Ex post information rents and disclosure in sequential screening

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Abstract

We study ex post information rents in sequential screening models where the agent receives private ex ante and ex post information. The principal has to pay ex post information rents for preventing the agent to coordinate lies about his ex ante and ex post information. When the agent's ex ante information is discrete, these rents are positive, whereas they are zero in continuous models. Consequently, full disclosure of ex post information is generally suboptimal. Optimal disclosure rules trade off the benefits from adapting the allocation to better information against the effect that more information aggravates truth-telling.

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1 Introduction

Economic theory recognizes that private information is a crucial determinant of economic inefficiencies. In adverse selection models where a principal faces a privately informed agent at the contracting stage, such as in monopolistic price discrimination (e.g, Baron and Myerson, 1982) or public procurement (e.g. Laffont and Tirole, 1986), the agent's private information forces the seller to concede information rents to the agent. As a result, a trade–off between rent extraction and efficiency emerges which leads to economic inefficiencies.

In contrast, Harris and Raviv (1978) show that, when the agent's private information arrives only ex post, after the agent's decision to participate in the relationship, this trade-off between rent extraction and efficiency does not emerge: the principal can extract the full surplus from the relation without leaving rents to the agent.[1] Esö and Szentes (2007a, b) extend this result to a framework where the agent receives both ex ante and ex post private information. In particular, they show that at the optimal mechanism, the agent's additional private ex post information does not add to the information rents that he receives already from his ex ante information.[2] These results suggest the general insight that only ex ante private information is a source of information rents, whereas ex post private information is not. The purpose of this paper is to qualify this insight and to offer a more comprehensive perspective of the role of private ex post private information in dynamic adverse selection problems.

Our main contribution is to show that in smooth continuous models such as Esö and Szentes (2007a,b) ex post information rents are zero, whereas they are strictly positive in models where the agent's ex ante information is discrete. In order to demonstrate this result, we provide a formal decomposition of the agent's total information rents into a part that accrues from his ex ante information and a part that accrues from his ex post information. Using this decomposition, we show that, in general, the seller has to concede the agent additional rents for eliciting his ex post private information, because the agent may benefit from *coordinating* lies about his ex ante information with lies about his ex post information. We show, however, that when the difference between ex ante types diminishes, the agent's potential benefits from coordinating his lies tend to zero at a faster speed than the difference between types. As a result, marginal ex post information

[1] This holds only in a framework with quasi–linear preferences where the agent is effectively risk neutral.

[2] Esö and Szentes (2007a) shows this for a single agent, whereas Esö and Szentes (2007b) demonstrates this result for the case with many agents.

rents vanish in the limit of a smooth continuous model.

We further show that, as an implication of our result, full disclosure of ex post information is no longer generally optimal when the agent's ex ante private information is discrete, whereas, as shown by Esö and Szentes (2007a, b), full disclosure is optimal in continuous models. We identify two conflicting effects of disclosure. More disclosure is beneficial to the seller because it allows him to choose among a larger set of allocations, as allocations can indirectly depend on the agent's message about the disclosed information. The negative effect of more disclosure is however that, as already noted in Myerson (1991, p.297), "revealing more information to players makes it harder to prevent them from finding ways to gain by lying". The seller's optimal disclosure rule trades off these two effects.

The rest of the paper is organized as follows. We introduce the formal model in the next section. In Section 3, we set up the seller's problem for the case where the agent's ex post information is private and public and discuss how these two problems relate to the question of ex post information rents. In Section 4, we study the case of two ex ante types and demonstrate that positive ex post information rents are necessary to prevent coordinated lying. In Section 5, we study ex post information rents for the general case, distinguishing between smooth continuous models, where the ex post information rent vanishes, and discrete models, where it does not. In Section 6 we explain with an explicit example that the seller is strictly better off by disclosing ex post information only partially rather than fully. Section 7 concludes. All proofs are relegated to an appendix.

2 The Model

There is a seller (she) and a buyer (he). The seller's costs to produce the good are commonly known and normalized to zero. The buyer's valuation is θ and takes values in $[\underline{\theta}, \overline{\theta}]$. The terms of trade are the probability with which the good is sold, $x \in [0, 1]$, and a payment $t \in \mathbb{R}$ from the buyer to the seller. Parties are risk–neutral and have quasi–linear utility functions. That is, under the terms of trade x and t, the seller receives utility t, and the buyer receives utility $\theta x - t$. The seller's objective is to design a selling mechanism that maximizes her expected revenue where a selling mechanism specifies the terms of trade, possibly contingent on communication between the parties.

When the seller offers a mechanism to the buyer, no party knows the true valuation, θ, but the buyer is better informed about the distribution from which θ is drawn than the seller. Formally, it is common knowledge that θ is distributed according to the distribution function $F(\cdot|s)$, where s is drawn from the support $S \subseteq \mathbb{R}$ with distribution function P. We follow the literature and assume "non–shifting support", that is, the support of $F(\cdot|s)$ is the interval $[\underline{\theta}, \overline{\theta}]$ for all $s \in S$. The distributions $\{F(\theta|s)|s \in S\}$ are ranked according to first order stochastic dominance, that is, $s > \hat{s}$ implies $F(\theta|s) < F(\theta|\hat{s})$ for all $\theta \in (\underline{\theta}, \overline{\theta})$. We further assume that the density $f(\theta|s) = dF(\theta|s)/d\theta$ exists for each $s \in S$ and is bounded away from zero, i.e., there exists some $\varepsilon > 0$ such that $f(\theta|s) > \varepsilon$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ and $s \in S$. Hence, $F(\theta|s)$ is strictly increasing in θ and decreasing in s.

Before the seller offers the buyer a mechanism, the buyer privately observes the signal s so that we interpret s as the buyer's *ex ante private information*. If the buyer accepts the mechanism, then, before production takes place, θ is drawn according to $F(\cdot|s)$ and the buyer observes this realization privately. In order to arrive at a proper definition of the agent's ex post private information embodied in the private observation of θ, we follow the idea of Esö and Szentes (2007a, b) and write the buyer's valuation as a compound of ex ante information and additional, orthogonal ex post information. Define the random variable $\gamma = F(\theta|s)$. Then γ is uniformly distributed on the unit interval and is stochastically independent of s.[3] The buyer's valuation can be backed out as a function of s and γ by

$$\theta = \theta(s, \gamma) \equiv F^{-1}(\gamma|s), \tag{1}$$

where F^{-1} is the inverse of F with respect to θ and exists because $F(\theta|s)$ is strictly increasing in θ. Hence, instead of assuming that s and θ is observed, we can equivalently assume that s and γ is observed. Because γ is independent of s, the formulation in terms of s and γ allows us to interpret γ as the agent's *ex post private information*.

Thus we consider the following timing:

1. The buyer privately observes s.
2. The seller offers a mechanism.
3. The buyer accepts or rejects.

   If he rejects, both parties receive their outside option normalized to zero.
4. If the buyer accepts, the buyer privately observes γ.

[3] Note that for all s, it holds: $\text{Prob}(\gamma < \bar{\gamma}) = F(F^{-1}(\bar{\gamma}|s)|s) = \bar{\gamma}$.

5. The terms of trade are implemented according to the mechanism.

We now introduce a distinction which will be key for our analysis:

Definition 1 *(a) We say that the* model is continuous *if S is an interval $[\underline{s}, \bar{s}]$ and for all $s \in (\underline{s}, \bar{s})$:*

$$\frac{\partial F(\theta|s)}{\partial s} \text{ exists and is bounded.} \tag{2}$$

(b) We say that the model is discrete *if S is a discrete set $\{\underline{s}, \ldots, s, \ldots, \bar{s}\}$.*[4]

For the case that the model is continuous, our setting is identical to the setup in Esö and Szentes (2007a, b)[5] who show that the privacy of the ex post information γ is irrelevant in the sense that the seller can fully extract the value of the buyer's ex post information. The main point of our paper is to qualify this result for the case that the model is discrete, and to identify the source of the buyer's ex post information rent. To make this point, we follow Esö and Szentes (2007a, b) and compare the optimal selling mechanism when γ is observed privately by the buyer to the optimal mechanism in the benchmark case when γ is publicly observable.

In what follows, it will often be more convenient to work with the function $\theta(s, \gamma)$ rather than with $F(\theta|s)$. Our assumptions on $F(\theta|s)$ translate directly into the following properties of $\theta(s, \gamma)$.

Lemma 1 *(a) For all $s > \hat{s}$:*

$$\theta(s, \gamma) > \theta(\hat{s}, \gamma) \quad \forall \gamma \in (0, 1). \tag{3}$$

(b) For all $s, \hat{s} \in S$ and $\gamma \in [0, 1]$, there exists a unique $\gamma^* \in [0, 1]$ such that

$$\theta(\hat{s}, \gamma^*) = \theta(s, \gamma).$$

(c) For all $s \in S, \gamma \in (0, 1)$:

$$\frac{\partial \theta(s, \gamma)}{\partial \gamma} \text{ exists, is strictly positive, and bounded.} \tag{4}$$

(d) If the model is continuous, then for all $s \in S, \gamma \in (0, 1)$:

$$\frac{\partial \theta(s, \gamma)}{\partial s} \text{ exists and is bounded.} \tag{5}$$

[4] Alternatively, we could equivalently define a discrete model as a model in which S is an interval and $F(\theta|s)$ is piece-wise constant in s. Clearly, then $F(\theta|s)$ would not be differentiable for some $s \in S$ and the model would not be continuous in our sense. We believe that our formulation is more in line with standard practice in mechanism design.

[5] Esö and Szentes (2007b) consider the auction case when the seller faces multiple potential buyers. Allowing for multiple buyers would not change our results.

3 The seller's problem

In describing the seller's problem, we begin with the case that the ex post information γ is the buyer's private information, and then describe the case when the ex post information is publicly observable.

3.1 Privately observable ex post information

When both ex ante and ex post information are the buyer's private information, we can apply the revelation principle for dynamic games (Myerson, 1986) which states that the optimal mechanism is in the class of direct and incentive compatible mechanisms. A direct and incentive compatible mechanism specifies the terms of trade as functions of reports by the buyer about his ex ante and ex post information and, moreover, induces the buyer, on the equilibrium path, to report his information truthfully.

Formally, a *direct mechanism* (x,t) is a combination of an allocation rule x and a transfer schedule t with

$$x : S \times [0,1] \to [0,1], \quad t : S \times [0,1] \to \mathbb{R}$$

that requires the buyer to report a message $\hat{s} \in S$ at the outset (before having observed γ) and to subsequently (after having observed γ) report a message $\hat{\gamma} \in [0,1]$. If a buyer who has observed s and γ reports $\hat{s}$ and $\hat{\gamma}$, his utility from the mechanism is

$$u(\hat{s},\hat{\gamma};s,\gamma) = \theta(s,\gamma)x(\hat{s},\hat{\gamma}) - t(\hat{s},\hat{\gamma}).$$

With slight abuse of notation, we define the buyer's utility from truth-telling as $u(s,\gamma) = u(s,\gamma;s,\gamma)$.

We say the mechanism (x,t) is *ex post incentive compatible* if it induces the buyer to report his ex post information truthfully after having truthfully reported his ex ante information:

$$u(s,\gamma) \geq u(s,\hat{\gamma};s,\gamma) \quad \forall s,\gamma,\hat{\gamma}. \tag{6}$$

Note that the revelation principle requires truth-telling in the second stage only after truth-telling in the first stage (see Myerson, 1986). Hence, it might (in fact, it will) be optimal for the buyer, after he lied about s to lie also about γ. Thus, the buyer's expected utility from reporting $\hat{s}$ at the initial stage when he has observed s is

$$U(\hat{s};s) = \int_0^1 \max_{\hat{\gamma}} u(\hat{s},\hat{\gamma};s,\gamma)\, d\gamma. \tag{7}$$

The "max" operator under the integral accounts for the fact that the mechanism may induce the buyer to lie about γ after lying about s. Again abusing notation, we define the buyer's utility from truth-telling at the initial stage as $U(s) = U(s;s)$.

We say that the mechanism is *ex ante incentive compatible* if it induces the buyer to report his ex ante information truthfully:

$$U(s) \geq U(\hat{s};s) \quad \forall s, \hat{s}. \tag{8}$$

A mechanism (x, t) is *incentive compatible with privately observable* γ if it is ex ante and ex post incentive compatible.

We also assume that the seller wants to ensure the buyer's participation in the mechanism, and therefore the mechanism needs to give the buyer his outside option of zero. A mechanism is *individually rational* if:

$$U(s) \geq 0 \quad \forall s. \tag{9}$$

A mechanism is *feasible with privately observable* γ if it is incentive compatible with privately observable γ and individually rational. We say that the allocation rule x is *implementable with privately observable* γ if there are transfers t so that (x, t) is feasible with privately observable γ. We define by[6]

$$W(x) = \max_{t} \int_S \int_0^1 t(s,\gamma)\, d\gamma\, dP(s) \quad s.t. \quad (6), (8), (9) \tag{10}$$

the principal's maximal expected revenue from an implementable allocation rule x.

The seller's problem when γ is private information, referred to as $\mathscr{P}$, is to choose a feasible mechanism that maximizes her expected revenue:

$$\mathscr{P}: \qquad \max_{x,t} \int_S \int_0^1 t(s,\gamma)\, d\gamma\, dP(s) \quad s.t. \quad (6), (8), (9). \tag{11}$$

We refer to a solution to $\mathscr{P}$ as (x^*, t^*) and denote the seller's value from problem $\mathscr{P}$ by W^*.

3.2 Publicly observable ex post information

When the ex post information γ is publicly observable, the only private information of the buyer is his initial information s. The revelation principle then implies that the optimal selling mechanism is a direct and incentive compatible mechanism $(\tilde{x}, \tilde{t})$ which requires the buyer to report a

[6]If S is discrete, the integral is to be understood as an integral over the discrete measure P and is thus a sum.

message $\hat{s} \in S$ about his ex ante information only, and which induces the buyer to report truthfully. In addition to the buyer's report, the mechanism can condition directly on the ex post information γ, as γ is contractible by assumption.

Formally, if the buyer has observed s and reports $\hat{s}$ at the outset, his expected utility from the mechanism $(\tilde{x}, \tilde{t})$ is

$$\tilde{U}(\hat{s}; s) = \int_0^1 \{\theta(s,\gamma)\tilde{x}(\hat{s},\gamma) - \tilde{t}(\hat{s},\gamma)\}\, d\gamma. \tag{12}$$

Note that, whatever the buyer's report $\hat{s}$, the terms of trade are enforced according to the true value of γ, as now γ does not need to be elicited from the buyer any more. We define, again with slight abuse of notation, $\tilde{U}(s) = \tilde{U}(s; s)$.

We say that a mechanism $(\tilde{x}, \tilde{t})$ is *incentive compatible with publicly observable* γ if it induces the buyer to report his initial information truthfully:

$$\tilde{U}(s) \geq \tilde{U}(\hat{s}; s) \quad \forall s, \hat{s}. \tag{13}$$

As before, the seller wants to ensure the buyer's participation in the mechanism, and therefore the mechanism needs to give the buyer his outside option of zero. We say a mechanism is *individually rational with publicly observable* γ if:

$$\tilde{U}(s) \geq 0 \quad \forall s. \tag{14}$$

A mechanism is *feasible with publicly observable* γ if it is incentive compatible and individually rational with publicly observable γ. We say that the allocation rule $\tilde{x}$ is *implementable with publicly observable* γ if there are transfers $\tilde{t}$ so that $(\tilde{x}, \tilde{t})$ is feasible with publicly observable γ. We define by

$$\tilde{W}(\tilde{x}) = \max_{\tilde{t}} \int_S \int_0^1 \tilde{t}(s,\gamma)\, d\gamma\, dP(s) \quad s.t. \quad (13), (14) \tag{15}$$

the principal's maximal expected revenue from an implementable allocation rule $\tilde{x}$.

The seller's problem when γ is public information, referred to as $\tilde{\mathscr{P}}$, is to choose a feasible mechanism that maximizes her expected revenue:

$$\tilde{\mathscr{P}}: \qquad \max_{\tilde{x}, t} \int_S \int_0^1 \tilde{t}(s,\gamma)\, d\gamma\, dP(s) \quad s.t. \quad (13), (14). \tag{16}$$

We refer to a solution to $\tilde{\mathscr{P}}$ as $(\tilde{x}^*, \tilde{t}^*)$ and denote the seller's value from problem $\tilde{\mathscr{P}}$ by $\tilde{W}^*$.

3.3 Ex post information rents and revenue–irrelevance

A direct comparison of the cases with privately and publicly observable ex post information allows us to study the role of the privacy of ex post information in the design of the optimal mechanism. Clearly, a given allocation rule x that is implementable with privately observable γ is implementable with publicly observable γ, and, moreover, the seller obtains a (weakly) higher revenue from implementing x when γ is publicly observable. If the seller can attain the same revenue when γ is private information, then the seller does not need to leave any rent for eliciting the buyer's ex post information when she wants to implement x.

Moreover, comparing the values W^* and $\tilde{W}^*$ reveals how the privacy of the ex post information bears on the seller's ability to extract surplus. If the seller attains the same revenue when γ is private as when γ is public information, then private ex post information is irrelevant in the sense that it does not affect inefficiencies. These considerations motivate the following definitions.

Definition 2 *(a) Let x be implementable with privately observable γ. We say that x is* implementable without ex post information rents *if $W(x) = \tilde{W}(x)$.*
(b) Private ex post information is said to be revenue-irrelevant *if $W^* = \tilde{W}^*$.*

Private ex post information may fail to be revenue–irrelevant for two reasons. First, the allocation rule $\tilde{x}^*$ that is optimal when γ is publicly observable may simply not be implementable with privately observable γ. Second, even if $\tilde{x}^*$ is implementable, it may not be implementable without ex post information rents.

Esö and Szentes (2007a, b) have shown that if the model is continuous and $\tilde{x}^*$ is implementable with privately observable γ, then private ex post information is revenue-irrelevant. As as a consequence, $\tilde{x}^*$ is implementable without ex post information rents. In what follows, we generalize the result of Esö and Szentes (2007a, b) and show that in continuous models, *any* allocation rule that is implementable with privately observable γ is implementable without ex post information rents. More interestingly, we also show that this result is not true for discrete models. Indeed, in discrete models, an allocation rule that is implementable with privately observable γ is implementable without ex post information rents only in degenerate cases. This directly implies that in discrete models, ex post information is revenue–irrelevant only in degenerate cases. In particular, even if the allocation rule $\tilde{x}^*$ is implementable with privately observable γ, ex post information is typically not revenue-irrelevant.

To illustrate the basic features of our general analysis, we now analyze a discrete model with two types and show that private ex post information is not revenue-irrelevant.

4 Two ex ante types

Let $S = \{s_L, s_H\}$ be the set of the buyer's ex ante types, occurring with $P(s_L) = 1 - p \in (0,1)$ and $P(s_H) = p$. To simplify notation, we replace the function arguments s_L and s_H by appending function symbols with the subindices L and H. We assume that type s_L corresponds to the *low valuation type* and type s_H to the *high valuation type*:

$$\theta_L(\gamma) < \theta_H(\gamma) \quad \forall \gamma \in (0,1). \tag{17}$$

4.1 Public ex post information

We begin by deriving the optimal selling mechanism when γ is public information. With two ex ante types, a selling mechanism is a quadruple $(\tilde{x}, \tilde{t}) = (\tilde{x}_L(\gamma), \tilde{t}_L(\gamma), \tilde{x}_H(\gamma), \tilde{t}_H(\gamma))$, and the incentive compatibility and participation constraints (13) and (14) write

$$\begin{aligned}
IC_H: &\quad \int_0^1 \{\theta_H(\gamma)\tilde{x}_H(\gamma) - \tilde{t}_H(\gamma)\}\, d\gamma \geq \int_0^1 \{\theta_H(\gamma)\tilde{x}_L(\gamma) - \tilde{t}_L(\gamma)\}\, d\gamma; \\
IC_L: &\quad \int_0^1 \{\theta_L(\gamma)\tilde{x}_L(\gamma) - \tilde{t}_L(\gamma)\}\, d\gamma \geq \int_0^1 \{\theta_L(\gamma)\tilde{x}_H(\gamma) - \tilde{t}_H(\gamma)\}\, d\gamma; \\
IR_H: &\quad \int_0^1 \{\theta_H(\gamma)\tilde{x}_H(\gamma) - \tilde{t}_H(\gamma)\}\, d\gamma \geq 0; \\
IR_L: &\quad \int_0^1 \{\theta_L(\gamma)\tilde{x}_L(\gamma) - \tilde{t}_L(\gamma)\}\, d\gamma \geq 0.
\end{aligned}$$

Standard arguments can be employed to show that at the optimal selling mechanism the incentive constraint for the high valuation type and the participation constraint for the low valuation type are binding and that the allocation rule has to satisfy a monotonicity property with respect to the ex ante information.[7]

Lemma 2 *With two ex ante types, the mechanism $(\tilde{x}, \tilde{t})$ which is optimal with publicly observable γ maximizes the seller's objective subject to (IC_H) and (IR_L) being binding, and* $\int_0^1 [\theta_H(\gamma) - \theta_L(\gamma)][\tilde{x}_H(\gamma) - \tilde{x}_L(\gamma)]\, d\gamma \geq 0$.

[7] The condition $\int_0^1 [\theta_H(\gamma) - \theta_L(\gamma)][\tilde{x}_H(\gamma) - \tilde{x}_L(\gamma)]\, d\gamma \geq 0$ is a monotonicity condition because, as $\theta_H(\gamma) - \theta_L(\gamma) > 0$ for all $\gamma \in (0,1)$, the condition is satisfied if $\tilde{x}_H(\gamma) \geq \tilde{x}_L(\gamma)$ "on average".

The binding constraints pin down the (expected) transfers as a function of the allocation rule according to

$$\int_0^1 \tilde{t}_L(\gamma)\, d\gamma = \int_0^1 \theta_L(\gamma)\tilde{x}_L(\gamma)\, d\gamma; \tag{18}$$

$$\int_0^1 \tilde{t}_H(\gamma)\, d\gamma = \int_0^1 \theta_H(\gamma)\tilde{x}_H(\gamma)\, d\gamma + \int_0^1 [\theta_L(\gamma) - \theta_H(\gamma)]\tilde{x}_L(\gamma)\, d\gamma. \tag{19}$$

Substituting out the transfer schedules in the seller's objective, the seller's problem simplifies to

$$\max_{\tilde{x},\tilde{t}} \quad \int_0^1 \{p\theta_H(\gamma)\tilde{x}_H(\gamma) + [\theta_L(\gamma) - p\theta_H(\gamma)]\tilde{x}_L(\gamma)\}\, d\gamma$$

$$s.t. \quad \int_0^1 [\theta_H(\gamma) - \theta_L(\gamma)][\tilde{x}_H(\gamma) - \tilde{x}_L(\gamma)]\, d\gamma \geq 0. \tag{20}$$

To solve this problem, we can adopt the standard approach and first ignore the monotonicity constraint (20). We then obtain the solution to this relaxed problem by point-wise maximization. It is then easy to verify that the solution to the relaxed problem always satisfies the monotonicity constraint so that it is also a solution to the original problem. We summarize this in the next lemma.

Lemma 3 *With two types, the solution $(\tilde{x}^*, \tilde{t}^*)$ to problem $\tilde{\mathscr{P}}$ is given by*

$$\tilde{x}_H^*(\gamma) = 1 \quad \forall \gamma \in [0,1], \qquad \tilde{x}_L^*(\gamma) = \begin{cases} 0 & if \qquad \theta_L(\gamma) < p\theta_H(\gamma) \\ 1 & else. \end{cases} \tag{21}$$

The optimal transfers $\tilde{t}_H^*$ and $\tilde{t}_L^*$ are only pinned down in expectation by (18) and (19).

Thus, under the optimal mechanism with publicly observable γ, the high valuation type s_H always consumes the good ("no distortion at the top"), and the low valuation type s_L consumes the good only if $\theta_L(\gamma) \geq p\theta_H(\gamma)$. The high valuation type s_H obtains an information rent of

$$\tilde{U}_H = \int_0^1 [\theta_H(\gamma) - \theta_L(\gamma)]\tilde{x}_L^*(\gamma)\, d\gamma = \int_{\gamma:\theta_L(\gamma)\geq p\theta_H(\gamma)}^1 \{\theta_H(\gamma) - \theta_L(\gamma)\}\, d\gamma. \tag{22}$$

4.2 Private ex post information

We now show that when γ is the buyer's ex post private information, the seller is strictly worse off than when γ is publicly observable. We derive this result by showing that, with private ex post information, the seller has to concede to the buyer strictly more information rents if she wants to implement the same allocation rule $\tilde{x}^*$ that is optimal when γ is publicly observable. That is, $\tilde{x}^*$ is not implementable without ex post information rents.

To prove our claim, we can use the following result from the literature: a *deterministic* allocation rule[8] is implementable with privately observable γ if, and only if, two properties hold: first, for each ex ante type, the allocation rule is non–decreasing in the ex post information; second, if the allocation rule prescribes consumption at a certain valuation for some ex ante type, then consumption is prescribed at all higher valuations for all larger ex ante types.[9,10] We now apply this result to the allocation rule $\tilde{x}^*$ in (21). Note that the second requirement is trivially met since $\tilde{x}^*_H(\gamma) = 1$ for all γ. The first requirement is met for type $s = s_H$ since $\tilde{x}^*_H(\gamma)$ is clearly non–decreasing in the ex post information γ. Thus, all that is needed to implement $\tilde{x}^*$ is that $\tilde{x}^*_L$ is non–decreasing in γ:

Lemma 4 *The allocation rule $\tilde{x}^*$ in (21) is implementable with privately observable γ if, and only if, $\tilde{x}^*_L(\gamma)$ is non–decreasing in γ.*

Hence, if $\tilde{x}^*_L$ is not non–decreasing, private ex post information is revenue–relevant for the simple reason that the optimal allocation rule $\tilde{x}^*$ is not implementable with privately observable γ. More interestingly, we now show that even if $\tilde{x}^*$ is implementable with privately observable γ, private ex post information is not revenue-irrelevant. Therefore, we focus from now on on the case that $\tilde{x}^*_L$ is monotone in the ex post information γ. A sufficient condition for this is that the ratio $\theta_L(\gamma)/\theta_H(\gamma)$ is strictly increasing because this guarantees a unique solution γ_L to the equation $\theta_L(\gamma_L) = p\theta_H(\gamma_L)$, and (21) then prescribes the good to be sold to the low valuation buyer type whenever $\gamma \geq \gamma_L$.[11] We now show that to implement $\tilde{x}^*$ with privately observable γ the seller has to concede a higher information rent to the buyer than in the case with publicly observable γ.

Indeed, ex post incentive compatibility requires that the buyer type s_L, after having reported his ex ante information truthfully, reports γ truthfully in the second stage. This implies that the

[8]An allocation rule x is deterministic if $x(s,\gamma) \in \{0,1\}$ for all s and γ.

[9]Formally: for $s < \hat{s}$, if $x(s,\gamma) = 1$, then $x(\hat{s},\hat{\gamma}) = 1$ for all $\hat{\gamma}$ with $\theta(\hat{s},\hat{\gamma}) \geq \theta(s,\gamma)$.

[10]The "if" part holds more generally also for non–deterministic allocation rules and was first proven in Courty and Li (2000). For a proof of the "only if" part, see Krähmer and Strausz (2011, Lemma 6).

[11]An example for which $\theta_L(\gamma)/\theta_H(\gamma)$ is strictly increasing, is: $\theta_H(\gamma) = \gamma$, and $\theta_L(\gamma) = \gamma^2$.

transfers t_L are piece-wise constant, and that at the critical realization γ_L, the buyer type s_L is indifferent between consuming and not consuming the good:

$$t_L(\gamma) = \begin{cases} t_L^0 & if \quad \gamma < \gamma_L \\ t_L^1 & else \end{cases}, \quad \text{and} \quad -t_L^0 = \theta_L(\gamma_L) - t_L^1. \tag{23}$$

To see that ex post incentive compatibility implies these restrictions on the transfer schedule, note that if t_L were not constant on $[0, \gamma_L)$, then since the allocation rule is constant on $[0, \gamma_L)$, there would be some realizations in $[0, \gamma_L)$ for which the buyer would benefit from reporting the value of γ that maximizes t_L on $[0, \gamma_L)$ instead of reporting his true value. For the same reason, t_L needs to be constant on $[\gamma_L, 1]$. On a related note, if the buyer were not indifferent between consuming and not consuming the good at the value γ_L, but, say, strictly preferred consumption, then he would still strictly prefer consumption at a value γ slightly smaller than γ_L and would thus benefit from pretending to have observed γ_L in this case.

Condition (23) pins down only the difference $t_L^1 - t_L^0 = \theta_L(\gamma_L)$, but not the absolute magnitude of the transfers. As is usual, we may interpret $r_L \equiv t_L^1 - t_L^0$ as an exercise price at which the buyer is allowed to buy the product after learning γ. Likewise, t_L^0 corresponds to an upfront payment by the buyer that gives him the option to make his consumption decision after he has fully learned his valuation. Note that by adjusting the upfront payment t_L^0, while keeping the exercise price fixed, the seller determines the buyer's utility without affecting the allocation. The expected utility of the buyer type s_L if he reports his ex ante information truthfully is

$$U_L = -t_L^0 + \int_{r_L}^{1} \{\theta_L(\gamma) - r_L\}\, d\gamma. \tag{24}$$

Now observe that $\tilde{x}_L^*$ and the payments t_L fully determine the high valuation buyer's expected utility U_{LH} when he pretends to be type s_L. Since ex ante incentive compatibility requires that the high valuation buyer attains at least a rent of U_{LH}, we can now obtain a lower bound on the rent that the seller has to concede to the buyer. To compute U_{LH}, note that after having reported s_L, the buyer type s_H exercises the option whenever

$$\theta_H(\gamma) > r_L \quad \Leftrightarrow \quad \gamma > \theta_H^{-1}(r_L). \tag{25}$$

The key observation is that when the high valuation type s_H reports s_L, he exercises the option more frequently than when the low valuation type s_L reports s_L. This is simply so because for any realization γ of ex post information, he displays a higher valuation than the buyer type s_L. Hence, for any value of γ so that $\theta_H(\gamma) > r_L > \theta_L(\gamma)$ the buyer type s_H does, and the buyer type s_L does not exercise the option. Therefore,

$$U_{LH} = -t_L^0 + \int_{\theta_H^{-1}(r_L)}^{1} \{\theta_H(\gamma) - r_L\}\, d\gamma \tag{26}$$

$$= -t_L^0 + \int_{\theta_H^{-1}(r_L)}^{\gamma_L} \{\theta_H(\gamma) - r_L\}\, d\gamma + \int_{\gamma_L}^{1} \{\theta_H(\gamma) - r_L\}\, d\gamma \tag{27}$$

$$= U_L + \int_{\theta_H^{-1}(r_L)}^{\gamma_L} \{\theta_H(\gamma) - r_L\}\, d\gamma + \int_{\gamma_L}^{1} \{\theta_H(\gamma) - \theta_L(\gamma)\}\, d\gamma, \tag{28}$$

where we have used (24) in the last line.

To obtain a lower bound on the rent the seller has to concede to the high valuation buyer, note that ex ante incentive compatibility requires $U_H \geq U_{LH}$ and individual rationality requires $U_L \geq 0$. As a result, by (28), the high valuation type s_H obtains a rent of at least[12]

$$U_H \geq \int_{\theta_H^{-1}(r_L)}^{\gamma_L} \{\theta_H(\gamma) - r_L\}\, d\gamma + \int_{\gamma_L}^{1} \{\theta_H(\gamma) - \theta_L(\gamma)\}\, d\gamma. \tag{29}$$

Now, reconsider the high valuation type's rent when γ is publicly observable. For the case that $\tilde{x}_L$ is monotone, we obtain from (22):

$$\tilde{U}_H = \int_{\gamma_L}^{1} \{\theta_H(\gamma) - \theta_L(\gamma)\}\, d\gamma. \tag{30}$$

Comparing expression (29) to expression (30) makes clear that the buyer obtains an additional information rent from the privacy of the ex post information of

$$\Delta U_H = U_H - \tilde{U}_H \geq \int_{\theta_H^{-1}(r_L)}^{\gamma_L} \{\theta_H(\gamma) - r_L\}\, d\gamma.$$

The expression also identifies the source of this additional rent. When ex post information is public, the contract enforces consumption directly, and irrespective of his true valuation, the buyer consumes only for $\gamma \geq \gamma_L$. In contrast when ex post information is private, the buyer effectively makes the consumption decision himself. As a result of his higher valuation, the high valuation buyer consumes the good more frequently than the low valuation buyer, and this is an additional source of rents: relative to the low valuation buyer, the high valuation buyer obtains not only the rent $\theta_H(\gamma) - \theta_L(\gamma)$ whenever both buyer types decide to consume but he also gets

[12] It may be shown that this lower bound is actually attainable.

the rent $\theta_H(\gamma) - r_L$ when the high valuation buyer does and the low valuation buyer does not decide to consume ex post.

More generally, the transfers $\tilde{t}^*$ that prevent the buyer from lying when ex post information is public are not sufficient to prevent the buyer from lying when ex post information is private. This is so because when ex post information is private, a lie about his ex ante information is more valuable to the buyer since he can coordinate it with a lie about his ex post information. Put differently, while $\tilde{x}^*$ is implementable with private ex post public information "on the path", it is not implementable with private ex post information "off the path" after a lie by the high valuation buyer. Even though a lie by the buyer is a zero probability event, the fact that the buyer can use his private ex post information to obtain a strictly higher utility off the path relative to when ex post information is public implies that the seller has to pay a higher rent to prevent the buyer from lying than with public ex post information.

To see that the strictly higher rents imply that private ex post information is not revenue–irrelevant, observe that $\tilde{x}^*$ is the unique optimal allocation rule when γ is publicly observable. Therefore, implementing any different allocation rule $x^* \neq \tilde{x}^*$ yields a smaller payoff than $\tilde{W}^*$ already when γ is observable. Because, as shown, $\tilde{x}^*$ requires the seller to pay larger rents to the buyer, the seller's revenue when γ is private is smaller than $\tilde{W}^*$ so that private ex post information is revenue–relevant.

5 The general case

In this section, we generalize the findings for the two types case. We first provide a decomposition of the buyer's rent into ex ante and ex post information rent. This is the crucial step to study the role of ex post information.

5.1 Ex ante and ex post information rents

We begin with two auxiliary lemmas. The first lemma states the well–known characterization of ex post incentive compatibility (we omit the proof).

Lemma 5 *The mechanism (x, t) is ex post incentive compatible if, and only if, $u(s, \gamma)$ is differentiable* *in γ for almost every $\gamma \in [0,1]$, and it holds for all $s \in S$:*

$$\frac{\partial u(s,\gamma)}{\partial \gamma} = \frac{\partial \theta(s,\gamma)}{\partial \gamma} x(s,\gamma) \quad a.e., \tag{31}$$

$$x(s,\gamma) \text{ is non–decreasing in } \gamma. \tag{32}$$

We next characterize the buyer's ex post reporting strategy. We define by

$$\Gamma^*(s,\hat{s},\gamma) = \arg\max_{\hat{\gamma}} u(\hat{s},\hat{\gamma};s,\gamma) \tag{33}$$

all optimal second stage reports by a buyer who has observed s and γ and who has reported $\hat{s}$ at the outset. The next lemma describes how the buyer optimally coordinates a lie about his ex ante information with a lie about his ex post information. It says that a buyer type who has observed s and γ and who has reported $\hat{s}$ submits a "corrected" report γ^* so that he has the same valuation and ends up with the same utility as a buyer type who has observed $\hat{s}$ and γ^* and who has reported truthfully.

Lemma 6 *If a mechanism is ex post incentive compatible, then there is a $\gamma^* = \gamma^*(s,\hat{s},\gamma) \in \Gamma^*(s,\hat{s},\gamma)$ so that:*
(a) $\theta(s,\gamma) = \theta(\hat{s},\gamma^*)$;
(b) $u(\hat{s},\gamma^*;s,\gamma) = u(\hat{s},\gamma^*) = \max_{\hat{\gamma}} u(\hat{s},\hat{\gamma};s,\gamma)$.

In light of the lemma, we can, without loss of generality, assume that a buyer who has observed s and γ and who has reported $\hat{s}$ at the outset, reports $\gamma^* = \gamma^*(s,\hat{s},\gamma)$ in the second stage.

We can now provide a decomposition of the buyer's information rent in ex ante and ex post rent. For an arbitrary allocation rule x, let

$$R^{xa}(s,\hat{s},x) \equiv \int_0^1 [\theta(s,\gamma) - \theta(\hat{s},\gamma)] x(\hat{s},\gamma)\, d\gamma; \tag{34}$$

$$R^{xp}(s,\hat{s},x) \equiv \int_0^1 \int_\gamma^{\gamma^*(s,\hat{s},\gamma)} \frac{\partial \theta(\hat{s},c)}{\partial \gamma} [x(\hat{s},c) - x(\hat{s},\gamma)]\, dc\, d\gamma. \tag{35}$$

Lemma 7 *(a) A mechanism $(\tilde{x},\tilde{t})$ is incentive compatible with publicly observable γ if, and only if, for all s, $\hat{s}$:*

$$\tilde{U}(s) - \tilde{U}(\hat{s}) \;\; \geq \;\; R^{xa}(s,\hat{s},\tilde{x}). \tag{36}$$

(b) A mechanism (x,t) is incentive compatible with privately observable γ if, and only if, (31) and (32) are satisfied and for all s, $\hat{s}$, it holds:

$$U(s) - U(\hat{s}) \;\; \geq \;\; R^{xa}(s,\hat{s},x) + R^{xp}(s,\hat{s},x). \tag{37}$$

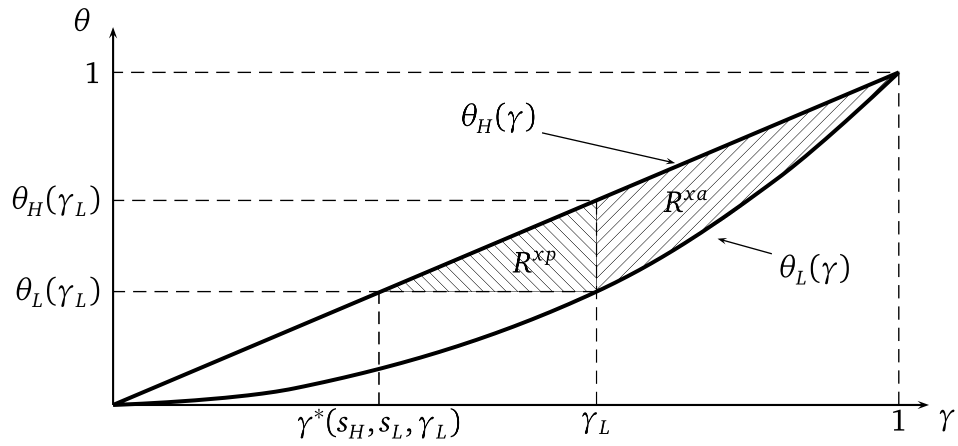

Figure 1: Ex ante vs. ex post information rents.

Moreover, $R^{xp}(s,\hat{s},x) \geq 0$*, where the inequality is strict if* $x(\hat{s},\gamma)$ *is not constant in* γ *on* $(0,1)$*.*

Lemma 7 provides, for both the case that ex post information is publicly and privately observable, lower bounds for the information rents a buyer of type s has to obtain so as not to pretend to be of type $\hat{s}$. When ex post information is private, the lower bound can be decomposed into a part R^{xa} and a part R^{xp}. The part R^{xa} is the same as the lower bound when ex post information is public and, therefore, accrues from the buyer's ex ante private information. Consequently, the part R^{xp} is the buyer's rent that accrues from his ex post private information.

Figure 1 illustrates the decomposition graphically for the two types case and the allocation rule $\tilde{x}^*$ given by (21) when $\tilde{x}^*_L$ is monotone. When γ is publicly observable, the high valuation buyer type s_H can, by announcing type $\hat{s} = s_L$, secure himself the same utility as the low valuation buyer plus the utility increment $\theta_H(\gamma) - \theta_L(\gamma)$ he receives due to his higher valuation whenever consumption is enforced by the mechanism with positive probability. To prevent type s_H from misreporting type $\hat{s} = s_L$, the seller needs to pay him at least a rent corresponding to the expected utility increment, as expressed by R^{xa}. Because the allocation rule $\tilde{x}^*$ in the two type case prescribes consumption exactly when γ exceeds the cutoff γ_L, the ex ante information rent R^{xa} corresponds to the area in between the curves $\theta_H(\gamma)$ and $\theta_L(\gamma)$ over the range γ_L to 1 as depicted in the figure.

When γ is privately observable, the high valuation buyer can increase his utility from a misreport $\hat{s} = s_L$ by combining it with a misreport $\hat{\gamma}$ about his ex post information that induces his most preferred allocation $x(\hat{s},\hat{\gamma})$ that is available ex post. This additional source of rent is captured by the term R^{xp}. The ex post information rent is strictly positive if, conditional on the first report $\hat{s}$, the allocation is responsive to the second report $\hat{\gamma}$. Only in this case, there are realizations of γ so that the high valuation buyer can use the private ex post information to induce a better allocation for himself than the allocation that is enforced when γ is public. Note that the allocation rule $\tilde{x}^*$ with two ex ante types s_H and s_L is responsive, as it prescribes consumption only if γ exceeds γ_L. As illustrated in Figure 1 this leads to a strictly positive ex post information rent R^{xp}. It corresponds to the area between $\theta_H(\gamma)$ and $\theta_L(\gamma_L)$ over the range from γ^* to γ_L, because exactly for these realization of γ a high valuation type would obtain a utility increase of $\theta_H(\gamma) - \theta_L(\gamma_L)$ by reporting $\gamma > \gamma_L$ in comparison to reporting γ truthfully and ending up with no consumption.

We now use the decomposition to study the relevance of ex post information. We show that ex post information does not give rise to additional rents and is revenue–irrelevant in continuous models, but for discrete models this is true only in the degenerate case that the allocation is not responsive to the second report $\hat{\gamma}$ for the lowest type $\underline{s}$.

5.2 Continuous model

We first show that for continuous models, the information rents are the same with public and private ex post information, because the marginal ex post information rent, R^{xp} disappears.

Lemma 8 *Suppose the model is continuous, then for an arbitrary allocation rule x, it holds:*
(a)

$$\lim_{\delta \to 0} \frac{R^{xp}(s+\delta, s, x)}{\delta} = 0. \tag{38}$$

(b) Moreover, if x is implementable with privately observable γ, then $\tilde{U}$ and U are differentiable for almost all $s \in S$, and whenever the derivatives exist, it holds:

$$\tilde{U}'(s) = U'(s) \geq 0. \tag{39}$$

The reason for why the marginal ex post information rent disappears can be intuitively seen as follows. The ex post information rent of a high valuation buyer relative to a low valuation buyer is essentially the product of two factors: the valuation increment and the probability with which the high valuation buyer benefits from misrepresenting his ex post information when he pretended to be the low valuation buyer. As the distance between the two ex ante types diminishes, both of these factors go to zero, because the valuation difference diminishes, and, crucially, because

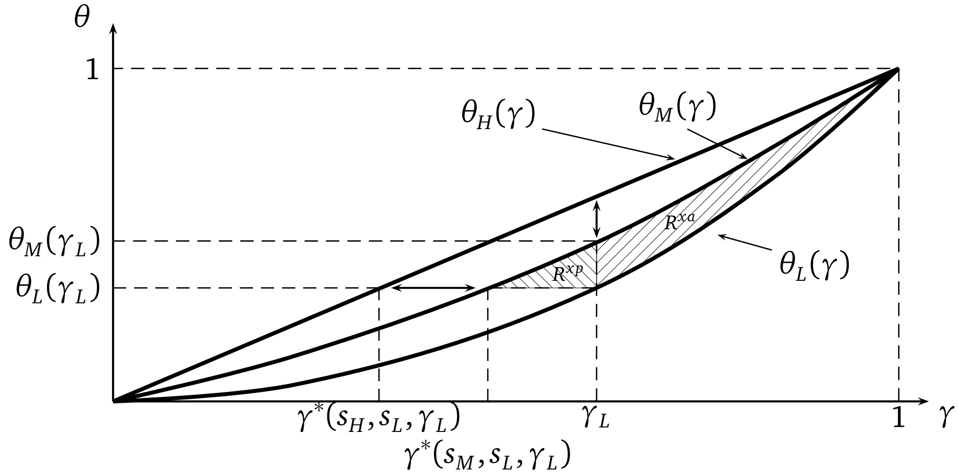

Figure 2: Quadratic reduction of R^{xp} vs. linear reduction of R^{xa}.

the set of realizations γ for which a misreport is still beneficial becomes smaller and smaller. Therefore, as the distance between ex ante types goes to zero, the ex post rent converges to zero at a quadratic speed.

In contrast, the ex ante rent converges to zero only at a linear speed, because only the difference in valuations diminishes, but the probability with which consumption is enforced by the mechanism does not diminish as the distance between ex ante types diminishes.

Figure 2 illustrates this graphically. As a result of introducing an additional intermediate ex ante type s_M, the area R^{xp} shrinks both in the vertical dimension, because the difference in the valuations between type s_L and the next higher type s_M becomes smaller, and in the horizontal dimension, because when having announced to be type $\hat{s} = s_L$, type s_M manipulates the ex post private information for a smaller range of ex post information than type s_H. In contrast, the area R^{xa} shrinks only in the vertical dimension, because the probability with which the contract enforces consumption upon an announcment of type $\hat{s} = s_L$ does not change by the introduction of the intermediate type s_M.

To complete the argument for why ex post information rents are zero in continuous models, notice that since $\tilde{U}'(s) = U'(s) \geq 0$, both with publicly and privately observable ex post information, the individual rationality constraints (9) and (14) are met if and only if they are met for the lowest valuation type $\underline{s}$. Therefore, both with publicly and privately observable ex post information, the revenue maximizing way for the seller to implement an allocation rule is to have the individual rationality constraint for the lowest type binding: $\tilde{U}(\underline{s}) = U(\underline{s}) = 0$. Because $\tilde{U}'(s) = U'(s)$, the buyer's utility is consequently the same with privately and with publicly observable ex post information, establishing our first main result:

Proposition 1 *Suppose the model is continuous. Then it holds:*
(a) Any allocation rule x that is implementable with privately observable γ, is implementable without ex post information rents.
(b) (Esö and Szentes (2007a, b)) If $\tilde{x}^*$ is implementable with privately observable γ, ex post information is revenue–irrelevant.

Part (a) of the proposition says that when the model is continuous, the agent never gets any information rents from his ex post information. This extends Esö and Szentes (2007a, b) who show this result only with respect to the allocation rule that is optimal when ex post information is public, as stated in part (b). In particular, if revenue–irrelevance fails in the continuous model, then it fails for implementability reasons, not because the principal has to concede more rents with private than with public ex post information.

5.3 Discrete model

We now turn to the discrete model and show that, only in rather special cases, ex post information does not increase the agent's rent. Recall that (x^*, t^*) is an the optimal mechanism with privately observable γ.

Proposition 2 *Suppose the model is discrete. Then it holds:*
(a) Any allocation rule x that is implementable with privately observable γ, is implementable without information rents only if $x(\underline{s}, \gamma)$ is constant in γ on $(0, 1)$.
(b) Ex post information is revenue–irrelevant only if $x^*(\underline{s}, \gamma)$ is constant in γ on $(0, 1)$.

To understand the result, note that if $x(\underline{s}, \gamma)$ is not constant in γ, then Lemma 7b implies that the implementation of x with privately observable γ requires that all types larger than $\underline{s}$ are paid a strictly positive ex post rent R^{xp} in order to prevent them from claiming to be type $\underline{s}$. By Lemma 7a, when γ is publicly observable, the seller can dispense with these extra rents when implementing x and only pay the ex ante rents R^{xa} to the buyer. This applies to any x that is implementable with privately observable γ and, in particular, for x^*. Hence, the seller can implement the optimal allocation x^* with strictly smaller information rents when γ is publicly observable. This means that ex post information is not revenue-irrelevant in discrete models.

For sequential screening models with discrete ex ante types where the so called first order approach works, it is generally known that the optimal sequential mechanism, for any ex ante type s, prescribes consumption and hence does not display a distortion for the most efficient ex post type $\gamma = 1$ (see, e.g., Courty and Li, 2000, or Dai et al., 2006).[13] In this case, we can strengthen Proposition 2b by stating that ex post information is revenue–irrelevant only if the optimal mechanism implements the efficient outcome where the buyer always consumes the good so that there are no distortions.

6 Disclosure of ex post information

In this section, we show that our findings have relevant implications for the seller's incentives to disclose ex post information to the buyer. As observed already by Esö and Szentes (2007a, b), when ex post private information is revenue–irrelevant, the seller optimally discloses any additional information to the buyer ex post even if the buyer observes such information privately. This is because the seller can extract the full value of information as if ex post information was publicly observable. We now show that full disclosure is, in fact, not optimal in general. We provide an example of a discrete model in which there exists a partial disclosure policy that allows the seller to attain the same revenue $\tilde{W}^*$ as when ex post information is public.

We reconsider the model with two types from Section 4 and assume that the buyer can only observe the ex ante information s, but that the extent of the ex post information the buyer observes is controlled by the seller. More specifically, as in Esö and Szentes (2007a, b), we assume that the seller can disclose to the buyer, without observing herself, the realization of any signal that is correlated with the underlying ex post information γ.[14] In particular, the seller may adopt a "binary" disclosure rule which reveals to the buyer only whether γ is below or above some cutoff value γ_0. We now consider a specific example in which there is such a partial disclosure rule which allows the seller to attain the revenue $\tilde{W}^*$. Let

$$\theta_L(\gamma) = \gamma^2, \qquad \theta_H(\gamma) = \gamma. \tag{40}$$

[13] There are good reasons to conjecture that this is a general property of optimal sequential mechanisms in any context, but proving this conjecture would go beyond the scope of this paper.

[14] Disclosure is modeled in this way also in, e.g., Bergemann and Pesendorfer (2007), Ganuza and Penalva (2010), or Ivanov (2010).

By Lemma 3, the optimal mechanism with publicly observable γ for specification (40) is given by the monotone allocation rule

$$\tilde{x}_H^*(\gamma) = 1 \quad \forall \gamma \in [0,1], \qquad \tilde{x}_L^*(\gamma) = \begin{cases} 0 & if \quad \gamma < p \\ 1 & else, \end{cases} \tag{41}$$

and expected payments (by (18) and (19))

$$\bar{t}_L = \int_p^1 \theta_L(\gamma)\, d\gamma; \qquad \bar{t}_H = \int_0^1 \theta_H(\gamma)\, d\gamma + \int_p^1 [\theta_L(\gamma) - \theta_H(\gamma)]\, d\gamma. \tag{42}$$

Now suppose that the seller commits to a partial disclosure mechanism that reveals to the buyer ex post only whether γ is below or above the cutoff value $\gamma_0 = p$. In addition, she offers the buyer the following menu of option contracts:

- By announcing type s_H, the buyer pays the price $\bar{t}_H$ and receives the good for sure.
- By announcing the type s_L, the buyer receives the right to buy the good at a price $r_L = \bar{t}_L/(1-\gamma_0)$ after having learned the additional information.

We claim that this partial disclosure mechanism implements the allocation rule $\tilde{x}^*$ and yields the seller the revenue $\tilde{W}^*$. Therefore, under the partial disclosure rule, the ex post private information is revenue-irrelevant, which, as we have seen in Section 4, is not the case under full disclosure.

More precisely, we show that, under the partial disclosure role, the high valuation buyer does no longer benefit from a lie ex post after having lied initially. In other words, the menu of option contracts does not only implement the allocation rule $\tilde{x}^*$ "on the path", after the buyer has announced his type s truthfully, but also "off the path", after a lie by the buyer. Note that if this is true, then, on the path, the high valuation buyer consumes the good with probability 1, inducing the payment $\bar{t}_H$ for sure, and the low valuation buyer consumes the good with probability $P[\gamma > \gamma_0] = 1 - \gamma_0$, inducing expected payments $\bar{t}_L$. Therefore, not only the allocation rule, but also the expected payments under the partial disclosure mechanism are the same as under the optimal mechanism with publicly observable γ. This directly implies that the partial disclosure mechanism is ex ante incentive compatible and individually rationally and yields the seller the same revenue as the optimal mechanism with publicly observable γ.

To see that the partial disclosure mechanism implements $\tilde{x}^*$ on and off the path, observe first that after an announcement s_H, the good is consumed with probability 1 no matter which buyer type made the announcement. Next consider what happens after an announcement s_L, and take first the case that the low valuation buyer has made this announcement. Observe that the exercise price r_L exactly matches the expected valuation of the low valuation buyer when he learns that γ exceeds γ_0:

$$E[\theta_L(\gamma)|\gamma > \gamma_0] = \int_{\gamma_0}^{1} \frac{\theta_L(\gamma)}{1-\gamma_0} d\gamma = \frac{\bar{t}_L}{1-\gamma_0} = r_L. \tag{43}$$

It is therefore optimal for the low valuation buyer s_L to exercise the option if, only if, he learns that γ exceeds γ_0.

Next, consider the case that the high valuation buyer announced s_L. Since $\theta_H(\gamma) > \theta_L(\gamma)$ for $\gamma \in (0,1)$, it follows $E[\theta_H(\gamma)|\gamma > \gamma_0] > E[\theta_L(\gamma)|\gamma > \gamma_0] = r_L$, and thus the high valuation buyer exercises the option when he learns that $\gamma > \gamma_0$. On the other hand, when he learns that $\gamma < \gamma_0$, then using (40), his expected payoff is

$$E[\theta_H(\gamma)|\gamma < \gamma_0] - \bar{t}_L/(1-\gamma_0) = \gamma_0/2 - (1-\gamma_0^3)/(3-3\gamma_0) = -(2\gamma_0^2 - \gamma_0 + 2)/6 < 0. \tag{44}$$

The inequality follows because the expression attains a maximum of $-5/16$ at $\gamma_0 = 1/4$.

This shows that the partial disclosure mechanism implements $\tilde{x}^*$ on and off the path, and, as a result, the seller obtains the expected revenue $\tilde{W}^*$. As shown in Section 4, this is more than what the seller can obtain under full disclosure, and therefore partial disclosure is optimal.

Intuitively, partial disclosure eliminates the ex post information rent, because it affects the way how the buyer uses his ex post information. Even though the high valuation buyer, after having announced s_L, is still free to misrepresent his ex post information, the disclosure policy is designed so that it is no longer optimal for the buyer to do so, but rather to choose the same allocation that would be enforced when the ex post information were public. In this case, ex post information does not yield additional rents.

That there is a disclosure rule that implements the same outcome as if ex post information was public, is a special feature of our example. On a more general note, restricting the ex post information available to the buyer has two countervailing effects. On the one hand, it reduces ex post information rents, as it diminishes the value from misrepresenting ex post information off the path. On the other hand, it also reduces the set of allocations that can be implemented on the path. We conjecture that, unlike in our example, there is in general no disclosure rule that fully resolves this trade–off and permits the seller to attain $\tilde{W}^*$.

The effect that revealing less information reduces information rents and may therefore be beneficial to the seller, is reminiscent of a general insight in mechanism design that "revealing more information to players makes it harder to prevent them from finding ways to gain by lying" (Myerson 1991, p.297) so that one should reveal "to each player only the minimum information needed to guide his action" (Myerson 1991, p.260). Indeed in our example, revealing only whether γ is larger or smaller than $\gamma_0 = p$ is the minimum amount of information revelation that is needed to implement the allocation rule. Disclosing more information only makes it more difficult to satisfy incentive compatibility, leading to higher implementation costs in the form of positive ex post information rents.

7 Conclusion

In this paper we show that, in general, a principal has to pay information rents for inducing an agent to reveal his private ex post information in excess of the rents that she needs to pay for eliciting the agent's private ex ante information. We interpret these additional rents as ex post information rents. The principal has to pay such rents, because in their absence the agent would benefit from coordinating his ex ante and ex post lies. The agent's potential benefits from such coordinated lying are strictly positive when the ex ante private information is discrete and the ex post private information is continuous. In such a model, the classical trade-off between rents and efficiency is more subtle, because the rents consist of both ex ante and ex post information rents. An interesting question for future research is to investigate how the potential problem of coordinated lying manifests itself in dynamic mechanism design models with more than two periods.

Appendix

Proof of Lemma 1: (a) The claim follows directly from the assumption that $F(\theta|s)$ first order stochastically dominates $F(\theta|s)$ for $s > \hat{s}$.

(b) The claim follows from the non–shifting support assumption, and because $F(\theta|s)$ is strictly increasing and continuous in θ for all $s \in S$.

(c) Existence of $\partial\theta(s,\gamma)/\partial\gamma$ follows because F has a density f. In particular, $\partial\theta(s,\gamma)/\partial\gamma = 1/f(\theta(s,\gamma)|s)$ is strictly positive and bounded because $f(\theta,s)$ is strictly positive and bounded away from zero.

(d) Finally, if the model is continuous, then $\partial F(\theta|s)/\partial s$ exists and, because in addition $f(\theta|s) > 0$, we can apply the implicit function theorem to obtain $\partial\theta(s,\gamma)/\partial s$ by differentiating the identity $F(\theta(s,\gamma)|s) = \gamma$ with respect to s:

$$\frac{\partial F(\theta|s)}{\partial\theta}\frac{\partial\theta(s,\gamma)}{\partial s} + \frac{\partial F(\theta|s)}{\partial s} = 0 \quad\Leftrightarrow\quad \frac{\partial\theta(s,\gamma)}{\partial s} = -\frac{\partial F(\theta|s)}{\partial s}\frac{1}{f(\theta|s)}. \tag{45}$$

Boundedness of $\partial\theta(s,\gamma)/\partial s$ follows because $\partial F(\theta|s)/\partial s$ is bounded and $f(\theta|s)$ is bounded away from zero. Q.E.D.

Proof of Lemma 2: Adding up (IC_H) and (IC_L) and re–arranging yields:

$$\int_0^1 [\theta_H(\gamma) - \theta_L(\gamma)][\tilde{x}_H(\gamma) - \tilde{x}_L(\gamma)]\, d\gamma \geq 0. \tag{46}$$

Moreover, (IC_H) and (IR_L) imply (IR_H) because of (17). Therefore, (IR_H) can be ignored, and it follows that at the optimum, (IR_L) and (IC_H) are binding, because otherwise, without affecting any other constraint, the expected transfers to type s_L could be reduced until (IR_L) binds, and the expected transfers to type s_H could be reduced until (IC_H) binds. Since (IC_H) is binding, (46) implies

$$\int_0^1 \tilde{t}_L(\gamma) - \tilde{t}_H(\gamma)\, d\gamma = \int_0^1 \theta_H(\gamma)[\tilde{x}_L(\gamma) - \tilde{x}_H(\gamma)]\, d\gamma \leq \int_0^1 \theta_L(\gamma)[\tilde{x}_L(\gamma) - \tilde{x}_H(\gamma)],$$

but this inequality is the same as (IC_L). Hence, also (IC_L) can be ignored, and the claim follows. Q.E.D.

Proof of Lemma 3 Follows directly from the main text. Q.E.D.

Proof of Lemma 4 Follows directly from the main text. Q.E.D.

Proof of Lemma 5 Standard and therefore omitted. Q.E.D.

Proof of Lemma 6 By Lemma 1b, there is a unique γ^* so that $\theta(s,\gamma^*) = \theta(\hat{s},\gamma^*)$. If type s has reported type $\hat{s}$ and observed γ, he chooses $\hat{\gamma}$ to maximize

$$\theta(s,\gamma)x(\hat{s},\hat{\gamma}) - t(\hat{s},\hat{\gamma}) = \theta(\hat{s},\gamma^*)x(\hat{s},\hat{\gamma}) - t(\hat{s},\hat{\gamma}). \tag{47}$$

By ex post incentive compatibility for type $\hat{s}$, the report $\hat{\gamma} = \gamma^*$ maximizes the right hand side of (47). Thus, γ^* is an optimal report in the second stage when type s has reported type $\hat{s}$ and observed γ.

To see (b), note that because of (a), we have

$$u(\hat{s},\gamma^*;s,\gamma) = \theta(s,\gamma)x(\hat{s},\gamma^*) - t(\hat{s},\gamma^*) = \theta(\hat{s},\gamma^*)x(\hat{s},\gamma^*) - t(\hat{s},\gamma^*) = u(\hat{s},\gamma^*). \tag{48}$$

This shows the result. Q.E.D.

Proof of Lemma 7 (a) Immediate from (12) and (13).

(b) By Lemma 5, it is sufficient to show that ex ante incentive compatibility (8) is equivalent to (37). Indeed, by ex post incentive compatibility we have that $U(\hat{s}) = \int_0^1 u(\hat{s},\gamma)\,d\gamma$. Hence, ex ante incentive compatibility (8) is equivalent to

$$\begin{aligned} U(s) - U(\hat{s}) &\geq \int_0^1 \max_{\hat{\gamma}} u(\hat{s},\hat{\gamma};s,\gamma)\,d\gamma - \int_0^1 u(\hat{s},\gamma)\,d\gamma && (49)\\ &= \int_0^1 u(\hat{s},\gamma^*) - u(\hat{s},\gamma)\,d\gamma && (50)\\ &= \int_0^1 \int_\gamma^{\gamma^*} \frac{\partial u(\hat{s},c)}{\partial \gamma} dc\,d\gamma && (51)\\ &= \int_0^1 \int_\gamma^{\gamma^*} \frac{\partial \theta(\hat{s},c)}{\partial \gamma} x(\hat{s},c)\,dc\,d\gamma, && (52) \end{aligned}$$

where the first equality follows from Lemma 6b and the final equality follows from Lemma 5.

Moreover, by Lemma 6a, we have that

$$\begin{aligned} R^{xa}(s,\hat{s},x) &= \int_0^1 [\theta(s,\gamma) - \theta(\hat{s},\gamma)]x(\hat{s},\gamma)\,d\gamma && (53)\\ &= \int_0^1 [\theta(\hat{s},\gamma^*) - \theta(\hat{s},\gamma)]x(\hat{s},\gamma)\,d\gamma && (54)\\ &= \int_0^1 \int_\gamma^{\gamma^*} \frac{\partial \theta(\hat{s},c)}{\partial \gamma} x(\hat{s},\gamma)\,dc\,d\gamma. && (55) \end{aligned}$$

Adding the last term to $R^{xp}(s,\hat{s},x)$ yields expression (52), and hence, (37) holds.

Moreover, to see that $R^{xp}(s,\hat{s},x)$ is positive, observe that $\gamma \gtreqless \gamma^*(s,\hat{s},\gamma) \Longleftrightarrow s \lesseqgtr \hat{s}$ for all $\gamma \in (0,1)$ by (3) and Lemma 6a. Hence, since $\partial\theta/\partial\gamma$ is positive by Lemma 1c, and $x(\hat{s},\gamma)$ is non–decreasing in γ according to Lemma 5, R^{xp} is positive.

Finally, suppose that $x(\hat{s},\gamma)$ is not constant in γ on $(0,1)$, and consider the case $s > \hat{s}$. (The case $s < \hat{s}$ can be treated analogously.) Then, because $x(\hat{s},\gamma)$ is non–decreasing in γ according to Lemma 5, there exist $0 < \gamma_1 < \gamma_2' < 1$ such that $x(\hat{s},\gamma_1) < x(\hat{s},\gamma_2')$ and $x(\hat{s},\gamma)$ is not constant on the open interval (γ_1,γ_2'). Define

$$\gamma_2 = \inf\{\gamma \mid x(\hat{s},\gamma) = x(\hat{s},\gamma_2')\}. \tag{56}$$

Because $x(\hat{s},\gamma)$ is not constant on the open interval (γ_1,γ_2'), we have that $\gamma_1 < \gamma_2$. Let

$$\Delta\gamma = \min_{\gamma\in[\gamma_1,\gamma_2]} \gamma^*(s,\hat{s},\gamma) - \gamma. \tag{57}$$

Note that $\Delta\gamma$ exists and, since $s > \hat{s}$ and $0 < \gamma_1 < \gamma_2 < 1$, we have $\Delta\gamma > 0$.

Now let

$$\gamma_0 = \gamma_2 - 1/2 \cdot \Delta\gamma. \tag{58}$$

Then, by definition of γ_2, we have for all $\gamma \in [\gamma_0,\gamma_2)$ that $\gamma < \gamma_2$ and $\gamma + \Delta\gamma > \gamma_2$ so that

$$\int_{\gamma}^{\gamma+\Delta\gamma} \{x(\hat{s},c) - x(\hat{s},\gamma)\}\, dc > 0. \tag{59}$$

Therefore,

$$\int_{\gamma_0}^{\gamma_2} \int_{\gamma}^{\gamma+\Delta\gamma} \{x(\hat{s},c) - x(\hat{s},\gamma)\}\, dc d\gamma > 0. \tag{60}$$

Moreover, let

$$k = \min_{c\in[\gamma_0,\gamma_2]} \frac{\partial\theta(\hat{s},c)}{\partial\gamma} > 0, \tag{61}$$

which is strictly positive by Lemma 1c.

Applying these properties together with $x(\hat{s},\gamma)$ being non–decreasing in γ, we can infer:

$$R^{xp}(s,\hat{s},x) = \int_0^1 \int_{\gamma}^{\gamma^*(s,\hat{s},\gamma)} \frac{\partial\theta(\hat{s},c)}{\partial\gamma}[x(\hat{s},c) - x(\hat{s},\gamma)]\, dc\, d\gamma \tag{62}$$

$$\geq \int_{\gamma_0}^{\gamma_2} \int_{\gamma}^{\gamma^*(s,\hat{s},\gamma)} \frac{\partial\theta(\hat{s},c)}{\partial\gamma}[x(\hat{s},c) - x(\hat{s},\gamma)]\, dc\, d\gamma \tag{63}$$

$$\geq k\int_{\gamma_0}^{\gamma_2} \int_{\gamma}^{\gamma+\Delta\gamma} [x(\hat{s},c) - x(\hat{s},\gamma)]\, dc\, d\gamma > 0. \tag{64}$$

where the final inequality follows from (60). This shows that $R^{xp}(s,\hat{s},x) > 0$ if $x(\hat{s},\gamma)$ is not constant in γ on $(0,1)$ and completes the proof. Q.E.D.

Proof of Lemma 8 (a) Note that $\gamma^*(s,s,\gamma)=\gamma$. Hence,

$$\lim_{\delta\to 0}\frac{R^{xp}(s+\delta,s,x)}{\delta} = \int_0^1 \left[\lim_{\delta\to 0}\frac{1}{\delta}\int_{\gamma^*(s,s,\gamma)}^{\gamma^*(s+\delta,s,\gamma)} \frac{\partial\theta(s,c)}{\partial\gamma}[x(s,c)-x(s,\gamma)]\,dc\right]d\gamma, \tag{65}$$

because we can interchange integration and taking the limit since all functions are bounded. Now observe that because the model is continuous, we have that $\theta(s,\gamma)$ is differentiable in s, and thus Lemma 6a, implies that $\gamma^*(s,\hat{s},\gamma)$ is differentiable in s. Hence, by Leibniz' rule, the term in the square bracket in(65) is equal to

$$\frac{\partial\theta(s,\gamma^*(s+\delta,s,\gamma))}{\partial\gamma}[x(s,\gamma^*(s+\delta,s,\gamma))-x(s,\gamma)]\cdot\frac{\partial\gamma^*(s+\delta,s,\gamma)}{\partial s}\Bigg|_{\delta=0}=0, \tag{66}$$

because $x(s,\gamma^*(s+\delta,s,\gamma))|_{\delta=0} = x(s,\gamma^*(s,s,\gamma)) = x(s,\gamma)$ so that the difference in the squared brackets in (66) is zero, and this shows part (a).

To see part (b), observe first that $\tilde{U}$ and U are increasing in s since $s>\hat{s}$ implies $R^{xa}(s,\hat{s},x)\geq 0$ by Lemma 1a, and Lemma 7 then implies $\tilde{U}(s)-\tilde{U}(\hat{s})\geq 0$ and $U(s)-U(\hat{s})\geq 0$. Therefore, $\tilde{U}$ and U are differentiable almost everywhere. Consider $s\in S$ at which $\tilde{U}'$ exists. Then, by Lemma 7a:

$$\lim_{\delta\to 0}1/\delta\cdot[\tilde{U}(s+\delta)-\tilde{U}(s)] \;\;\geq\;\; \lim_{\delta\to 0}1/\delta\cdot R^{xa}(s+\delta,s,x) = \int_0^1\frac{\partial\theta(s,\gamma)}{\partial s}x(s,\gamma)\,d\gamma, \tag{67}$$

and

$$\lim_{\delta\to 0}1/\delta\cdot[\tilde{U}(s)-\tilde{U}(s-\delta)] \;\;\leq\;\; \lim_{\delta\to 0}-1/\delta\cdot R^{xa}(s-\delta,s,x) = \int_0^1\frac{\partial\theta(s,\gamma)}{\partial s}x(s,\gamma)\,d\gamma, \tag{68}$$

where the limits on the right hand side exist, because $\partial\theta(s,\gamma)/\partial s$ exists and is bounded. The two inequalities imply that

$$\tilde{U}'(s) = \int_0^1\frac{\partial\theta(s,\gamma)}{\partial s}x(s,\gamma)\,d\gamma. \tag{69}$$

With analogous reasoning, we can deduce by Lemma 7b that for all $s\in S$ at which U' exists, we have

$$U'(s) \;=\; \lim_{\delta\to 0}1/\delta\cdot R^{xa}(s+\delta,s,x)+\lim_{\delta\to 0}1/\delta\cdot R^{xp}(s+\delta,s,x) \tag{70}$$

$$= \int_0^1\frac{\partial\theta(s,\gamma)}{\partial s}x(s,\gamma)\,d\gamma + 0, \tag{71}$$

where the final equality follows from part (a). Hence we have shown that $\tilde{U}'(s) = U'(s)$ for all $s \in S$ at which the derivatives exist. Moreover, as remarked earlier, $\tilde{U}$ and U are increasing, thus the derivatives are positive. This completes the proof of part (b). Q.E.D.

Proof of Proposition 2 Note that (b) directly follows from (a) with $x = x^*$. Hence, we only show (a). If x is implementable with privately observable γ, then there exists a transfer schedule t such that (x,t) is feasible with privately observable γ. Denote by $\hat{t}$ a solution to (11), that is, $(x,\hat{t})$ is feasible and

$$W(x) = \int_S \int_0^1 \hat{t}(s,\gamma)\, d\gamma\, dP(s). \tag{72}$$

We now show that if $x(\underline{s},\gamma)$ is not constant in γ on $(0,1)$, then $W(x) < \tilde{W}(x)$, which implies that x is not implementable without ex post information rents. We prove this result by constructing an alternative set of transfers $\tilde{t}$ such that $(x,\tilde{t})$ is feasible with publicly observable γ and satisfies

$$W(x) < \int_S \int_0^1 \tilde{t}(s,\gamma)\, d\gamma\, dP(s). \tag{73}$$

Indeed, define

$$R \equiv \min_{s \neq \underline{s}} R^{xp}(s,\underline{s},x). \tag{74}$$

For the discrete model, S is finite and, therefore, the minimum exists. Since $x(\underline{s},\gamma)$ is not constant in γ on $(0,1)$, Lemma 7b, implies that $R > 0$. Define now the alternative payment scheme $\tilde{t}$ by

$$\tilde{t}(s,\gamma) = \begin{cases} \hat{t}(\underline{s},\gamma) & \text{if } s = \underline{s} \\ \hat{t}(s,\gamma) + R & \text{otherwise.} \end{cases} \tag{75}$$

Clearly, $\tilde{t}$ satisfies (73). In order to show that $(x,\tilde{t})$ is also feasible with publicly observable γ, let $\tilde{U}(s)$ be the expected utility of buyer type s from the mechanism $(x,\tilde{t})$ when he truthfully reports his type s, and γ is publicly observable, as defined after equation (12). Likewise, let $U(s)$ be the expected utility of buyer type s from the mechanism $(x,\hat{t})$ when he truthfully reports his type s, and γ is privately observable, as defined after equation (7). By construction of the payments, we have

$$\tilde{U}(s) = U(s) - R \quad \forall s \neq \underline{s}, \qquad \tilde{U}(\underline{s}) = U(\underline{s}). \tag{76}$$

We now show that $(x,\tilde{t})$ is incentive compatible with publicly observable γ. Indeed, since $(x,\hat{t})$ is incentive compatible with private ex post information, Lemma 7b, implies for all $s \neq \underline{s}, \hat{s} \neq \underline{s}$,

$$\tilde{U}(s)-\tilde{U}(\hat{s}) = U(s)-U(\hat{s}) \geq R^{xa}(s,\hat{s},\tilde{x})+R^{xp}(s,\hat{s},\tilde{x}) \geq R^{xa}(s,\hat{s},\tilde{x}), \tag{77}$$

and for all $s \neq \underline{s}$,

$$\tilde{U}(s)-\tilde{U}(\underline{s}) = U(s)-U(\underline{s})-R \geq R^{xa}(s,\underline{s},\tilde{x})+R^{xp}(s,\underline{s},\tilde{x})-R \geq R^{xa}(s,\underline{s},\tilde{x}), \tag{78}$$

and for all $\hat{s} \neq \underline{s}$,

$$\tilde{U}(\underline{s})-\tilde{U}(\hat{s}) = U(\underline{s})-U(\hat{s})+R \geq R^{xa}(\underline{s},\hat{s},\tilde{x})+R^{xp}(\underline{s},\hat{s},\tilde{x})+R \geq R^{xa}(\underline{s},\hat{s},\tilde{x}). \tag{79}$$

Hence, $\tilde{U}$ satisfies the inequalities in Lemma 7a, and thus $(x,\tilde{t})$ is incentive compatible with public ex post information.

It remains to show that $(x,\tilde{t})$ is individually rational with public ex post information. Indeed, since $(x,\hat{t})$ is individually rational with private ex post information, we have $\tilde{U}(\underline{s}) = U(\underline{s}) \geq 0$. Since we established in the proof of Lemma 8b that $\tilde{U}(s)$ is increasing in s whenever the mechanism is incentive compatible with publicly observable γ, we have $\tilde{U}(s) \geq \tilde{U}(\underline{s}) \geq 0$ for all $s \in S$ so that $(x,\tilde{t})$ is individually rational. And this completes the proof. Q.E.D.

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8 Remark on Continuity

• Consider a continuous model.

• P_0 be a measure on $\{\underline{s}, \bar{s}\}$ and consider the corresponding discrete model with $F(\cdot|\underline{s})$ and $F(\cdot|\bar{s})$.

• Let P_ε be a measure with support S which converges to P_0, as $\varepsilon \to 0$.

• Consider an allocation rule x in the continuous model which is implementable with privately observable ex post information and which is not constant in γ on $(0,1)$ for $\underline{s}$.

⋄ Then for all ε: $W_\varepsilon(x) - \tilde{W}_\varepsilon(x) = 0$.

• Consider the same allocation rule x restricted to the domain of the discrete model:

⋄ Then: $W_0(x) - \tilde{W}_0(x) < 0$.

• To understand the discontinuity, we argue that $\tilde{W}_\varepsilon(x) \not\to \tilde{W}_0(x)$.

• Indeed, in the discrete model, we know that under the revenue maximizing mechanism that implements x when γ is publicly observable, the incentive constraint for type $\bar{s}$ and the participation constraint for type $\underline{s}$ are binding, leading to a utility $\tilde{U}_0(\bar{s})$ for type $\bar{s}$.

• Suppose now that in the continuous model, the type $\bar{s}$ strictly prefers to announce the truth rather than $\underline{s}$ (note that whether this is the case or not is independent of the measure P_ε). Hence, for all ε, the utility of type $\bar{s}$ in teh continuous model, $\tilde{U}_\varepsilon(\bar{s})$ is strictly larger than in the discrete model. $\tilde{U}_\varepsilon(\bar{s}) > U_0(\bar{s}) + \Delta$ for some $\Delta > 0$. In this case, of course, $\tilde{W}_\varepsilon(x) \not\to \tilde{W}_0(x)$.

• Therefore, continuity requires that also in the continuous model, type $\bar{s}$ is indifferent between announcing the truth and announcing $\underline{s}$.

⋄ But this will imply a contradiction to the assumption that x is implementable with privately observable γ.

• This is so, because since $x(\underline{s}, \gamma)$ is not constant in γ on $(0,1)$, type $\bar{s}$ obtains an additional ex post information rent from announcing type $\underline{s}$. Hence, it is impossible to implement x with the same rent when γ is publicly and privately observable. This contradicts the fact that, in the continuous model, $W_\varepsilon(x) - \tilde{W}_\varepsilon(x) = 0$.