e)$ subject to the constraints in (16) when e is treated as a fixed parameter. The latter is a linear optimization problem since $V(\gamma, e)$ and all constraints are linear in γ, and the Kuhn-Tucker conditions are both necessary and sufficient for a maximum.

Following a standard method, we temporarily ignore that γ has to satisfy the inequality $V_L(\gamma, \hat{x}_L) \geq V_L(\gamma, \hat{x}_H)$ in (8), and show later that this constraint is automatically satisfied in the proof of part (d) below. Consider the Lagrangian

$$L \equiv V(\gamma, e) + \lambda\left(V_H(\gamma, \hat{x}_H) - V_H(\gamma, \hat{x}_L)\right) + \mu\left(U_H(\gamma) - U_L(\gamma) - c'(e)\right) \tag{35}$$

with $\lambda \geq 0$. Note that $\mu > 0$ as the agent's incentive constraint must be binding.

Straightforward differentiation shows that $w_{HL} = 0$ because

$$\frac{\partial L}{\partial w_{HL}} = -(1-e)(1-\sigma) + \lambda\sigma - \mu(1-\sigma) < \frac{\partial L}{\partial b_{HL}} = -(1-e)(1-\sigma) + \lambda\sigma. \tag{36}$$

Similarly, $w_{LL} = 0$ because

$$\frac{\partial L}{\partial w_{LL}} = -(1-e)\sigma + \lambda(1-\sigma) - \mu\sigma < \frac{\partial L}{\partial b_{LL}} = -(1-e)\sigma + \lambda(1-\sigma). \tag{37}$$

Moreover, $\theta_{HH} = b_{HH} = \theta_{LH} = b_{LH} = 0$ because

$$\frac{\partial L}{\partial \theta_{HH}} = \alpha x_H \frac{\partial L}{\partial b_{HH}} = -\alpha\sigma x_H (e+\lambda) < 0, \tag{38}$$

$$\frac{\partial L}{\partial \theta_{LH}} = \alpha x_H \frac{\partial L}{\partial b_{LH}} = -\alpha(1-\sigma) x_H (e+\lambda) < 0. \tag{39}$$

Finally, $b_{LL} = 0$ because $\sigma > 1/2$ implies that

$$\frac{\partial L}{\partial b_{LL}} = -(1-e)\sigma + \lambda(1-\sigma) < \frac{\partial L}{\partial b_{HL}} = -(1-e)(1-\sigma) + \lambda\sigma. \tag{40}$$

(c) If $b_{HL} > 0$ then

$$\frac{\partial L}{\partial b_{HL}} = -(1-e)(1-\sigma) + \lambda\sigma = 0, \tag{41}$$

and thus $\lambda = (1-e)(1-\sigma)/\sigma$. This implies

$$\frac{\partial L}{\partial \theta_{HL}} = -(1-e)(1-\sigma)\alpha x_L + \lambda\sigma\alpha x_H \tag{42}$$

$$= (1-e)(1-\sigma)\alpha\left(x_H - x_L\right) > 0. \tag{43}$$

Therefore $b_{HL} > 0$ implies $\theta_{HL} = 1$.

Finally, if $\theta_{LL} > 0$ then

$$\frac{\partial L}{\partial \theta_{LL}} = -(1-e)\sigma\alpha x_L + \lambda(1-\sigma)\alpha x_H \geq 0, \tag{44}$$

and thus $\lambda \geq \left[(1-e)\sigma x_L\right] / \left[(1-\sigma)x_H\right]$. By (42) this implies

$$\frac{\partial L}{\partial \theta_{HL}} \geq -(1-e)(1-\sigma)\alpha x_L + \frac{(1-e)\sigma x_L}{(1-\sigma)x_H}\sigma\alpha x_H \tag{45}$$

$$= (1-e)\alpha x_L \frac{2\sigma - 1}{1-\sigma} > 0. \tag{46}$$

Therefore $\theta_{LL} > 0$ implies $\theta_{HL} = 1$.

(d) By part (b), the principal's incentive constraint $V_H(\gamma, \hat{x}_H) \geq V_H(\gamma, \hat{x}_L)$ in (8) simplifies to

$$\sigma w_{HH} + (1-\sigma) w_{LH} \leq \sigma\left(\theta_{HL}\alpha x_H + b_{HL}\right) + (1-\sigma)\theta_{LL}\alpha x_H. \tag{47}$$

Suppose this constraint is not binding. Then θ_{HL} must be strictly positive, since by limited liability the left hand side of (47) is non-negative, and by part (c) the right hand side can be positive only if $\theta_{HL} > 0$. Thus one must have $\partial L / \partial \theta_{HL} \geq 0$, and by (42) this implies $\lambda > 0$. This proves that the constraint (47) must be binding and $V_H(\gamma, \hat{x}_H) \geq V_H(\gamma, \hat{x}_L)$ must hold with equality.

Using part (b) the inequality $V_L(\gamma, \hat{x}_L) \geq V_L(\gamma, \hat{x}_H)$ in (8) reduces to

$$(1-\sigma)\left(\theta_{HL}\alpha x_L + b_{HL}\right) + \sigma\theta_{LL}\alpha x_L \leq \sigma w_{LH} + (1-\sigma) w_{HH}. \tag{48}$$

Since (47) is binding,

$$\begin{aligned}\sigma w_{LH} + (1-\sigma) w_{HH} &= \frac{2\sigma - 1}{\sigma} w_{LH} + \frac{1-\sigma}{\sigma}\left[\sigma\left(\alpha\theta_{HL}x_H + b_{HL}\right) + (1-\sigma)\alpha\theta_{LL}x_H\right] \\ &\geq (1-\sigma)\left(\alpha\theta_{HL}x_H + b_{HL}\right) + \frac{(1-\sigma)^2}{\sigma}\alpha\theta_{LL}x_H \end{aligned} \tag{49}$$

where the inequality follows from $w_{LH} \geq 0$. Subtracting the left hand side of (48) shows that

$$\begin{aligned}&\sigma w_{LH} + (1-\sigma) w_{HH} - \left((1-\sigma)\left(\theta_{HL}\alpha x_L + b_{HL}\right) + \sigma\theta_{LL}\alpha x_L\right) \\ &\geq (1-\sigma)\alpha\theta_{HL}\left(x_H - x_L\right) + \left[(1-\sigma)^2 x_H/\sigma - \sigma x_L\right]\alpha\theta_{LL}.\end{aligned} \tag{50}$$

If $\theta_{LL} = 0$, this implies that (48) is satisfied. Similarly, if $(1-\sigma)^2 x_H/\sigma \geq \sigma x_L$, (48) is satisfied.

To complete the argument, we show that one cannot have that $(1-\sigma)^2 x_H/\sigma < \sigma x_L$ and $\theta_{LL} > 0$. Indeed, $\theta_{LL} > 0$ implies by (44) that

$$\lambda \geq \frac{(1-e)\sigma x_L}{(1-\sigma)x_H}. \tag{51}$$

By the first equality in (41) this implies

$$\frac{\partial L}{\partial b_{HL}} \geq (1-e)\left[-(1-\sigma) + \frac{\sigma x_L}{(1-\sigma)x_H}\sigma\right] > 0, \tag{52}$$

where the second inequality holds if $(1-\sigma)^2 x_H/\sigma < \sigma x_L$. Since this would imply $b_{HL} = \infty$, we have shown that $\theta_{LL} = 0$ if $(1-\sigma)^2 x_H/\sigma < \sigma x_L$.

(e) By part (b), the agent's incentive constraint (13) reduces to

$$\sigma w_{HH} + (1-\sigma) w_{LH} = c'(e). \tag{53}$$

Combining this with (47), which holds with equality as shown above, completes the proof. Q.E.D.

Proof of Lemma 2 Let (γ, e) solve (16). By Lemma 1 part (b), the agent's incentive constraint (13) reduces to (19). Changing w_{HH} and w_{LH} such that equation (19) continues to hold leaves the the principal's payoff constant and does not interfere with any of the constraints. Therefore, the optimal wages w_{HH} and w_{LH} are not unique. Q.E.D.

Proof of Lemma 3 (a) As shown in the last part of the proof of Lemma 1 (d), $\theta_{LL} = 0$ if $(1-\sigma)^2 x_H/\sigma < \sigma x_L$. As this inequality is equivalent to $\sigma > \bar{\sigma}$, this proves part (a).

(b) Let $b_{HL} > 0$ and $\sigma < \bar{\sigma}$. Then $\partial L/\partial b_{HL} = 0$ and so by the second equality in (36)

$$\lambda = \frac{(1-e)(1-\sigma)}{\sigma}. \tag{54}$$

By the equality in (44) this implies

$$\frac{\partial L}{\partial \theta_{LL}} = -(1-e)\sigma\alpha x_L + \frac{(1-e)(1-\sigma)}{\sigma}(1-\sigma)\alpha x_H \tag{55}$$

$$= (1-e)\alpha\left[-\sigma x_L + \frac{(1-\sigma)}{\sigma}(1-\sigma)x_H\right] > 0, \tag{56}$$

where the last inequality holds because $\sigma < \bar{\sigma}$. Therefore, $\theta_{LL} = 1$. Q.E.D.

Proof of Proposition 2 We substitute out all choice variable except e from the principal's problem, and then optimize with respect to e. By Lemma 1 (b) and equation (19), the principal's profit $V(e,\gamma)$ equals

$$ex_H + (1-e)x_L - ec'(e) - (1-e)\left[(1-\sigma)\left(\alpha\theta_{HL}x_L + b_{HL}\right) + \sigma\theta_{LL}\alpha x_L\right]. \tag{57}$$

By Lemma 3, $\theta_{LL} = 0$ and hence by Lemma 1 (e),

$$\sigma\left(\alpha\,\theta_{HL}\,x_H + b_{HL}\right) = c'(e). \tag{58}$$

There are two possible cases. First, suppose that $\sigma\alpha x_H \geq c'(e)$. This is equivalent to $e \leq \hat{e}$, where $\hat{e} \equiv c'^{-1}\left(\sigma\alpha x_H\right)$. In this case, (58) and Lemma 1 (c) imply that $b_{HL} = 0$ and $\theta_{HL} = c'(e)/\left(\sigma\alpha x_H\right)$. Profit equals

$$\phi_1(e) \equiv ex_H + (1-e)x_L - \left[ec'(e) + (1-e)(1-\sigma)\frac{c'(e)x_L}{\sigma x_H}\right]. \tag{59}$$

Second, suppose that $\sigma\alpha x_H < c'(e)$, or, equivalently, $e > \hat{e}$. Then $\theta_{HL} = 1$ and so by (58) $b_{HL} = c'(e)/\sigma - \alpha x_H > 0$. In this case, the principal's payoff is

$$\phi_2(e) \equiv ex_H + (1-e)x_L - ec'(e) - (1-e)(1-\sigma)\left[\frac{c'(e)}{\sigma} - \alpha\left(x_H - x_L\right)\right]. \tag{60}$$

Note that $\phi_1(e) \leq \phi_2(e)$ if and only if $c'(e) \leq \sigma\alpha x_H$. Therefore, the principal's payoff as a function of e can be written as

$$\tilde{V}(e) \equiv \min\left\{\phi_1(e), \phi_2(e)\right\}. \tag{61}$$

The functions ϕ_1 and ϕ_2 are strictly concave in e.[14] The minimum of two strictly concave functions is strictly concave, hence $\tilde{V}$ is strictly concave.

Differentiating ϕ_2 yields

$$\phi_2'(e) = x_H - x_L - \frac{2\sigma - 1}{\sigma} c'(e) - \left[e + (1-e) \frac{(1-\sigma)}{\sigma} \right] c''(e) - \alpha (1-\sigma) \left(x_H - x_L \right). \quad (62)$$

Define e_2^* implicitly by $\phi_2'\left(e_2^*\right) = 0$. Since ϕ_2 is strictly concave, e_2^* is unique. Since $c'''(e) \geq 0$ and $c'(0) = 0$, $ec''(e) \geq c'(e)$ for all e. By (1) therefore $c''(1) \geq c'(1) > x_H - x_L$. This implies $\phi_2'(1) < 0$ and so $e_2^* < 1$.

If $e_2^* > \hat{e}$, then e_2^* maximizes the principal's payoff $\tilde{V}(e)$. Moreover, if the optimal contract involves $b_{HL} > 0$, then $e_2^* > \hat{e}$. We use the intermediate value theorem to show that $e_2^* > \hat{e}$ if and only if α is strictly smaller than a critical value $\bar{\alpha} \in (0,1)$. The argument proceeds in three steps:

1. At $\alpha = 0$, $\phi_2'(0) > 0$ by assumptions (1) and (2). Moreover, if $\alpha = 0$, the critical value $\hat{e}$ equals zero. Therefore, if $\alpha = 0$, then $e_2^* > \hat{e}$.

2. The critical value $\hat{e}$ is continuous and strictly increasing in α. Moreover, e_2^* is continuous and strictly decreasing in α:

$$\frac{de_2^*}{d\alpha} = -\frac{\partial \phi_2'\left(e_2^*\right)/\partial \alpha}{\phi_2''\left(e_2^*\right)} = \frac{(1-\sigma)\left(x_H - x_L\right)}{\phi_2''\left(e_2^*\right)} < 0. \quad (63)$$

3. If $\alpha = 1$, e_2^* solves

$$\frac{2\sigma - 1}{\sigma} c'\left(e_2^*\right) + \left[e_2^* + \left(1 - e_2^*\right) \frac{(1-\sigma)}{\sigma} \right] c''\left(e_2^*\right) = \sigma \left(x_H - x_L\right). \quad (64)$$

Since $c'''(e) \geq 0 = c'(0)$, we have $ec''(e) \geq c'(e)$ and thus

$$\left[\frac{2\sigma - 1}{\sigma} + 1 \right] c'\left(e_2^*\right) < \sigma \left(x_H - x_L\right). \quad (65)$$

By $\sigma > 1/2$, it follows that $c'\left(e_2^*\right) < \sigma x_H$. We conclude that, if $\alpha = 1$, $c'\left(e_2^*\right) < \sigma \alpha x_H$ and thus $e_2^* < \hat{e}$.

From steps 1–3, it follows that there exists a critical value $\bar{\alpha} \in (0,1)$ such that $e^* > \hat{e}$ holds if and only if $\alpha < \bar{\alpha}$. As argued above, this implies that $b_{HL} > 0$ if and only if $\alpha < \bar{\alpha}$. Q.E.D.

[14] This can be shown by differentiating them twice and using $x_H > x_L$, $\sigma > 1/2$, and $c'''(e) \geq 0$.

Proof of Proposition 3 There are three cases corresponding to statements (a), (b) and (c). First, suppose that $c'(e) \leq \sigma\alpha x_H$, or equivalently, $e \leq \hat{e} = c'^{-1}(\alpha\sigma x_H)$. Then, by Lemma 1 (c) and (e), $\theta_{HL} = c'(e)/(\sigma\alpha x_H)$ and $\theta_{LL} = b_{HL} = 0$. Profit equals $\phi_1(e)$, as defined in the proof of Proposition 2. Second, suppose that $\sigma\alpha x_H < c'(e) \leq \alpha x_H$. This is equivalent to $e \in (\hat{e}, \bar{e}]$, where $\bar{e} := c'^{-1}(\alpha x_H)$. In this case $\theta_{HL} = 1$, and $b_{HL} = 0$, and $\theta_{LL} = [c'(e) - \sigma\alpha x_H] / [(1-\sigma)\alpha x_H]$. Using (57), the principal's profit is

$$\phi_3(e) \equiv e x_H + (1-e) x_L - e c'(e) - (1-e)\left[\frac{\sigma x_L}{(1-\sigma) x_H} c'(e) - \alpha x_L \frac{2\sigma - 1}{1-\sigma}\right]. \tag{66}$$

Third, suppose that $c'(e) > \alpha x_H$, or, equivalently, $e > \bar{e}$. Then $\theta_{LL} = \theta_{HL} = 1$ and $b_{HL} = (c'(e) - \alpha x_H)/\sigma$, and profit equals

$$\phi_4(e) \equiv e x_H + (1-e) x_L - e c'(e) - (1-e)\left[\alpha x_L + \frac{(1-\sigma)(c'(e) - \alpha x_H)}{\sigma}\right]. \tag{67}$$

Note that $\phi_1(e) \leq \phi_3(e)$ if and only if $c'(e) \leq \sigma\alpha x_H$. Moreover $\phi_3(e) \leq \phi_4(e)$ if and only if $c'(e) \leq \alpha x_H$. Thus $c'(e) \leq \alpha\sigma x_H$ if and only if $\phi_1(e) = \min\{\phi_1(e), \phi_3(e), \phi_4(e)\}$. Moreover, $\sigma\alpha x_H < c'(e) \leq \sigma x_H$ if and only if $\phi_3(e) < \phi_1(e)$ and $\phi_3(e) \leq \phi_4(e)$. Therefore, the principal's payoff can be written

$$\tilde{V}(e) = \min\{\phi_1(e), \phi_3(e), \phi_4(e)\}. \tag{68}$$

The functions ϕ_1, ϕ_3, and ϕ_4 are strictly concave in e.[15] Therefore, $\tilde{V}$ is strictly concave in e.

Define e_4^* implicitly by $\phi_4'(e_4^*) = 0$. Since ϕ_4 is strictly concave, e_4^* is unique. By assumption (1), $e_4^* < 1$. If $e_4^* > \bar{e}$, then e_4^* maximizes $\tilde{V}(e)$. Moreover, if the optimal contract involves $b_{HL} > 0$, then $e_4^* > \bar{e}$.

We use the intermediate value theorem to show that $e_4^* > \bar{e}$ if and only if α is strictly smaller than a critical value $\bar{\alpha}_1 \in (0,1)$. The argument proceeds in three steps:

1. If $\alpha = 0$, $\phi_4'(0) > 0 = \bar{e}$. Thus if $\alpha = 0$, $e_4^* > \bar{e}$.

2. e_4^* is continuous and strictly decreasing in α. Moreover, $\bar{e} = c'^{-1}(\alpha x_H)$ is continuous and strictly increasing in α.

3. At $\alpha = 1$, e_4^* solves

$$x_H = c'(e_4^*) + \left[e_4^*\sigma + (1 - e_4^*)(1-\sigma)\right] c''(e_4^*) \frac{\sigma}{2\sigma - 1}. \tag{69}$$

   Therefore, at $\alpha = 1$, $\alpha x_H > c'(e_4^*)$ and so $e_4^* < \bar{e}$.

[15]This can be shown by differentiating them twice and using $1/2 < \sigma < \bar{\sigma} < x_H/(x_H + x_L)$, $c''(e) > 0$ and $c'''(e) \geq 0$.

Thus there exists an $\bar{\alpha}_1 \in (0,1)$ such that if $\alpha = \bar{\alpha}_1$, $e_4^* = \bar{e}$. For all $\alpha < \bar{\alpha}_1$, the optimal effort is $e_4^* > \bar{e}$; moreover $b_{HL} > 0$ and $\theta_{HL} = \theta_{LL} = 1$. On the other hand, for all $\alpha \geq \bar{\alpha}_1$, $b_{HL} = 0$.

It remains to show that there exists $\bar{\alpha}_2 \in (\bar{\alpha}_1, 1)$ such that $\theta_{LL} > 0$ if and only if $\alpha < \bar{\alpha}_2$. Note that $\theta_{LL} > 0$ if and only $\phi_3'(\hat{e}) > 0$. To see this, first suppose that $\phi_3'(\hat{e}) \leq 0$. Then $\tilde{V}(e) < \tilde{V}(\hat{e})$ for all $e > \hat{e}$ since $\tilde{V}(\cdot)$ is strictly concave. Hence the optimal effort is no larger than $\hat{e}$ and $\theta_{LL} = 0$. Second, if $\phi_3'(\hat{e}) > 0$, then the optimal effort is strictly bigger than $\hat{e}$ and $\theta_{LL} > 0$.

We use the intermediate value theorem to show that there exists $\bar{\alpha}_2 \in (\bar{\alpha}_1, 1)$ such that $\phi_3'(\hat{e}) > 0$ if and only if $\alpha < \bar{\alpha}_2$. The argument proceeds in three steps:

1. By definition of $\bar{\alpha}_1$, if $\alpha = \bar{\alpha}_1$, then $\phi_4'\left(e_4^*\right) = \phi_4'(\bar{e}) = 0$. Since ϕ_3 and $\tilde{V}$ are strictly concave, and $\bar{e} > \hat{e}$, it follows that $\phi_4'(\bar{e}) \leq \phi_3'(\bar{e}) < \phi_3'(\hat{e})$. Therefore, if $\alpha = \bar{\alpha}_1$, then $\phi_3'(\hat{e}) > 0$.

2. $\phi_3'(\hat{e})$ is continuous and strictly decreasing in α:
$$\frac{d}{d\alpha}\phi_3'(\hat{e}) = \left[\frac{\partial}{\partial\alpha}\phi_3'(e)\right]_{e=\hat{e}} + \phi_3''(\hat{e})\frac{d\hat{e}}{d\alpha} < 0 \tag{70}$$

3. Suppose $\alpha = 1$. Since $ec''(e) \geq c'(e)$,
$$\begin{aligned}
\phi_3'(\hat{e}) &= x_H - x_L - x_L\frac{2\sigma-1}{1-\sigma} - \left[1 - \frac{\sigma x_L}{(1-\sigma)x_H}\right]c'(\hat{e}) &(71)\\
&\quad - \left[\hat{e} + (1-\hat{e})\frac{\sigma x_L}{(1-\sigma)x_H}\right]c''(\hat{e}) &(72)\\
&< x_H - x_L - x_L\frac{2\sigma-1}{1-\sigma} - \left[2 - \frac{\sigma x_L}{(1-\sigma)x_H}\right]\sigma x_H &(73)\\
&= x_H - 2\sigma x_H - \sigma x_L < 0. &(74)
\end{aligned}$$

   Hence if $\alpha = 1$, then $\phi_3'(\hat{e}) < 0$.

Q.E.D.

Proof of Proposition 4 The principal's expected payoff $\tilde{V}(e)$ defined in (61) and (68) is strictly increasing in σ because the functions ϕ_1, ϕ_2, ϕ_3 and ϕ_4 are strictly increasing in σ. If $\theta_{HL} < 1$, then $\tilde{V}(e) = \phi_1(e)$, as defined in (59), and therefore $\partial\tilde{V}(e)/\partial\alpha = 0$. If $\theta_{HL} = 1$, then $\tilde{V}(e) = \phi_2(e)$ in the case of Proposition 2 and $\partial\tilde{V}(e)/\partial\alpha > 0$ because ϕ_2 is strictly increasing in α. In the case of Proposition 3, $\tilde{V}(e) = \min\{\phi_3(e), \phi_4(e)\}$. As $\sigma < \bar{\sigma}$, ϕ_3 and ϕ_4 are strictly increasing in α and so $\partial\tilde{V}(e)/\partial\alpha > 0$. Q.E.D.

Proof of Proposition 5 Since (γ, e) solves problem (16), γ satisfies (17). This reduces the constraints in (29) to

$$\begin{aligned}
V_L(\gamma, \hat{x}_L|s_L) - V_L(\gamma, \hat{x}_H|s_L) &= w_{LH} - \alpha\theta_{LL}x_L \geq 0, & (75)\\
V_L(\gamma, \hat{x}_L|s_H) - V_L(\gamma, \hat{x}_H|s_H) &= w_{HH} - \alpha\,\theta_{HL}\,x_L - b_{HL} \geq 0, & (76)\\
V_H(\gamma, \hat{x}_H|s_H) - V_H(\gamma, \hat{x}_L|s_H) &= \alpha\,\theta_{HL}\,x_H + b_{HL} - w_{HH} \geq 0, & (77)\\
V_H(\gamma, \hat{x}_H|s_L) - V_H(\gamma, \hat{x}_L|s_L) &= \alpha\theta_{LL}x_H - w_{LH} \geq 0. & (78)
\end{aligned}$$

Further, (18) and (19) imply

$$\sigma w_{HH} + (1-\sigma)w_{LH} = \sigma(\alpha\,\theta_{HL}\,x_H + b_{HL}) + (1-\sigma)\alpha\,\theta_{LL}\,x_H. \tag{79}$$

By (78) $w_{LH} \leq \alpha\theta_{LL}x_H$. Suppose that this inequality is strict, i.e. $w_{LH} < \alpha\theta_{LL}x_H$. Then (79) implies that $w_{HH} > \alpha\,\theta_{HL}\,x_H + b_{HL}$, a contradiction to (77). This proves that the constraints in (29) cannot be satisfied if $w_{LH} \neq \alpha\theta_{LL}x_H$.

Now let $w_{LH} = \alpha\theta_{LL}x_H$. Then (75) and (78) hold because $x_L < x_H$. As $w_{LH} = \alpha\theta_{LL}x_H$, (79) implies

$$w_{HH} = \alpha\,\theta_{HL}\,x_H + b_{HL}. \tag{80}$$

Thus the equality holds in (77), and the strict inequality holds in (76) because $x_L < x_H$. This proves that (75)–(78) are satisfied if (γ, e) solves problem (16) with $w_{LH} = \alpha\theta_{LL}x_H$ and $w_{HH} = \alpha\,\theta_{HL}\,x_H + b_{HL}$. Q.E.D.

References

BAKER, G. P. (1992): "Incentive Contracts and Performance Measurement," *Journal of Political Economy* 100, 598-614.

BAKER, G. P., R. GIBBONS AND K. J. MURPHY (1994): "Subjective Performance Measures in Optimal Incentive Contracts," *Quarterly Journal of Economics* 109(4), 1125-1156.

BESTER, H. AND D. KRÄHMER (2012): "Exit Options in Incomplete Contracts with Asymmetric Information," *Journal of Economic Theory* 147, 1947-1968.

BOLTON, P. AND D. S. SCHARFSTEIN (1996): "Optimal Debt Structure and the Number of Creditors," *Journal of Political Economy* 104, 1-25.

CHWE, M. S.-Y. (1990): "Why Were Workers Whipped? Pain in a Principal-Agent Model," *Economic Journal* 100, 1109-1121.

FUCHS, W. (2007): "Contracting with Repeated Moral Hazard and Private Evaluations," *American Economic Review* 97, 1432-1448.

GROSSMANN S. AND O. HART (1983): "An Analysis of the Principal Agent Problem," *Econometrica* 51, 7-46.

HARRIS, M. AND A. RAVIV (1979): "Optimal Incentive Contracts with Imperfect Information," *Journal of Economic Theory* 20, 231-259.

HOLMSTROM, B. (1979): "Moral Hazard and Observability," *Bell Journal of Economics* 10, 1979, 74-91.

KAHN, C. AND G. HUBERMAN (1988): "Two-Sided Uncertainty and 'Up-or-Out' Contracts," *Journal of Labor Economics* 6, 423-444.

KHALIL, F., J. LAWAREE AND T. SCOTT (2012): "Private Monitoring, Collusion, and the Timing of Information," Working Paper (University of Washington).

LANG, M. (2013): "Contracting with Subjective Evaluations and Communication," Working Paper (MPI Bonn).

LERNER, J. AND U. MALMENDIER (2010): "Contractibility and the Design of Research agreements," *American Economic Review* 100, 214-246.

LEVIN, J. (2003): "Relational Incentive Contracts," *American Economic Review* 93, 835-857.

LEWIS, T. R. AND D. E. SAPPINGTON (1997): "Information Management in Incentive Problems," *Journal of Political Economy* 105, 796-821.

MacLeod, B. (2003): "Optimal Contracting with Subjective Evaluation," *American Economic Review* 93, 216-240.

Myerson, R. (1979): "Incentive Compatibility and the Bargaining Problem," *Econometrica* 47, 61-73.

Myerson, R. (1986): "Multistage Games with Communication," *Econometrica* 54, 323-58.

Pearce, D. G. and E. Stacchetti (1998): "The Interaction of Implicit and Explicit Contracts in Repeated Agency," *Games and Economic Behavior* 23, 75-96.

Prendergast, C. (1999): "The Provision of Incentives in Firms," *Journal of Economic Literature* 37, 7-63.

Sappington, D. E. (1983): "Limited Liability Contracts between Principal and Agent," *Journal of Economic Theory* 29, 1-21.

Schmitz, P. (2002): "On the Interplay of Hidden Action and Hidden Information in Simple Bilateral Trading Problems," *Journal of Economic Theory* 103, 444-460.

Sherstyuk, K. (2000): "Performance Standards and Incentive Pay in Agency Contracts," *Scandinavian Journal of Economics* 102, 725-736.

Sonne, M. and A. Sebald (2012): "Personality and Conflict in Principal-Agent Relations based on Subjective Performance Evaluations," Working Paper (University of Copenhagen).