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License auctions with exit (and entry) options: Alternative remedies for the exposure problem☆

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Abstract

Inspired by some spectrum auctions, we consider a stylized license auction with incumbents and one entrant. Whereas the entrant values only the bundle of several units (synergy), incumbents are subject to non-increasing demand. The seller proactively encourages entry and restricts incumbent bidders. In this framework, an English clock auction gives rise to an exposure problem that distorts efficiency and impairs revenue. We consider three remedies: a (constrained) Vickrey package auction, an English clock auction with exit option that allows the entrant to annul his bid, and an English clock auction with exit and entry option that lifts the bidding restriction if entry failed.

Keywords: Auctions, package auctions, combinatorial clock auctions, spectrum auction, bundling, synergies.
2000 MSC: D21, D43, D44, D45G34.

1. Introduction

Radio spectrum is an essential input to deliver mobile voice and data service and is required for capacity and coverage purposes. If an operator does not gain sufficient access to spectrum, he may not be able to deliver service and recover his cost. During the early years of the mobile phone industry, operators were awarded radio spectrum either by a managed assignment process or a beauty contest. However, during the past 15 years, regulators all over the world have successfully employed auctions to award spectrum which however pose their own challenges.

In spectrum auctions the regulator typically faces several concerns that are difficult to reconcile:

- the regulator favors entry, but not at all costs

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- entrants require a minimum endowment of spectrum, a spectrum floor, which gives rise to increasing demand (synergy effect)
- synergies may give rise to an exposure problem which in turn leads to cautious bidding, a low probability of entry, and low revenue
- revenue matters, not only because debt-ridden governments are short of revenue but also because license fees are a distortion free method of taxation.

Typically, the regulator responds to these concerns by restricting the bidding rights of incumbent bidders or by adding provisions that deal with the exposure problem, for example by adopting package or combinatorial clock auctions (CCA).[1] Although some regulators proactively encouraged entry without regard of cost by reserving some designated lots for entrants.[2]

An interesting alternative response to these concerns was adopted in the 3G and 4G spectrum auction in Germany in the years 2000 and 2010. There bidders were entitled to state a minimum requirement for spectrum, with the provision that if a bidder ends up winning less than his stipulated minimum, his bids are annulled. In addition, provisions were made to re-auction any left-over (or "stranded") spectrum to minimize the risk that spectrum is wasted.[3]

The present paper is inspired by these rules and contributes to evaluate their merit. In particular, we ask the questions: does an "exit option" that allows an entrant to annul his bids if a minimum requirement is not satisfied resolve the entrants' exposure problem?; does an "entry option" that allows an incumbent to acquire stranded spectrum if entry has failed assure that no spectrum is wasted without compromising the preference for entry? And how does an auction that includes exit and entry options perform in terms of entry and seller's revenue compared to a (constrained) Vickrey package auction that is usually recommended as a remedy for the exposure problem?

We address these issues in the framework of a stylized model with one entrant and two incumbents. The entrant is subject to increasing demand (synergy), due his need for a minimum endowment, whereas incumbents already own complementary spectrum and are subject to flat (or decreasing) demand. The seller favors entry and restricts incumbent bidders. The seller auctions two blocks of spectrum of which each incumbent prefers one (because they may be neighbors to already owned spectrum).

We first consider a constrained English auction and show that it leads to a double exposure problem for the entrant, because in equilibrium he may win

[1] Package auctions have been used recently, for example in Austria, Denmark, France, India, the Netherlands, Nigeria, the UK, and Switzerland. Some of these were sealed-bid first-price auctions (Austria, France, Nigeria) and others (like Switzerland) employed the CCA format (for a detailed survey of auction formats employed in recent 4G auctions, see Nett and Stumpf, 2011).

[2] This policy was adopted in the 3G auction in the U.K. in the year 2000, and more recently in post-3G auctions in the Czech Republic and in the Netherlands.

[3] In the 3G auction that minimum requirement was 2 blocks of 2×5 MHz spectrum in the 1900-2025 MHz band. This requirement was requested by the industry on the ground that building a 3G network involves a fixed cost in the order of 8 billion DM (roughly 4 billion Euros), which can only be recovered if with sufficient capacity. The 4G auction did not attract entrants and incumbents did not stipulated a minimum requirement.

only one object which has no value or he may win both objects but pay more than the bundle value.

Both problems can be remedied by allowing the entrant to annul his bid if he failed to win a stipulated number of units ("exit option"). Moreover, we show that adding an entry option that lifts bidding restrictions if entry has failed, further improves the performance. Our main result is that the thus amended English auction performs better than the constrained Vickrey package auction.

There is a small literature on synergies in auctions (see Krishna and Rosenthal, 1996; Rosenthal and Wang, 1996). Package auctions were designed to deal with synergies in auctions and the resulting exposure problems. The classical reference is Ausubel and Milgrom (2002); a comprehensive collection of contributions is in Cramton et al. (2006).

For an account of the 3G auction in Germany see Grimm et al. (2004), van Damme (2002), and Klemperer (2002, 2004), and for the rules of the 4G auction see Nett and Stumpf (2011).

The plan of the paper is as follows: After stating the model in section 2 we analyze a standard two-clock English auction in Section 3 and the (constrained) Vickrey package auction in section 4. The English clock auction is amended by an exit option for the entrant in Section 5, and by adding an entry option for the successful incumbent in Section 6. The robustness of our analysis is discussed in Section 7. The paper concludes with a brief discussion in Section 8.

2. Model

Consider a stylized license auction with two incumbent bidders (A and B), one potential entrant bidder (E), and two objects for sale (a and b). The entrant E is free to bid on both objects and values only the bundle of a and b. In other words, E is subject to a strong synergy (the value of two objects is greater than the sum of the values for each object alone). Incumbents are subject to a spectrum cap which restricts them to bid on only one object (A wants a and B wants b), and if that restriction is lifted they are subject to flat (or decreasing) demand.

Bidders' valuations V_A, V_B, V_E are *i.i.d.* random variables[4] drawn from the c.d.f. F with support $(0, 1)$, and their realizations are their private information. V_A, V_B denotes incumbents' valuations for their preferred object (their valuations for the the bundle are $2V_A, 2V_B$); the entrant's valuation for the bundle is $2V_E$ whereas his valuation for a single object is equal to zero. F is continuously differentiable and with *p.d.f.* $f(v) > 0$ for all $v \in (0, 1)$.

The seller's own valuation is normalized to zero and the seller neither uses strategic reserve prices nor entry fees.

The two objects are auctioned simultaneously, using either an English clock auction with two clocks or a Vickrey package auction.

Two variations of the base model English clock auction are considered:

1) English clock auction with exit option: the entrant has the option to exit if he fails to win both objects. If that option is exercised, one object is not

[4] As a rule, random variables are denoted by capital letters and their realization by lower case letters.

sold and the seller collects payments only from the one incumbent who won one object.[5]

2) English clock auction with exit plus entry options: the auctioneer lifts the restriction on bidding if the entrant exercised his exit option. In that case, the incumbent who won the first object has the option to buy the second object at the price he paid for the first.

In spectrum auctions, the assumed synergy applies because entrants can only operate profitably if they acquire a minimum amount of spectrum, due to the high fixed cost of building a network of radio stations, customer base, and billing system. The regulator often has a preference for entry and thus restricts the bidding rights of incumbents.[6] However, the regulator wants to award all available spectrum and therefore makes provisions to reallocate stranded spectrum among incumbents if entry has failed.

3. English (two-)clock auction

In the English (two-)clock auction bidders indicate their demand for object a or b at given clock prices. Price clocks start at zero, and go up continuously in response to excess demand. If initially there is excess demand for both objects, both price clocks go up in tandem; but once there is excess demand for only one object, one price clock keeps going up while the other stands still. The auction terminates if there is no excess demand. Then, the remaining active bidders are awarded the respective object at prices equal to the terminal reading of the respective price clock.

There are several possible outcomes:

The entrant may be the first to quit. In that case, each incumbent wins his favored object and pays the price at which the entrant quit.

One incumbent, say A, may be the first to quit. Then, price clock a stops and E is preliminary winner of a, while bidding continues on b. If thereafter the entrant is the first to quit auction b, the auction ends: B wins b at the price at which E quit and E wins a at the price at which A quit and E suffers a loss. Whereas if B quits auction b before E quits, E wins both objects and pays the prices at which the two incumbents had quit.

The entrant suffers a winner's curse if he wins only one object, which is useless for him, and if he wins both objects but pays more than the bundle value. As we will show below, in equilibrium both kinds of winner's curse occur with positive probability, which in turn induces the entrant to play a cautious initial stop rule, to the dismay of the seller.

The details of the solution of the game are as follows:

Bidders' strategies are stop rules. Incumbents bid only on their favored object; therefore, their bid strategy is a stop rule for their favored object that prescribes incumbent $i \in \{a, b\}$ to quit auction i at price $\beta_i(v_i)$. The entrant's strategy is a collection of stop rules $(s(v_E), \sigma(v_E, p))$: 1) the initial stop rule that prescribes to quit bidding at price $s(v_E)$ as long as no incumbent has quit

[5] Alternatively, the exit option can be replaced by a rule that automatically annuls a bid if the entrant did not meet the stated minimum requirement.

[6] An alternative to such spectrum caps has been used recently in Greece. There, entrants were granted to right to buy an essential endowment of spectrum at fixed prices prior to the auction.

before and 2) the continuation stop rule that prescribes to quit the remaining active auction at price $\sigma(v_E, p)$, after one incumbent has quit first at price p.

Evidently,

Proposition 1. *Incumbents have the dominant strategy: bid on your favored object up to its value, $\beta_i(v_i) = v_i, i \in \{a, b\}$, and the entrant has the dominant continuation strategy: having won one object at price p, bid on the other object up to the bundle value, $\sigma(v_E, p) = 2v_E$ for all p.*

Once the entrant has won one object, he is driven to "overbid" on the second object. The entrant is thus exposed to a double winner's curse problem: he risks winning only one object and he risk winning the bundle at a price that exceeds its value,[7] and in either event suffers a loss.

The entrant responds to this double exposure problem by playing a cautious initial stop rue, $s(v_E)$, that exhibits bid shading to counteract the equilibrium overbidding for the second object, as follows:

Proposition 2. *The entrant plays a cautious initial stop rule that exhibits bid shading, $s(v_E) < v_E$. That stop rule is strictly increasing, approaches v_E as v_E approaches 1, and solves the "break-even" condition:*

$$s(v_E) + E\left[V \mid V \in (s(v_E), 2v_E)\right] = 2v_E, \quad \textit{for all } v_E. \tag{1}$$

That condition prescribes to bid up to that level at which the updated expected cost of the bundle matches the bundle value.

Proof. Given the continuation strategy $\sigma(v_E) = 2v_E$ and incumbents' equilibrium strategy, the entrant's expected payoff is a function of the initial bid s, as follows (where $f_{(12)}(y, z)$ denotes the joint *p.d.f.* of the two order statistics of incumbents' *i.i.d.* valuations)[8]:

$$\begin{aligned}\pi_E(s) = 2v_E \int_0^s \int_z^{\min\{2v_E,1\}} f_{(12)}(y,z)dydz - \int_0^s \int_z^1 zf_{(12)}(y,z)dydz \\ - \int_0^s \int_z^{\min\{2v_E,1\}} yf_{(12)}(y,z)dydz.\end{aligned} \tag{2}$$

The equilibrium strategy $s(v_E)$ must solve the best-reply condition: $s(v_E) = \arg\max_s \Pi_E(s)$. Using the first-order condition of these maximization problems yields (1).

To show that $s(v_E)$ exhibits bid shading and approaches v_E from below as v_E approaches 1, suppose $v_E \geq 1/2$. Then, the first-order condition of the best-reply problem, $\partial_s \pi_E(s) = 0$, can be assessed as follows:

$$\begin{aligned}0 &= 2v_E\left(1 - F(s)\right) - \int_s^1 (s + y)dF(y) \\ &< 2v_E\left(1 - F(s)\right) - 2s\left(1 - F(s)\right) \\ &= 2\left(v_E - s\right)\left(1 - F(s)\right).\end{aligned} \tag{3}$$

[7]Note, this can only happen if $v_E < 1/2$; for, if $v_E \geq 1/2$, $2v_E$ exceeds incumbents' valuation with probability 1.

[8]That joint *p.d.f.* is $f_{(12)}(y,z) = 2f(y)f(z)$ for $y \geq z$ and equal to zero otherwise.

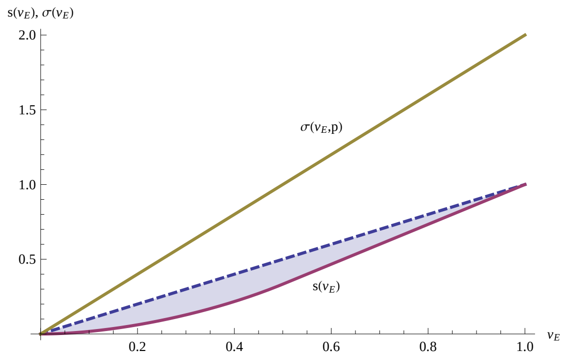

Figure 1: Entrant's equilibrium strategies, $s(v_E), \sigma(v_E, p)$, for the uniform distribution

Hence, $s(v_E) < v_E$ for all $v_E \geq 1/2$ (the proof for $v_E < 1/2$ is similar and hence omitted). Moreover, as $v_E \to 1$ the first-order condition (3) implies $s(v_E) \to v_E$, because by L'Hôpital's rule $\lim_{s\to 1}\left(\frac{1}{1-F(s)}\int_s^1 ydF(y)\right) = 1$.

Differentiating the identity (1) in v_E with respect to v_E confirms the strict monotonicity of s:

$$s'(v_E) = \frac{2\left(F(2v_E) - F(s(v_E))\right)}{1 - F(s(v_E)) + 2\left(v_E - s(v_E)\right) f\left(s(v_E)\right)} > 0.$$

□

If F is the uniform distribution, $s(v_E)$ takes the form:

$$s(v_E) = \begin{cases} \frac{1}{3}\left(1 + 2v_E - \sqrt{4v_E(1-2v_E)+1}\right) & \text{if} \quad v_E \in [0, 1/2] \\ \frac{1}{3}\left(4v_E - 1\right) & \text{if} \quad v_E \geq 1/2. \end{cases}, \tag{4}$$

which is plotted in Figure 1. There, the shaded area indicates the extent of bid shading during the first phase of the auction.

To compute the seller's equilibrium expected revenue, one needs to distinguish between three kinds of events and associated equilibrium prices: 1) $s(V_E) < \min\{V_A, V_B\} \Rightarrow$ the entrant quits first at prices equal to $s(v_E)$; therefore each incumbent wins one object; seller's revenue $\Pi_0 = 2s(V_E)$.

2) $s(V_E) > \min\{V_A, V_B\}$, $2V_E < \max\{V_A, V_B\} \Rightarrow$ one incumbent quits first and the entrant quits second; therefore, one object is won by the entrant and one by an incumbent; seller's revenue $\Pi_0 = \min\{V_A, V_B\} + 2V_E$.

3) $s(V_E) > \min\{V_A, V_B\}$, $2V_E > \max\{V_A, V_B\} \Rightarrow$ the entrant wins both objects; seller's revenue $\Pi_0 = V_A + V_B$.

Therefore, the seller's equilibrium expected revenue, π_0, and equilibrium probability of entry, ρ, are equal to

$$\begin{aligned}\pi_0 = &\int_0^1 \int_{s(v)}^1 \int_z^1 2s(v) f_{(12)}(y,z) f_E(v) dydzdv \\ &+ \int_0^1 \int_0^{s(v)} \int_{\min\{2v,1\}}^1 (z+2v) f_{(12)}(y,z) f_E(v) dydzdv \\ &+ \int_0^1 \int_0^{s(v)} \int_z^{\min\{2v,1\}} (z+y) f_{(12)}(y,z) f_E(v) dydzdv\end{aligned} \tag{5}$$

$$\begin{aligned}\rho &= \Pr\{V_{(2:2)} \leq s(V_E) \wedge V_{(1:2)} \leq 2V_E\} \\ &= \int_0^1 \int_{s(v)}^{\min\{2v,1\}} \int_0^{s(v)} f_{(12)}(y,z) f(v) dz dy dv.\end{aligned} \tag{6}$$

If F is the uniform distribution, the seller's expected revenue is $\Pi_0 \approx 0.598$ and the probability of entry is $\rho = 1/2$.

Altogether, the English clock auction is unfavorable for the entrant because it exposes him to a double winner's curse problem. Because the entrant responds to this by playing a cautious initial stop rule, it also tends to be unfavorable for the seller's revenue. Therefore, it is in the interest of the entrant as well as of the seller to remedy the exposure problem.

4. (Constrained) Vickrey package auction

In the Vickrey package auction the auctioneer selects the allocation that maximizes the sum of valuations, and the winner(s) have to pay a price equal to the sum of valuations crowded out by their participation; like in a single-object Vickrey auction, truthful bidding is an equilibrium (see Ausubel and Milgrom, 2002).[9]

In order to be comparable we consider this auction subject to the regulatory constraint that incumbents cannot be awarded more than one object. As a benchmark we will also consider the unconstrained Vickrey package auction without regulatory constraint.

4.1. Constrained Vickrey package auction

In order to characterize the equilibrium outcome and compute the seller's equilibrium expected revenue, Π_0, one needs to distinguish between the following events and associated equilibrium prices:

1. $2V_E > V_A + V_B \Rightarrow$ the entrant wins the bundle and pays the incumbents' valuations which he crowds out; seller's revenue $\Pi_0 = V_A + V_B$.

2. $2V_E < V_A + V_B \Rightarrow$ the incumbents win their favored objects.

a) $\min\{V_A, V_B\} > 2V_E \Rightarrow$ no single incumbent crowds out the entrant; seller's revenue $\Pi_0 = 0$.

b) $\max\{V_A, V_B\} > 2V_E > \min\{V_A, V_B\} \Rightarrow$ only the high value incumbent crowds out the entrant; seller's revenue $\Pi_0 = 2V_E - \min\{V_A, V_B\}$.

c) $\max\{V_A, V_B\} < 2V_E \Rightarrow$ each incumbent crowds out the entrant; seller's revenue $\Pi_0 = 4V_E - (V_A + V_B)$.

Therefore, the seller's equilibrium expected revenue, π_0, and the equilibrium

[9] We do not consider the open ascending bid package auctions that were designed to simplify bidding and deter strategic manipulations. The most frequently used form of this auction is the combinatorial clock auction designed by dotecon. There, two stages are distinguished: In the first stage, bidders bid on their preferred bundle until there is no excess demand; in the second stage, bidders may make one final bid on all possible bundles, subject to complex restrictions that reflect their history of bidding during the first stage (see Marsden and Siong, 2010). A good example is the recent Swiss 4G auction (see BAKOM, 2010).

probability of entry, ρ, are equal to (the lines corresponds to the order of events):

$$\begin{aligned}\pi_0 = &\int_0^1 \int_0^{\min\{v,1\}} \int_z^{\min\{2v-z,1\}} (z+y) f_{(12)}(y,z) f(v) dy dz dv \\ &+ \int_0^1 \int_{\min\{v,1\}}^{\min\{2v,1\}} \int_z^{\min\{2v,1\}} (4v-z-y) f_{(12)}(y,z) f(v) dy dz dv \\ &+ \int_0^1 \int_0^{\min\{2v,1\}} \int_{\min\{2v,1\}}^1 (2v-z) f_{(12)}(y,z) f(v) dy dz dv \\ &+ \int_0^1 \int_0^{\min\{v,1\}} \int_{\min\{2v-z,1\}}^{\min\{2v,1\}} (4v-z-y) f_{(12)}(y,z) f(v) dy dz dv\end{aligned} \tag{7}$$

$$\begin{aligned}\rho &= \Pr\{V_A + V_B < 2V_E\} \\ &= \int_0^1 \int_0^1 \Pr\{V_A \leq 2v - x\} f(x) f(v) dx dv \\ &= \int_0^1 \int_0^1 F(v-x) f(x) f(v) dx dv.\end{aligned} \tag{8}$$

If F is the uniform distribution one finds $\pi_0 = 7/12 \approx 0.583$, which is less than the expected revenue in the above English clock auction, and $\rho = 1/2$.

Altogether, the Vickrey package auction eliminates the exposure problem because it assures that the entrant never wins only one object and never pays more than the bundle value, and it assures that no object is stranded. However, the example suggests that it performs poorly in terms of generating revenue to the seller. Moreover, as is well-known from the literature, the Vickrey package auction has other deficiencies such as its vulnerability to collusion and shill-bidding (see Ausubel and Milgrom, 2002; Bulow and Milgrom, 2009; Rothkopf, 2007).

4.2. Unconstrained Vickrey package auction (benchmark)

As a benchmark we also consider the unconstrained Vickrey package auction, which implements the value maximizing allocation without regard for entry. Due to the assumed symmetry, in that case both objects are awarded to the bidder who has the highest valuation. Because in equilibrium bidders bid truthfully, the seller earns twice the second highest valuation. Therefore, the seller's equilibrium expected revenue, π_0, and the equilibrium probability of entry, ρ, are[10]

$$\pi_0 = 2 \int_0^1 y f_{(2:3)}(y) dy \tag{9}$$

$$\rho = \Pr\{V_{(1:2)} \leq 2V_E\} = \int_0^1 \int_z^1 \int_y^1 f_{(12)}(y,z) f(v) dv dy dz \tag{10}$$

In order to compare the profitability of the constrained and the unconstrained Vickrey package auction it is useful to apply the same decomposition

[10]There, $f_{(2:3)}(y)$ denotes the *p.d.f.* of the second highest order statistic of a sample of three *i.i.d.* random variables.

of the state space as in (7), and one finds:

$$\pi_0 = \int_0^1 \int_z^1 \int_y^1 2y f_{(12)}(y,z) f(v) dv dy dz + \int_0^1 \int_z^1 \int_v^1 2v f_{(12)}(y,z) f(v) dy dv dz$$
$$+ \int_0^1 \int_v^1 \int_z^1 2z f_{(12)}(y,z) f(v) dy dz dv.$$

5. English (two-) clock auction with exit option

We now modify the above English clock auction by allowing the entrant to exit if he has won only one object. If that option is exercised, the entrant returns the object he won and pays nothing; in that event one object remains unsold.

The exit option induces the entrant to radically change his behavior in three ways: the entrant always exercise the exit option, gives up his cautious initial stop rule, and changes his continuation strategy, in such a way that he never suffers a winner's curse. Of course, incumbents' equilibrium strategy is unchanged.

Again, one finds by elimination of dominated strategies:

Proposition 3. *In the English clock auction with exit option, the entrant plays the bid strategy:*

$$s(v_E) = v_E \tag{11}$$
$$\sigma(v_E, p) = 2v_E - p, \tag{12}$$

and exercises his exit option if he won only one object.

Evidently, the exit option completely removes the exposure risk. The seller benefits from this in so far as the entrant no longer plays a cautious initial stop rule. However, his revenue may diminish because the entrant plays a less aggressive continuation strategy; for the same reason, the effect on the entry probability, $\rho = \Pr\{V_{(1:2)} < V_E \wedge 2V_E - V_{(1:2)} > V_{(2:2)}\}$, is ambiguous.

Figure 2 illustrates the impact of the exit option based on a Monte Carlo simulation that assumes uniformly distributed valuations. There, we plot the probability distributions of the seller's equilibrium revenue. Evidently, adding the exit option shifts the probability distribution of the seller's revenue in such a way that more probability mass is shifted to the tails of the distribution. At the same time, the expected revenue increases (see Table 1). Therefore, adding the exit option induces a second-order stochastic dominance shift of the seller's revenue.

6. English (two-)clock auction with exit and entry option

Now we further fine-tune the English clock auction by allowing the incumbent who won one object to also buy the second object at the price he paid for the first (or a fixed fraction of that price if demand is diminishing).[11]. In other words we supplement the entrant's exit option with an entry option. That entry

[11] See Section 7 below.

option can be exercised by the incumbent who won one object if the incumbent exercised his exit option.

Obviously, adding the entry option does not affect the equilibrium strategies. However, it assures that no spectrum is stranded, i.e., all goods are allocated, and it enhances the seller's revenue. Indeed, it enhances the seller's revenue to such an extent that it always exceeds that of the constrained Vickrey package auction:

Proposition 4. *The English clock auction with exit and entry option (E+) is superior to the constrained Vickrey package auction (CV): it gives rise to higher revenue of the seller, the same entry profile, and a superior allocation if entry fails to occur (and is also superior to the English clock auction with exit but without entry option and without exit and entry options).*

Proof. First we show that the seller's revenue in the E+ auction is never lower and almost always higher than in the CV auction:

1) $2V_E > V_A + V_B \Rightarrow$ the entrant wins both objects and pays $V_A + v_B$, just like in the CV auction.

2) $2V_E < V_A + V_B \Rightarrow$ the incumbents win just like in the CV auction.

2a) $\min\{V_A, V_B\} > V_E \Rightarrow$ the entrant quits first; therefore, the seller's revenue is equal to $\Pi_0 = 2v_E$. This exceeds the seller's expected revenue in the CV auction, Π_0', which is equal to

$$\Pi_0' = \begin{cases} 0 & \text{if} \quad 2V_E < \min\{V_A, V_B\} \\ 2V_E - \min\{V_A, V_B\} & \text{if} \quad 2V_E \in \big(\min\{V_A, V_B\}, \max\{V_A, V_B\}\big) \\ 2V_E - (V_A + V_B) & \text{if} \quad 2V_E > \max\{V_A, V_B\}. \end{cases} \tag{13}$$

2b) $\min\{V_A, V_B\} < V_E \Rightarrow$ the weaker incumbent stops first, and the entrant stops second; the entrant exercises his exit opion and the stronger incumbent exercises his entry option; therefore, the seller's revenue is equal to $\Pi_0 = 2\big(2v_E - \min\{V_A, V_B\}\big) = 4V_E - 2\min\{V_A, V_B\}$. This exceeds the seller's revenue in the CV auction, Π_0', which is equal to

$$\Pi_0' = \begin{cases} 2V_E - \min\{V_A, V_B\} & \text{if} \quad 2V_E < \max\{V_A, V_B\} \\ 4V_E - (V_A + V_B) & \text{if} \quad 2V_E > \max\{V_A, V_B\}. \end{cases} \tag{14}$$

Second, we show that entry occurs in the E+ auction if and only it occurs in the CV auction. Recall, in the E+ auction entry occurs if and only if $V_E > \min\{V_A, V_B\}$ and $2V_E - \min\{V_A, V_B\} > \max\{V_A, V_B\}$, whereas in the CV auction entry occurs if and only if $2V_E > V_A + V_B$. Suppose entry occurs in the E+ auction. Then, $2V_E - \min\{V_A, V_B\} > \max\{V_A, V_B\}$ and hence $2V_E > V_A + V_B$ which implies that entry occurs in the CV auction. Next, suppose entry does not occur in the E+ auction. Then, either (a) $2V_E - \min\{V_A, V_B\} < \max\{V_A, V_B\}$ or (b) $V_E < \min\{V_A, V_B\}$. (a) implies $2V_E < V_A + V_B$, and similarly, (b) implies $2V_E < 2\min\{V_A, V_B\} = V_A + V_B$; hence, entry does not occur in the CV auction.

Third, notice that if entry does not occur, the VC auction allocates one object to each incumbent, whereas the E+ auction allocates two objects to the stronger incumbent if $\min\{V_A, V_B\} > V_E$, which creates more surplus. □

	Constrained English clock auctions			Vickrey package auctions	
	basic	+exit option	+exit+entry options	constrained	unconstrained
π_0	0.598	0.667	0.750	0.583	1
Pr{entry}	0.500	0.500	0.500	0.500	0.333
Pr{1 stranded}	0.030	0.167	0	0	0

Table 1: Seller's expected revenue, π_0, entry and leftover probabilities

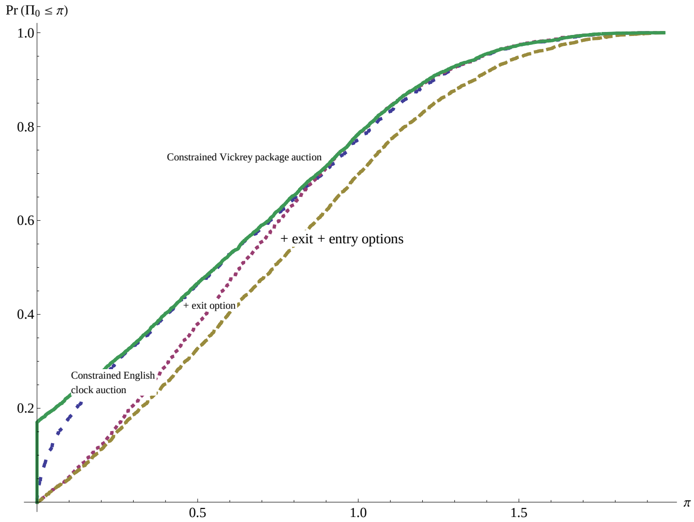

Figure 2: Simulation result: c.d.f.'s of the seller's equilibrium revenue

Table 1 illustrates the performance ranking of all considered auctions assuming uniformly distributed valuations.

In addition, in Figure 2 we summarizes a comparison of the c.d.f. of the seller's revenue for all considered auctions, based on simulation that assumes uniformly distributed values. The simulation was carried out using the Excel based simulation package by Myerson (2005). These results indicate that all considered English clock auctions yield a stochastically higher revenue than the constrained Vickrey package auction in the strong sense of first-order stochastic dominance. The English clock auction with exit option yields a higher expected revenue than the plain English clock auction and moreover exhibits less probability mass on the two tails of the distribution. Therefore, the English clock auction with exit option second order stochastically dominates the plain English clock auction. Finally, the English clock auction with exit and entry option first-order stochastically dominates the English clock auction with exit option.

7. Extensions and robustness

So far we used two extreme assumptions: 1) if incumbents are restricted to bid on one object, they bid only on their favored object, and 2) if incumbents are given an entry option that allows them to bid on more than one object, their demand is flat, i.e., they value objects the same. These assumptions seem to be overly restrictive and may even appear to contradict each other. However, as we show now, these assumptions should be viewed as facilitating the exposition only. Indeed, if we replace these restrictive assumptions, our analysis extends qualitatively and our results become even stronger.

7.1. When goods are substitutes for incumbents

As a first extension suppose incumbents view the two objects as perfect substitutes. In that case, prices always move up *in tandem*, because once an incumbent has quit the auction, the other incumbent will engage in arbitrage bidding, and alternate bidding on one and the other object. Therefore, if the entrant wins both objects he now has to pay $2\max\{V_A, V_B\}$, in lieu of $V_A + V_B$. Of course, after one incumbent has quit first, the other incumbent either bids on one object or quits. Therefore, the entrant knows that he will necessarily end up with one object if the remaining incumbent quits before he quits. However, this does not imply that he should bid the bundle value on each object, because as long as he continues to bid, he raises the price of the object that he is sure to win. This induces a strategic "demand reduction" which takes the form of a strategically reduced stop rule.

Proposition 5. *If goods are perfect substitutes, the entrant's exposure problem becomes even more severe and the entrant responds by quitting earlier, i.e., both the initial stop rule, $\hat{s}$, and the continuation game stop rule, $\hat{\beta}$, are lower than if objects cannot be substituted, for all v_E, p:*

$$\hat{s}(v_E) < s(v_E), \quad \text{and} \quad \hat{\beta}(v_E, p) \leq \beta(v_E, p). \tag{15}$$

In particular, if F is convex and $v_E < 1/2$, the entrant quits immediately after one incumbent has quit first, $\hat{\beta}(v_E, p) = p, \forall p$, and therefore the entrant does not bid in the first place, $\hat{s}(v_E) = 0, \forall v_E \leq 1/2$.

Proof. 1) In a first step we solve the continuation strategy $\hat{\beta}(v_E)$. For this purpose, suppose one incumbent has quit first at price p while the remaining incumbent plays his dominant strategy and bids his value (alternating between bidding on A and B so that prices move up in tandem). For simplicity we write

$$\hat{b} := \min\{\hat{\beta}(v_E), 1\}.$$

1a) If $v_E \geq 1/2$, the entrant's best reply is to bid up to the bundle value $2v_E$, because the remaining incumbent will quit with probability one at a price below 1, which is below the bundle value $2v_E \geq 1$.

1b) If $v_E < 1/2$, the entrant's best reply is to bid less than the bundle value $2v_E$. In particular, the entrant's expected profit is a function of his bid b (stopping point) and the price p, as follows:[12]

$$\Pi_E(b, p) = \int_p^b 2(v_E - v)\frac{f(v)}{1 - F(p)}dv - b\int_b^1 \frac{f(v)}{1 - F(p)}dv \tag{16}$$

$$\partial_b \Pi_E(b, p) = \frac{1}{1 - F(p)}\varphi(b), \quad \varphi(b) := f(b)\left(2v_E - b - \frac{1 - F(b)}{f(b)}\right). \tag{17}$$

[12]We assume without loss of generality $b \leq 1$.

The best reply is either a corner solution, $\hat{b} \in \{p, 1\}$ or an interior solution that solves the condition $\varphi(\hat{b}) = 0$ for some $\hat{b} \in (p, 1)$. Evidently, the corner solution $\hat{b} = 1$ cannot apply because $b = 1 \Rightarrow \partial_b \Pi_E(b, p) < 0$.

If there is an interior solution, one has $\varphi(b) = 0$ and hence

$$\hat{b} = 2v_E - \frac{(1 - F(\hat{b}))}{f(\hat{b})} < 2v_E, \tag{18}$$

as asserted.

However, typically the solution is the corner solution $\hat{b} = p$ (stop immediately). In particular, if F is convex (which includes the uniform distribution case), the best reply is $\hat{b} = p$ because there is no interior solution and $b = p \Rightarrow \partial_b \Pi_E(b, p) < 0$, because

$$\begin{aligned}
\varphi(b) :&= (2v_E - b) f(b) - \int_b^1 f(v) dv \\
&\leq (2v_E - b) f(b) - \int_b^1 f(b) dv \quad \text{(because } f \text{ is non-decreasing)} \\
&= f(b)(2v_E - 1) \\
&< 0 \quad \text{(because } v_E < 1/2\text{)}.
\end{aligned}$$

2) Next, we characterize the initial stop rule, $\hat{s}$, and show that $\hat{s}(v_E) < s(v_E)$, for all v_E.

Using the same qualitative distinction of events as in the proof of Proposition 2, the reduced form payoff function (incorporating the equilibrium of the continuation game) as a function of the initial stop rule, s, is

$$\begin{aligned}
\pi_E(s) =& 2v_E \int_0^s \int_z^{\hat{b}} f_{(12)}(y, z) dy dz - \int_0^s \int_z^{\hat{b}} 2y f_{(12)}(y, z) dy dz \\
&- \hat{b} \int_0^s \int_{\hat{b}}^1 f_{(12)}(y, z) dy dz.
\end{aligned}$$

2a) Suppose $v_E \geq 1/2$, then $\hat{b} = 1$ and $\hat{s}(v_E)$ must solve the condition $\partial_s \pi_E(s) = 0$. For all $s \in (0, 1]$ the sign of $\partial_s \pi_E(s)$ is the same as the sign of $2v_E - E[2V \mid V > s]$. Therefore, $\hat{s}(v_E)$ must solve the condition: $\hat{g}(s) := E[2V \mid V > s] = 2v_E$. Similarly, the initial stop rule in the model when goods are not substitutes, $s(v_E)$, solves the condition (see (3)): $g(s) := E[V + s \mid V > s] = 2v_E$. Obviously, $\hat{g}(s) > g(s)$; and because $\hat{g}, g$ are strictly increasing, it follows immediately that $\hat{s}(v_E) < s(v_E)$.

2b) Suppose $v_E < 1/2$. Then, $\hat{\beta}$ has either the corner solution to stop immediately or an interior solution characterized by (18).

If $\hat{\beta}$ has the corner solution, which occurs for example if F is convex, the entrant cannot ever win the bundle of goods, and therefore does not bid at all; in that case, trivially, $\hat{s}(v_E) = 0 < s(v_E)$.

Whereas if $\hat{\beta}$ has the interior solution, $\hat{s}$ solves the condition $\partial_s \pi_E(s) = 0$ which can be rewritten as

$$\int_{\hat{b}}^1 s dF(y) = \int_s^{b^*} (2v_E - y) dF(y) - \int_s^{b^*} y dF(y).$$

Adding $\int_s^{\hat{b}} sdF(y)$ to both sides of the equation one has

$$\begin{aligned} \int_s^1 sdF(y) &= \int_s^{\hat{b}} (2v_E - y)dF(y) - \int_s^{\hat{b}} (y-s)dF(y) \\ &< \int_s^{\hat{b}} (2v_E - y)dF(y) \\ &< \int_s^{2v_E} (2v_E - y)dF(y). \end{aligned} \tag{19}$$

Now recall that the initial stop rule when goods are not substitutable, $s(v_E)$, and $v_E < 1/2$, solves the condition

$$\int_s^1 sdF(y) = \int_s^{2v_E} (2v_E - y)dF(y). \tag{20}$$

Because the RHS of (19) is smaller than the RHS of (20) and $\int_s^{2v_E}(2v_E - y)dF(y)$ is decreasing in s, it follows that $\hat{s}(v_E) < s(v_E)$, as asserted. □

Similar reasoning applies if the two objects are imperfect substitutes and the favored object is worth only a multiple of the favored object.

7.2. When incumbents' demand is decreasing (rather than flat)

Our analysis also extends to the case when incumbents value the non-preferred object by a multiple γ of the preferred object's valuation. In that case the entry option must take the form that the incumbent who won the first object can buy the second at γ times the price of the first object (provided the entrant exercised his exit option).

8. Discussion

One limitation of the present analysis is that we consider the sale of only two blocks of spectrum.

If more than two blocks of the same kind are made available, it is typically the case that neighboring spectrum is more valuable because the operator can better deal with interferences if he controls neighboring spectrum. This gives rise to yet another exposure risk issue: the risk of acquiring a fragmented portfolio of spectrum. In that regard, the designers of the spectrum auctions in Germany innovated another nice idea: the sale of generic, unnamed lots.

If generic lots are traded, at the time of bidding bidders do not know which concrete lots will be acquired. The regulator promises to allocate neighboring frequencies to achieve a contiguous holding of spectrum, after the auction. The advantage of this procedure is that bidders do not need to worry about fragmentation and at the time of the auction, all lots of the same kind are homogeneous goods which greatly simplifies bidding.[13]

[13]Generic lots were employed in the German 2G, 3G, and in part in the recent 4G simultaneous multi-round auction. In the latter, the auctioned UMTS (2.2 GHz) frequencies were concrete (named) frequencies. These frequencies were resold in the year 2010 because two winners of the 3G auction in the year 2000 had unexpectedly returned their licenses shortly after the auction. Generic lots have also been employed in simultaneous multi-round spectrum auctions in other countries.

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