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Contests with multi-tasking

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Abstract

The standard contest model in which participants compete in a single dimension is well understood and documented. Multi-dimension extensions are possible but are liable to increase the complexity of the contest structure, mitigating one of its main advantages: simplicity. In this paper we propose an extension in which competition ensues in several dimensions and a competitor that wins a certain number of these is awarded a prize. The amount of information needed to run the contest is hence limited to the number of dimensions won by each player. We look at the design of this contest from the point of view of maximizing effort in the contest (per dimension and totally), and show that there will be a tendency to run small contests with few dimensions. The standard Tullock model and its results are encompassed by our framework.

Keywords: contest design, multi-tasking, effort incentives

JEL Classification: D72

1. Introduction

In many areas, including lobbying, internal labor markets, promotional competition, political campaigns, sports, litigation, international conflict and war and in many areas of biology, situations are characterized by one or several prizes that are allocated among a set of players as a function of the players' costly efforts. Such games have been called conflicts, tournaments, all-pay auctions, or wars, and the common underlying structure has been studied intensively.[1] Some more recent contributions focus on the problem of how such games should be designed if the designer pursues certain objectives. The role of the different types of contest success functions and reward functions (Kräkel 2003), the size of the prize, multiple prizes and their optimal structure (Clark and Riis 1998, Moldovanu and Sela 2001, 2005), resource constraints (Che and Gale 1997), spending limits (Che and Gale 1998), the contestants' choice of points of aspiration and the incentive to moderate their conflicting demands (Epstein and Nitzan 2004), timing (Baik and Shogren 1992, Leininger 1993), the role of fee-shifting rules in litigation contests (Farmer and Pecorino 1999, Baye, Kovenock and de Vries 2005), the role of tournaments with multiple, more complex structures (Amegashie 1999 and Gradstein and Konrad 1999) have been analysed. Related to this, researchers in industrial organization and in political economy have considered questions of sequential contests in which the same contestants interact repeatedly. In the industrial organization context, early influential work is by Harris and Vickers (1985, 1987) and Budd, Harris and Vickers (1993) who considered two structures of repeated contests, the race, and the tug-of-war. In the first type, the number of single-stage contests is finite and the prize is awarded to the contestant who first wins a certain threshold of single-stage contests. In the tug-of-war,

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[1] In the context of lobbying, some selective surveys are Nitzan (1994) and Lockard and Tullock (2001).

the number of single-stage contests is possibly infinite, and the tug-of-war ends if one contestant has gained a sufficiently large advantage. Konrad and Kovenock (2005) consider the tug-of-war in the context of all-pay auctions and offer complete solutions to the problem. Klumpp and Polborn (2006) consider the primaries in the U.S. presidential elections. They focus on the sequential nature of this process, and compare it with simultaneous contests in several states. The sequential nature causes considerable dynamics in terms of spending levels in the sequence of elections.

In a standard contest, participants are invited to try to win a prize of fixed value by making an irretrievable effort or outlay, with the winner being determined by a contest success function. The advantages and disadvantages of tournaments compared to a standard principal-agent contract are well understood in the context in which agents spend effort along one dimension only, and are surveyed by Kräkel (2004) who introduces limited liability as a further dimension along which standard contracts and tournaments should be compared. Generally, non-verifiability of output and the contractual problems for the principal that this may generate, systematic noise, or limits to the comparability of outputs on an ordinal scale are known to be main reasons that may make tournaments, or relative reward schemes more generally, superior to other, standard incentive mechanisms. Their wide use (see Lazear 1996 and Kräkel 2004 for discussions of examples) in firms, sports and other contexts suggests that the conditions for their superiority are often met.

Our paper considers a particular aspect of contest design: in many situations the number of prizes that can be awarded is much smaller than the number of dimensions along which contestants compete with each other. The prize(s) must be awarded as a function of the outcomes of a larger number of contests or tournaments, one for each of several tasks of similar importance, such that the contest designer or principal knows in how many tasks each player performed better than his competitor. When opening up the contest for several effort dimensions, Franckx et al. (2004) note that one risks losing one of the main advantages of this type of incentive mechanism, namely its simplicity. If the principal has access to data from several dimensions, then it is not obvious how this should be combined to determine the contest winner. Franckx et al. (2004) sum up the efforts in each dimension and then add a series of shocks to the output signal in the spirit of Lazear and Rosen (1981). Our analysis is close in spirit, and in simplicity, to the standard Tullock (rent-seeking) contest (Tullock, 1980). The principal has information only on the number of dimensions won by each contestant, but not the size of the winning margin. Hence effort aggregation is not an option.

This contest literature concentrates on problems in which the agents perform a single task, or, where performance is measured along one dimension. In many organizational problems, agents have to decide about their overall effort, and how to allocate this effort between different tasks. Often, the competitors will be rewarded according to some aggregate measure of overall performance, and not rewarded for performance in each task. The water regulating authority in the UK, OFWAT, has used price cap regulation to drive efficient water and sewerage provision, and have adopted eight improvement

criteria for drinking water quality. Those water authorities that are significantly better than the industry average have been allowed a price limit increase of 0.5% (OFWAT, 1999). Hence the prize can be thought of as being awarded to the winner of the majority of these criteria, and this is a case captured by our model. The importance of multi-tasking, and the problems this causes in the context of principal-agent theory was first formally studied by Holmstrom and Milgrom (1991). Principals may want agents to spend effort along several dimensions and to pursue several goals, whereas the correlation between input or output along these different dimensions need not be equal, and they need not be equally well observable or contractible.

The problem of multi-tasking may also come together with one or several of the contractual problems that make a contest or tournament the appropriate incentive tool. The agents may compete along several dimensions, providing several types of effort that generate several types of output, and the contest designer may need to decide about the structure of prizes as a function of relative performance along this set of outputs. If each type of effort leads to one different type of output, compared to the problem of the choice of the structure of prizes when contestants compete along one dimension, a tournament designer can essentially choose the number of tournament dimensions along which an agent needs to win in order to win a prize. This is the framework analysed in this paper.

In the analysis we focus on symmetric pure strategy equilibria with the particular property that the same effort is spread out along all dimensions of the contest.[2] If the

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[2] This is analogous to the procedure adopted by Klumpp and Polborn (2006) that they call Symmetric Uniform Campaign Equilibrium.

principal values the sum of agents' effort along each dimension equally and independently, but has decreasing marginal benefits from the agents' sum of efforts in each dimension, holding everything else equal, the principal would like agents to attribute equal effort to each dimension. We focus on the type of equilibria which this principal, loosely speaking, likes most.

In a contest with multiple dimensions, the principal may simply want to award one prize in each dimension, instead of one big prize. However, in some contexts, such as tournaments for promotion in internal labor markets, the prizes are indivisible and absolutely limited in number. Multi-tasking, compared to single-tasking, is then costly for the principal in the tournament context as well. We consider optimal contest design in such a framework, where the principal can optimize only along two design variables. Suppose that participants compete in n symmetric, mutually independent dimensions of output and effort, with a winner in each dimension being determined as in Tullock's (1980) lottery contest. In the contest that we present in this paper, participants compete in a number of dimensions, and the winner must beat the opponent in at least some pre-specified number. The problem is tractable for the case of identical players and equally important dimensions, for which we derive the symmetric pure strategy Nash equilibrium. We present existence conditions for this equilibrium, and look at how the specification of the rule affects incentives to exert effort. We look at the optimal contest design from the point of view of maximizing effort (per dimension or totally). The standard Tullock contest is encompassed in our framework, and provides a natural point of comparison. The principal may specify a rule in which the winner must beat the opponent in a least k

of these dimensions and may choose k to maximize his objective function. If *2k>n* then

this rule establishes that the winner must beat the opponent in the majority of the

dimensions to win, with the size of the required majority increasing as *k* gets closer to *n*.

In these cases there can be at most one winner, but there may be no winner at all. If the

principal chooses *k* with *n≥2k* then a majority is not needed to win, and situations can

arise in which one or both of the participants win a prize.

The multi-tasking contest is a structure that is relevant in many contexts, where the

choice of *k* or *n* may be a design problem in some context, like in sports tournaments, or

in deliberately designed research tournaments, but may also be exogenously given in

other contexts in which the decision is based on a whole set of relative performance

measures, which have been discussed in the theory of contests more generally.[3] Examples

of applications of the model are

 i) committee decision making with *n* members: effort will then be interpreted as the

amount of resources devoted to persuading each committee member. If *k>0.5n* then some

type of majority is needed to reach a decision; if *k=0.5(n+1)* then a simple majority is

sufficient, but decisions of a more fundamental nature – such as changes in constitution –

may need a larger majority.[4] This is also related to the literature on political competition

(e.g. Congleton 1984, Snyder, 1989, Amegashie, 2002).

ii) Beauty contest. In a competitive tendering situation, contestants are compared

according to a number of criteria, and a winner is specified as the firm that beats the

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[3] Examples are litigation (Farmer and Pecorino 1999, Baye, Kovenock and deVries 2005), campaigning (Skaperdas and Grofman 1995), lobbying (Tullock 1980, 1988), or bribing games (Clark and Riis 2000).

[4] Hence our framework is a considerable extension of some of the analysis in Congleton (1984).

opponent in a pre-specified majority of these. As a labour market example, one could consider annual wage negotiations for employees in the Norwegian state. One possibility of achieving a wage increase is in a beauty contest with others at the same institution according to a list of difficult to measure criteria, such as environment-building, research results, and teaching quality. The prize is scarce (and not completely divisible in practice) since only a small proportion of the applicants are awarded a salary increase.

iii) A model of sales. Consider two sales representatives with a list of n potential clients; the firm may use our contest structure to motivate effort by specifying a bonus to the seller with some pre-specified majority of sales.

Section 2 presents the model and solves for the symmetric pure strategy Nash equilibrium. The set of designs that satisfy the identified equilibrium is then investigated in Section 3 from the point of view of effort maximization. A brief conclusion follows in section 4.

2. The model

There are two risk neutral players who compete for a prize that they value at V by making irretrievable efforts at constant unit marginal cost. The effort of player $i = 1, 2$ is spread over n dimensions, and a player must win at least k of these dimensions in order to win the prize. Thus there are feasible contest designs in which both players may win the prize ($n \geq 2k$), or in which there is a single winner or no winner. The effort of player 1 (2) in dimension j is given by x_j (y_j), $j = 1, 2, ...n$, and the probability that player 1 wins dimension j takes a common form:

$$(1) \qquad p_j = \frac{x_j}{x_j + y_j}$$

if at least one effort is positive, and $p_j = (1 - p_j) = 1/2$ if both players choose zero effort along this dimension. From (1) it is clear that the competitions to secure each dimension are independent of each other. Players decide upon their efforts simultaneously at the start of the game, and the outcome of each dimension is then determined by (1) and payoffs are then awarded accordingly.

A key assumption of the analysis here is that a contestant can win at most one prize. The information available to the contest designer is very coarse; he observes only the identity of the contestant(s) that has (have) passed the criterion for the award of a prize. No finer grid of information is needed to implement the contest that we consider in this paper. Franckx et al (2004) state that a multi-task setting loses one of the advantages of contests/tournaments over other mechanisms, namely the limited informational requirement. Indeed, in their model the principal receives a signal in each of the competitive dimensions. In our model we retain the feature from single dimension contests, namely that the principal receives only information on the identity of the winner (rather than information specifying the winning margin in each dimension).

In the model we focus on a symmetric equilibrium in pure strategies. Let this symmetric pure strategy equilibrium be denoted by $\mathbf{x}^* = \mathbf{y}^* = \boldsymbol{\chi} = (\chi_1, \chi_2, ..., \chi_n)$. Then, as is shown in Appendix A, $\chi_1 = \chi_2 = ... = \chi_n$ must hold. This makes room for a simplification of the expected payoff function that can be used to characterize the

equilibrium.[5] We set up the dependence of the payoff for player 1 given that player 2 has a symmetric effort in all of the dimensions, and that player 1 has the same outlay in all dimensions but the first. We then differentiate this expression with respect to x_1 and solve the first order condition for x_1 to find the equilibrium.

Given an expenditure x_1 for player 1 in the first dimension, and a symmetric outlay χ for player 2 in all dimensions, and player 1 in all but the first, the probability that player 1 wins none of the competitions is:

$$(2) \qquad P(0) = \frac{\displaystyle\prod_{j=1}^{n} y_j}{\displaystyle\prod_{j=1}^{n} (x_j + y_j)} = \frac{\chi^n}{2^{n-1}\chi^{n-1}(x_1 + \chi)} = \frac{\chi}{2^{n-1}(x_1 + \chi)}.$$

If player 1 wins exactly one of the dimensions, then it could either be dimension 1 (with probability $\dfrac{x_1 \chi^{n-1}}{2^{n-1}\chi^{n-1}(x_1 + \chi)}$ or it could be one of the other n-1 dimensions with probability $\dfrac{(n-1)\chi^n}{2^{n-1}\chi^{n-1}(x_1 + \chi)}$. Hence the probability of winning exactly one of the dimensions is

$$(3) \qquad P(1) = \frac{x_1 \chi^{n-1} + (n-1)\chi^n}{2^{n-1}\chi^{n-1}(x_1 + \chi)} = \frac{x_1 + (n-1)\chi}{2^{n-1}(x_1 + \chi)}.$$

Proceeding in this way, one can write the total probability of winning exactly $j \geq 2$ dimensions as:

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[5] The general form of the expected payoff function would involve a specification of all of the combinations of winning at least k of n trials, and the corresponding probabilities.

$$(4) \qquad P(j) = \frac{\binom{n-1}{j-1} x_1 \chi^{n-1} + \binom{n-1}{j} \chi^n}{2^{n-1} \chi^{n-1}(x_1 + \chi)} \quad \text{for } j \geq 2$$

where $\binom{n}{j} = \frac{n!}{j!(n-j)!}$ is the formula for the number of combinations of j from n. There

are $\binom{n-1}{j-1}$ combinations of j from n that involve winning dimension 1, and $\binom{n-1}{j}$ that

do not. Let $P \equiv \sum_{j=0}^{k-1} P(j)$ be the probability that player 1 wins less than k of the

dimensions. Then the dependence of 1's payoff on his effort in dimension 1 is captured

by:

$$(5) \qquad \pi_1(x_1, \chi) = (1 - P(x_1, \chi))V - x_1 - (n-1)\chi$$

Maximization of this expression by choice of x_1 yields the following result:

Proposition 1

For contest design (n, k) a symmetric pure strategy Nash equilibrium is characterized as

follows. Each player's effort in each dimension is

$$(6) \qquad \chi(n,k) = \frac{(n-1)!V}{2^{n+1}(k-1)!(n-k)!} \equiv \frac{V\binom{n-1}{k-1}}{2^{n+1}}.$$

The total effort for each player is nχ(n, k) which can be written as

$$n\chi(n,k) = \frac{kV}{2^{n+1}} \binom{n}{k} \equiv \mathrm{X}(n,k).$$

The equilibrium probability of winning is given by

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$$P(n,k) = 1 - \frac{\sum_{j=0}^{k-1}\binom{n}{j}}{2^n}$$

and the equilibrium payoff is $\pi(n, k) = P(n, k)V - n\chi(n, k)$.

A necessary condition for this equilibrium to exist is

(7) $\quad \sum_{j=k+1}^{n}\binom{n}{j} - \frac{(k-2)}{2}\binom{n}{k} \geq 0$.

Proof

Differentiating (5) with respect to x_1 and cancelling terms yields:

(8) $\quad \dfrac{\partial \pi_1}{\partial x_1} = \dfrac{\binom{n-1}{k-1}\chi}{2^{n-1}(x_1 + \chi)^2}V - 1$

Setting (8) equal to zero and evaluating at a symmetric situation ($x_1 = \chi$) yields the effort indicated in the Proposition. Differentiating (8) with respect to x_1 and evaluating at the symmetric situation reveals that this effort maximizes 1's expected profit (5). The expressions for the equilibrium probability of winning and the expected payoff follow directly. The existence condition in (7) secures a non-negative profit in equilibrium. QED

The existence condition in (7) guarantees that the players achieve a non-negative profit in this equilibrium. Examination of this condition reveals that is sets limits for the relationship between the number of dimensions that must be won and the total number of dimensions in the competition. Intuitively, the probability of winning a prize in equilibrium must be large enough when weighed up against the total cost of effort; this

means that *n* must be sufficiently large in relation to *k*. There does not seem to be a

single relationship between *n* and *k* that satisfies (7), but Table 1 presents results of

simulations of the existence condition for *20≥n≥1*. Figure 1 presents numerical results

for larger values of *n*. For a given *n*, *k*(n)* represents the maximum number of

dimensions that must be won that is consistent with existence of the symmetric

equilibrium. In Figure 1, contest designs on and below the line are commensurate with

the equilibrium in Proposition 1. In the further analysis we shall refer to *E* as the set of

contest designs that satisfy the criterion in (7). Appendix B gives a summary of the

designs that satisfy the existence condition in (7) for *n≤20*.

Table 1

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
k*(n)	1	2	2	2	3	3	3	4	4	5	5	5	6	6	7	7	7	8	8	9

Figure 1

 About here

From Proposition 1 it can be noted that the contest design *(1, 1)* is the usual Tullock case,

yielding an effort per player of $\chi(1, 1) = V/4$. This contest design naturally satisfies the

existence condition in (7). We now turn to the issue of how the pair $(n, k) \in E$ affects the

amount of effort expended in the contest.

3. The optimal design

3.1 Fixed k

Suppose that the contest designer has specified a rule for how many dimensions a player must succeed in to win a prize. Given this k, we can calculate how the number of dimensions affects outlay per dimension, and the total outlay; furthermore we can calculate the k that maximizes these magnitudes within E. Holding k fixed and increasing the number of dimensions from n to $n+1$ has the following effect on effort per dimension, and the total effort:

$$(9) \qquad \chi(n+1,k) - \chi(n,k) = \frac{V(n-1)!(2k-2-n)}{2^{n+2}(n-k)!(k-1)!(n-k+1)}$$

$$(10) \qquad (n+1)\chi(n+1,k) - n\chi(n,k) = \frac{V(n-1)!n(2k-1-n)}{2^{n+2}(n-k)!(k-1)!(n-k+1)}.$$

From (9) it is clear that the change in effort per dimension is positive until *n=2k-2* upon which the addition of another dimension (to *2k-1*) does not change the effort, but further additions reduce the effort in each dimension. Equation (10) indicates that total effort increases in *n* until *n=2k-1* upon which the addition of another dimension (to *2k*) has no effect on the total effort; adding more dimensions than this reduces the total effort.

We must, however, check whether these maxima actually exist, i.e. if the efforts are consistent with the equilibrium so that the considered designs are in set *E*. Define *n*(k)* as the minimum number of dimensions that must be present for the equilibrium to exist as

defined by (7).[6] For *k=1*, equilibrium exists for *n≥1* so that *n=2k-1* indeed maximizes

effort per dimension, and *n=2k-1* and *n=2k* maximizes their sum. For *k=2* we have

n*(2)=2 so that the designs that maximize efforts are feasible. For *k=3* it is the case that

n*(3)=2k-1 so that this is the design that maximizes effort per dimension; the maximal

efforts for *k=3* are consistent with equilibrium. For *k≥4*, the existence condition in (7) is

not fulfilled at the *n* that maximizes efforts; hence *n* must be increased to the lowest level

that is consistent with equilibrium (since efforts are decreasing in *n* in this region). The

required *n* is thus *n*(k)*.

These results are summed up in Proposition 2, and in the table in Appendix B.

Proposition 2

Consider designs in set E and fix the number of dimensions that must be won to secure a

prize at k. If k=1 then the effort per dimension is maximal for n=1, and the total effort is

maximal for n=1 and n=2. If k = 2, the effort per dimension is maximized for n=2 and

n=3, and the total effort per player is maximized for n=3 and n=4. For k=3, then the

effort per dimension is maximized for n=5, and the total effort per player is maximized

for n=5 and n=6. These are all unconstrained maxima. For k≥4, only the constrained

maximum is attainable; total effort and the effort per dimension is maximized for n=n*(k).

Given the designs in set *E,* if only a single dimension must be won, then efforts per

dimension and total effort are maximized for the standard Tullock contest design with

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[6] This information can be read from Table 1 by looking at the first occurrence of a number in the second
row and finding the value for *n* directly above this in the table: *n*(1)=1, n*(2)=2, n*(3)=5, n*(4)=8* etc.

competition in a single dimension. Adding further dimensions increases the chances of winning for a given effort and this makes the competitors reduce their effort and cost in each dimension. Adding a second dimension reduces output in each but does not affect the total. Hence the Tullock contest yields an equivalent total effort to the design *(2, 1)*; the former only permits one winner, while the latter permits up to two. If a prize is awarded to a contestant that wins at least two dimensions, then the designs *(2, 2)* and *(3, 2)* yield the maximal effort per dimension and the designs *(3, 2)* and *(4, 2)* maximize the total effort. For *k=2* then one can conclude that the design *(3, 2)* is optimal in the sense that it maximizes both definitions of effort and secures that only a single prize is awarded. When *k=3* the corresponding optimal design is *(5, 3)* for the same reason.

For larger values of *k*, the existence condition constrains the maximum, so that the lowest *n* that is consistent with equilibrium maximizes efforts. Numerical simulations have shown that for *k≥4*, *n*(k)* can be written in the form *n*(k)=2k+i* where *i≥0* and *i* increases periodically as *k* increases.[7] This means that in order to maximize effort in the contest requires contest designs that potentially permit two winners. As the number of successes needed to win gets larger, the probability of winning for a given effort gets reduced; to balance this, the number of dimensions in the contest must be increased. To ensure existence it must be potentially possible for both players to win a prize.

For designs in *E,* maximizing effort per dimension gives rise to the emergence of a specific pattern when *k* is fixed; when faced with a choice between a single prize and a

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[7] It does not appear that the period is constant, nor does it follow a specific pattern, however. It is for this reason that we present numerical results.

two-prize system, it is optimal for the principal to instigate the single-prize structure if this is in *E*. If only two-prize systems are commensurate with equilibrium then the lowest possible *n* is chosen. To maximize total effort the division between the prize systems is not so sharp; in cases where it is feasible to implement a single- and a two-prize system, then the highest *n* that gives a single prize and the lowest value that involves two prizes both maximize total effort.

3.2 Fixed n

In many applications it is the number of dimensions in the competition that will be fixed at *n*; how should the contest designer set the number of successes on which to base the awarding of the prize? Treating *k* as a continuous variable, the sign of $\dfrac{\partial \chi(n,k)}{\partial k}$ is the same as the sign of *(n-2k+1)*; hence this function reaches its maximum at *k=0.5(n+1)*. When *n* is odd then this maximum can be reached exactly if the equilibrium exists, but when *n* is even, the efforts to either side of this *k* are maximal since *k* is an integer. This can also be seen directly since the sign of the change in effort (and total effort since *n* is fixed here) is given by

$$(11) \qquad \chi(n,k+1) - \chi(n,k) = \frac{V(n-1)!(n-2k)}{2^{n+1}(n-k)!k!} = \frac{V\binom{n-1}{k}(n-2k)}{2^{n+1}}$$

so that effort is increased by adding more dimensions if *k<0.5n*. The change is zero between *k=0.5n* and *k=0.5n+1*, and negative thereafter. Since the number of dimensions *n* is fixed, designs in *E* that maximize effort per dimension will also maximize effort in sum. However, the unconstrained maximum can only be achieved if the design satisfies

(7). Referring to Appendix B, we see that for $n=\{1,3,4,5,6,8,10\}$ there is a single k in each case that achieves the unconstrained effort maximum. From Table 1 and Appendix B it is apparent that the maximizing k in each of these cases is $k^*(n)$. When $n=2$ both $k=1$ and $k=k^*(n)=2$ give the same maximum effort level. For $n=\{7,9\}$ and $n\geq11$, the condition in (7) constrains candidates for k that maximize effort. Since $\chi(n,k)$ can easily be verified to be a function that is strictly concave in k and that is symmetric around its maximum, k in these cases should be as large as (7) allows to achieve the constrained maximum. The largest possible k in each case is $k^*(n)$.

Proposition 3 sums up this set of results.

Proposition 3

Suppose that n is fixed. For designs in E, effort per dimension and aggregate effort are maximized for k=k*(n); for n=2 the maximum can also be achieved for k=k*(2)-1(=1). For n={1,3,4,5,6,8,10} the unconstrained maximum can be achieved. For other n, equation (7) constrains the maximum that can be attained.

From table 1 we see that for $n\geq6$, $k^*(n)<0.5n$, and numerical simulations confirm this for larger n. Hence the effort maximizing choice of k in these contests is such that there must be the possibility that there can be two winners. For $n=6$, for instance, efforts are maximized for $k^*(6)=3$ so that two winners occur if each wins half of the dimensions. As n grows, the potential for two winners must increase for the constrained maximum for

effort to be consistent with equilibrium; *k*(50)=21* for example, so that there are many outcomes of the 50 dimensions in which both players will receive a prize.

Similar to the discussion around Proposition 2, there is a divide between single-prize systems and those that involve potentially two prizes. For a given *n*, *k* should be increased until a single-prize design is achieved. If this is not feasible then the largest available *k* should be chosen to minimize the probability that two prizes actually have to be awarded.

The figures in the table in Appendix B, together with Propositions 2 and 3 indicate that - for the designs in *E* - total effort can always be increased by increasing the size of the contest (i.e. simultaneous increases in *n* and *k*). This gives more total effort, but spread increasingly thinly across all dimensions. This leads to the following result:

Proposition 4

Let $\bar{n}$ be the largest possible value of n. Within E, to maximize total effort in the contest, the principal should choose the design $(\bar{n}, k*(\bar{n}))$. To maximize effort per dimension, the design (1,1) should be chosen.

Proposition 4 can be used to help explain the conundrum posed by Tullock (1988) that theoretical models predict more rent-seeking activity than is observed in practice. As analysts, we may only observe the identity of the winner of a contest, and have often assumed that this has resulted from competition in a single dimension. A principal

wanting to maximize total effort, however, will prefer a large contest with competition in several dimensions; the theoretical model from the one-dimensional case will overstate the true amount of rent-seeking.

4. Conclusion

The standard Tullock model of a contest has seen many applications and extensions. The limited amount of information needed by the principal to implement such an incentive system has made this structure popular in applied work also. When several effort dimensions are considered, one immediately opens up for many possibilities regarding the information known by the principal, and how this translates into a winning competitor. In this paper we have used a framework that is as close to the original as possible, given the extension of multiple effort dimensions. The information assumed available to the principal here can be very coarse – as in the Tullock model – consisting only of a number of dimensions won by each competitor; in addition, and again in common with the Tullock framework, the prizes are fixed in size. The optimal designs that we have presented ultimately rest on these assumptions. If the prizes are divisible and the principal knows which competitor has won each dimension, then he may design an incentive scheme that rewards competitors accordingly. If information is available on the winning margin in each dimension, then this may further be built in to the incentive scheme. We believe that these types of contests should be investigated in future work. However, one should bear in mind that the complexity of the contest will likely increase its administration cost. Our analysis has been an attempt to open up the multi-task issue in

contests, while at the same time preserving the main advantage if this type of structure:

simplicity.

Appendix A

We show the following

Lemma *If* **x** *and* **y** *with* **x** = **y** *are the effort vectors that characterize a symmetric equilibrium in pure strategies, then* $x_i = y_i = y_j = x_j$ *for all* i, j *in* $\{1,2,...,n\}$.

For a proof, note first that a symmetric equilibrium in pure strategies cannot have zero effort along all dimensions if the prize has a strictly positive value, as each contestant can increase the probability for winning the prize from some probability strictly smaller than 1 to 1 by an infinitely small amount of effort.

Note further that different effort along different dimensions can also not be an equilibrium. We show this by way of a contradiction. Consider a candidate equilibrium **x** = **y** = χ with $x_1 = y_1 < y_2 = x_2$ and $x_1 > 0$ (the case $x_1 = 0$ can be treated along similar lines) in a contest with n dimensions and a requirement to win at least $k \geq 2$ dimensions in order to win a prize. Let $\bar{p}(k-i)$ be the probability that player 1 wins $k-i$ of the contests along the dimensions 3,4,...n. The outcome in the contests along dimensions 1 and 2 are payoff relevant for player 1 only if the outcome can increase the number of dimensions which the player wins from below k to k or above. Accordingly, the candidate equilibrium can be an equilibrium only if there is no reallocation of the effort $x_1 + x_2$ between the two dimensions 1 and 2 by which the player can increase his overall probability of winning at least k contests. Hence, a necessary condition for the candidate to constitute an equilibrium is that

$$\frac{x_1}{x_1 + y_1} \frac{x_2}{x_2 + y_2} \bar{p}(k-2) + (1 - \frac{y_1}{x_1 + y_1} \frac{y_2}{x_2 + y_2})\bar{p}(k-1)$$

cannot be increased, for instance, by a small increase $dx_1 = -dx_2 > 0$. The impact of such

a shift of effort is

$$(\frac{y_1}{(x_1 + y_1)^2} \frac{x_2}{x_2 + y_2} - \frac{x_1}{x_1 + y_1} \frac{y_2}{(x_2 + y_2)^2}) \bar{p}(k - 2)$$

$$- (\frac{-y_1}{(x_1 + y_1)^2} \frac{y_2}{x_2 + y_2} - \frac{y_1}{x_1 + y_1} \frac{-y_2}{(x_2 + y_2)^2}) \bar{p}(k - 1).$$

Evaluating this term at $x_1 = y_1$ and $x_2 = y_2$ reduces it to

$$(\frac{1}{4x_1} \frac{1}{2} - \frac{1}{2} \frac{1}{4x_2}) \bar{p}(k - 2) +$$

$$+ (\frac{1}{4x_1} \frac{1}{2} - \frac{1}{2} \frac{1}{4x_2}) \bar{p}(k - 1) .$$

Accordingly, each of these terms is positive if $x_1 < x_2$. As this contradicts the optimality

of the strategy x in the candidate equilibrium, the efforts must be distributed uniformly

across all dimensions in a symmetric equilibrium in pure strategies.

Appendix B

Contest designs that satisfy (7), i.e $(n,k) \in E$ (for $n \leq 20$)

k,n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	$\frac{1}{4}, \frac{1}{4}$	$\frac{1}{8}, \frac{1}{4}$	$\frac{1}{16}, \frac{3}{16}$	$\frac{1}{32}, \frac{1}{8}$	$\frac{1}{64}, \frac{5}{64}$	$\frac{1}{128}, \frac{3}{64}$														
2	-	$\frac{1}{8}, \frac{1}{4}$	$\frac{1}{8}, \frac{3}{8}$	$\frac{3}{32}, \frac{3}{8}$	$\frac{1}{16}, \frac{5}{16}$															
3	-	-	-	-	$\frac{3}{32}, \frac{15}{32}$	$\frac{5}{64}, \frac{15}{32}$														
4	-	-	-	-	-	-	-	$\frac{35}{512}, \frac{280}{512}$												
5	-	-	-	-	-	-	-	-	-	$\frac{126}{2048}, \frac{1260}{2048}$										
6	-	-	-	-	-	-	-	-	-	-	-	-								
7	-	-	-	-	-	-	-	-	-	-	-	-	-	-						
8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-				
9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Combinations of *k* and *n* that do not satisfy (7) are denoted by -. The first figure in each cell is effort per player per dimension, and the second is total effort per player for some example designs.

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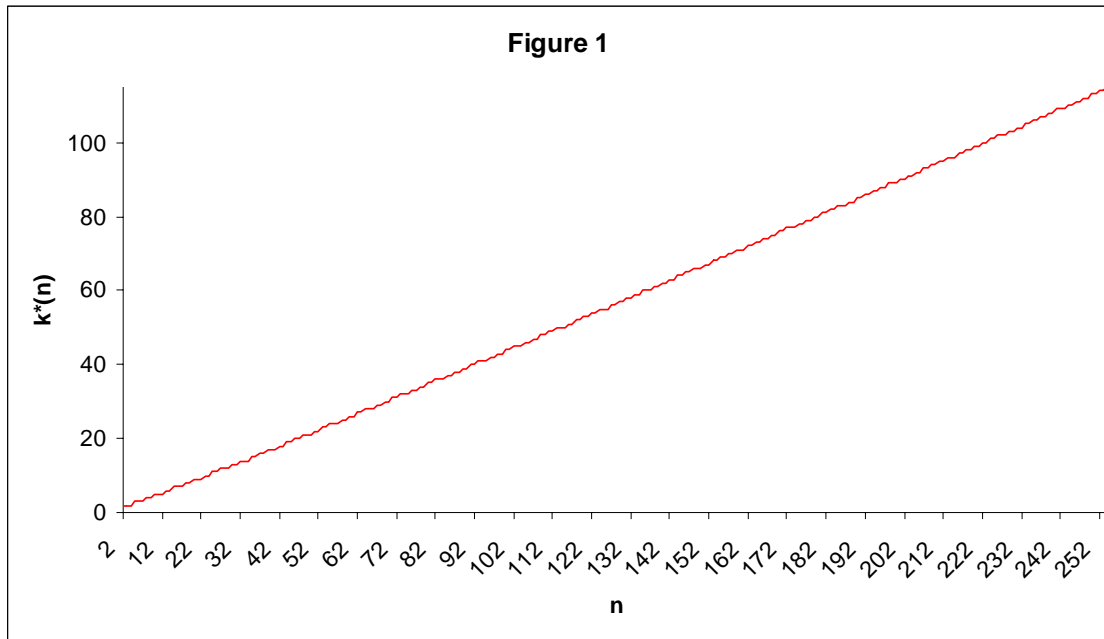
Figure 1: Relationship between the number of dimensions (n) and the maximum amount

of dimensions that must be won for existence of equilibrium ($k*(n)$)