GOVERNANCE AND THE EFFICIENCY OF ECONOMIC SYSTEMS
GESY

Discussion Paper No. 44

Are 18 holes enough for Tiger Woods?

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May 2005

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Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

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Abstract

This paper addresses the selection problem in promotion tournaments. I consider a situation with heterogeneous employees and ask whether an employer might be interested in repeating a promotion tournament. On the one hand, this yields a reduction in uncertainty over the employees' abilities. On the other hand, there are costs if a workplace stays vacant.

Key words: Promotion tournament, selection, heterogeneous employees, repetition.

JEL classification: D82, M51

* I would like to thank Michael Brei and Matthias Kräkel for helpful comments. Financial support from the Deutsche Forschungsgemeinschaft (DFG) through SFB-TR 15 (Governance and the efficiency of economic systems) is gratefully acknowledged.

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1. Introduction

In practice, tournaments are very famous, since they help to determine the most able competitor in a simple way and, therefore, mitigate problems due to informational asymmetries. Consider, for example, a company trying to fill a vacancy on a high hierarchy level, but not knowing the abilities of the lower-level employees. Clearly, this company wishes to fill the vacancy with a rather able employee, since an unable employee might perform badly and, hence, might influence the company's profit in an unfavourable way. One possibility for the company is to arrange an inner-company promotion tournament.[1] As an able employee is more likely to win the tournament than an unable employee, the asymmetry problem would be weakened. However, the tournament outcome might be affected by luck or random components. So, the probability that an unable employee wins the tournament and the company promotes the "wrong" one is positive.

There is only little literature discussing this selection problem in tournaments.[2] Meyer (1991) e.g. considers a series of promotion tournaments between two heterogeneous employees, where the number of tournaments is exogenously given. She demonstrates that the problem of incorrect promotion decisions may be mitigated by biasing the tournament results. If only ordinal information about the employees' performances is available, an optimal bias (that in most cases favours the actual leader in the tournament) will increase the tournament's information content such that the information becomes a sufficient statistic for cardinal information. Clark and Riis (2001) show that the problem of incorrect promotion decisions can be solved by combining a promotion tournament with several test standards. In their model, there are three tournament prizes, and the tournament's winner receives the highest prize only if he additionally passes two tests. By using the test standards, the employer receives further information about the employees' abilities. With this information, the selection problem might be solved completely. In contrast, Hvide and Kristiansen (2003) emphasise the relevance of the selection problem. They examine a promotion tournament, in which the employees are able to choose strategies of different risk. In this case the selection problem is quite relevant, as a low-ability employee might choose a very risky strategy and so may overturn his ability disadvantage.

This paper regards a practical instrument to weaken the selection problem that is very successful in sports. Most tournaments in sports are characterised by the existence of repeating competition, i.e., players compete more than one time. In this case, the tournament's winner is

[1] In this work, I do not analyse whether a tournament is optimal in the class of all contracts. I assume that the company uses a promotion tournament to fill a vacancy because of its practicability and ask how to improve it.

[2] The literature on rank-order tournaments mostly focuses on the use of tournaments as incentive scheme. See e.g. Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983) or Rosen (1986).

the player that has the highest success on average. For example, in golf[3], the competitors play 18 holes, and the winner is the player that needs the fewest shots to pocket the golf ball in all 18 holes.[4] Imagine an extreme situation, in which the competitors play only one hole to identify the most able player. In this situation, the quality of the tournament results is very doubtful. An able player might have unfavourable conditions (e.g. strong wind or rain) and, therefore, might need more shots than a less able competitor playing under good conditions. In the contrary extreme situation, the number of competitions between the golf players would be infinitely large. The law of large numbers then predicts that the tournament's winner is surely the most able player, so the selection problem would be solved. However, it is arguably impossible to golf an infinitely high number of holes.

In this paper, the repetition mechanism is transferred to an inner-company promotion tournament. An employer decides about the number of tournaments he arranges between two heterogeneous employees. His decision is thereby determined by the following trade-off: On the one hand, it is costly for the employer to operate more than one tournament, since he wishes to fill a vacant workplace. On the other hand, extending the number of tournaments reduces uncertainty about the employees` abilities and leads to higher expected future profits.

The remainder of the paper is organised as follows: Section 2 contains the description of the basic model. Thereafter, in section 3, the model solution is presented. In particular, it is shown under what circumstances the employer is interested in extending the tournament. While the primary aim of section 3 is to examine the selection properties of the above described and widely used mechanism, section 4 presents some instruments to further improve selection quality. Concluding remarks are offered in section 5.

2. Description of the model and notation

Consider a risk-neutral principal arranging a series of k one-period tournaments between two heterogeneous and non risk-loving employees. Without loss of generality, employee 1 is the more able one with ability a_H and employee 2 the less able one with ability a_L $(a_H > a_L)$. I define $a_H - a_L$ as Δa. A situation with asymmetric information is assumed such that each employee knows her own ability and the ability of her opponent, whereas the employer only knows that there is one able employee with ability a_H and one unable employee with ability a_L. Both employees might presently work in the same department of their company and, for this reason, are able to estimate the abilities of each other in a detailed way, while the employer

[3] Other examples include sports like skiing or cycling.
[4] The primary aim of repetition in sports is to entertain spectators for a certain period rather than to solve the selection problem. Yet clearly, repeating competition mitigates this problem.

naturally has less information about his employees' abilities. The winner of the overall tournament will be promoted to a vacant workplace at a higher level in the company. This workplace is already vacant when the first tournament starts. Therefore, the net profit of the company from this workplace is zero in each period during the tournament. If the able employee is promoted, this profit will be $\pi \in \{\pi_L, \pi_H\}$, with $\pi_H > \pi_L = 0$ and $\Pr ob\{\pi = \pi_H\} = p \in (0,1)$ in each period.[5] Otherwise, it will be $\pi \in \{\pi_L, \pi_H\}$, with $\Pr ob\{\pi = \pi_H\} = q \in (0, p)$.[6] That is, the able employee is more likely to achieve a high profit than the less able one. The company and the two employees discount future utilities with $r \le 1$. Their time horizon is infinite. Intuitively, r could be interpreted as the probability that employee i (i=1,2) continues to work for the company in the next period. Suppose that the performance of employee i in the tournament in period t (t=1,…,k), y_{it}, is given by the sum of the effort e_{it} he has chosen in this tournament, his ability a_i and a random noise ε_{it}.

(1) $\quad y_{it} = e_{it} + a_i + \varepsilon_{it}$.[7]

It is assumed that these performances do not increase the company's profits. They are only useful as a signalling instrument.[8] The random components ε_{it} are independently drawn from a normal distribution with mean zero and variance σ^2. Effort entails costs for an employee which are given by $C(e_{it})$ with $C(0) = 0$, $C'(e_{it}) > 0$ and $C''(e_{it}) > 0$. In case of promotion, the promoted employee in each period receives an income w. It is further assumed that an employee's utility is additively separable in income and costs. Therefore, the expected utility of employee i (i=1,2) is given by $EU_{ik} = P_{ik} \cdot U(w) \cdot (r^k/1-r) - \sum_{j=1}^{k} r^{j-1} \cdot C(e_{ij})$ where $U(0) = 0$, $U'(w) > 0$ and $U''(w) \le 0$. P_{ik} denotes the employee's winning-probability. I start by

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[5] It is assumed that an employee on the higher level receives a wage of w, while he receives a wage of zero on the lower level. At the beginning of the tournament, the high-level workplace is vacant and so, during the tournament, nothing is produced on the workplace and no wage is paid. Thus, the profit is clearly zero. Furthermore, one could think that after promotion, in the good state of the world output worth $\pi_H + w$ and in the bad state of the world output worth w is produced.

[6] Note that, from observing the output on the higher level, the principal cannot deduce the employee's type with certainty. Hence, testing the employees on the higher level is only possible to some degree. Throughout the paper it is assumed that the model parameters are such that the tournament scheme is preferred by the principal to a testing scheme. This will for example be the case, if π_H and (p-q) are rather low so that the costs of arranging a tournament (the potential profits to be lost) and the gains from testing the employees are negligible.

[7] This kind of production function has the analytical advantage that the tournament will lead to symmetric effort choices of the employees. Symmetry will help to state the results to be derived most clearly. As there may also be settings, where this production function is inappropriate, section 4.1 extends the model to encompass more general production functions.

[8] One could justify this assumption as follows: The profits that could be generated on the primary level are so low compared to the ones on the higher level that they are of (almost) no importance. Alternatively, as in Clark and Riis (2001), one could think of the model as a hiring process, in which the company arranges a series of (valueless) tests in order to experience the abilities of the potential employees.

assuming that the employee attaining the highest aggregate output becomes the tournament's winner, as this decision rule is quite frequent in practice. Different and more sophisticated promotion rules are considered in section 4. Further, suppose that the employees learn intermediate results, i.e., in a given tournament, they know the results of the previous tournaments. Finally, the employer determines the optimal number of tournaments to maximise his expected discounted profit, which is given by $\pi_k = \left(r^k/1-r\right)\cdot\left[\left(P_{1k}\cdot p + \left(1-P_{1k}\right)\cdot q\right)\cdot \pi_H\right]$.

3. Solution to the model

The model is solved by backward induction. Hence, we start by deriving the employees' efforts in the final period. These efforts are chosen to maximise (2) and (3), respectively:

$$(2)\qquad EU_{1k} = \Pr ob\left\{\sum_{t=1}^{k-1}\left(y_{1t}-y_{2t}\right)+\Delta a + e_{1k} - e_{2k} > \varepsilon_{2k}-\varepsilon_{1k}\right\}\cdot\left(U(w)/1-r\right) - C\left(e_{1k}\right),$$

$$(3)\qquad EU_{2k} = \left[1-\Pr ob\left\{\sum_{t=1}^{k-1}\left(y_{1t}-y_{2t}\right)+\Delta a + e_{1k} - e_{2k} > \varepsilon_{2k}-\varepsilon_{1k}\right\}\right]\cdot\left(U(w)/1-r\right) - C\left(e_{2k}\right).$$

Let F_k denote the cumulative distribution function of the composite random variable $\varepsilon_{2k}-\varepsilon_{1k}$, and f_k the corresponding density function. The first-order conditions to the employees' maximisation problems are given by (4) and (5).

$$(4)\qquad \frac{\partial EU_{1k}}{\partial e_{1k}} = f_k\left(\sum_{t=1}^{k-1}\left(y_{1t}-y_{2t}\right)+\Delta a + e_{1k} - e_{2k}\right)\cdot\left(U(w)/1-r\right) \overset{!}{=} C'\left(e_{1k}\right),$$

$$(5)\qquad \frac{\partial EU_{2k}}{\partial e_{2k}} = f_k\left(\sum_{t=1}^{k-1}\left(y_{1t}-y_{2t}\right)+\Delta a + e_{1k} - e_{2k}\right)\cdot\left(U(w)/1-r\right) \overset{!}{=} C'\left(e_{2k}\right).$$ [9]

From the first-order conditions, we see that both employees choose same effort, thus we have $e_{1k} = e_{2k} =: e_k$. Analogously, continuing backward induction up to the first round, we see that this symmetry holds in every tournament, so for all t=1,…,k we have $e_{1t} = e_{2t} =: e_t$. From an ex ante point of view, the winning-probability of the high-ability employee can therefore be written as

$$(6)\qquad \begin{aligned} P_{1k} &= \Pr ob\left\{\sum_{t=1}^{k} y_{1t} > \sum_{t=1}^{k} y_{2t}\right\} = \Pr ob\left\{\sum_{t=1}^{k} e_{1t} + \sum_{t=1}^{k}\varepsilon_{1t} + k\cdot a_H > \sum_{t=1}^{k} e_{2t} + \sum_{t=1}^{k}\varepsilon_{2t} + k\cdot a_L\right\} \\ &= \Pr ob\left\{\sum_{t=1}^{k}\left(\varepsilon_{2t}-\varepsilon_{1t}\right) - k\cdot\Delta a < \sum_{t=1}^{k}\left(e_{1t}-e_{2t}\right)\right\} =: G_k\left(\sum_{t=1}^{k}\left(e_{1t}-e_{2t}\right)\right) = G_k(0). \end{aligned}$$

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[9] As in Lazear and Rosen (1981), the second-order conditions will hold and so an equilibrium will exist if the variance σ^2 is sufficiently large. Intuitively, an equilibrium will only exist if luck plays a significant role. In what follows, the existence of an equilibrium is assumed.

In this context, G_k stands for the cumulative distribution function of the composite random variable $\sum_{t=1}^{k}(\varepsilon_{2t}-\varepsilon_{1t})-k\cdot\Delta a$, while g_k denotes the corresponding density function. $G_k(0)$ can be rewritten as $\Phi\left(\left(\sqrt{k}\cdot\Delta a/\sqrt{2}\cdot\sigma\right)\right)$, where $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal distribution. It is then straightforward to derive proposition 1:

Proposition 1: The winning-probability of the high-ability employee is strictly increasing in k.

Extending the number of tournaments from k to k+1 affects the winning-probability of the high-ability employee in two countervailing ways. On the one hand, abstracting from random factors, the difference between the two employees' performances increases from $k\cdot\Delta a$ to $(k+1)\cdot\Delta a$. Hence, employee 1 is more likely to be promoted. On the other hand, the influence of random factors increases, too. The variance of each individual's performance rises from $k\cdot\sigma^2$ to $(k+1)\cdot\sigma^2$, so employee 1 is less likely to be promoted. However, the first effect outbalances the second one, and the winning-probability of the able employee increases when the number of tournaments gets higher. For $k\rightarrow\infty$, $G_k(0)$ equals one. The selection problem would be completely eliminated by infinitely repeating the promotion tournament.

Extending the number of tournaments is advantageous for the employer, since an incorrect promotion decision becomes less likely. Yet, it is also disadvantageous. The employer looses potential payoffs, for the workplace stays vacant for a longer time. In order to clearly understand how the employer decides and how the model parameters influence his decision, we restrict his possible actions. Particularly, we assume that the employer has to decide between arranging m or m+1 tournaments, where m is an integer and positive number. In this case he prefers m+1 tournaments, if the condition $\pi_{m+1}>\pi_m$ holds. This condition is rewritten in (7):

(7) $$\left(\Phi\left(\sqrt{(m+1)/2}\cdot(\Delta a/\sigma)\right)\cdot r-\Phi\left(\sqrt{m/2}\cdot(\Delta a/\sigma)\right)\right)>\left(q\cdot(1-r)/(p-q)\right).$$

From (7), I derive proposition 2 where I define $y:=\Delta a/\sigma$:

Proposition 2. If the employer has the possibility to arrange m or m+1 promotion tournaments, there exists a cut-off $\tilde{r}(p,q,m)\in\left(((m+1)/m)^{-0.5},1\right)$ *such that the following will hold:*

(i) *For* $r<\tilde{r}$*, the employer always arranges m tournaments.*

(ii) *For* $r>\tilde{r}$*, there are two cut-offs* $\hat{y}(p,q,m)>0$ *and* $\tilde{y}(p,q,m)>0$ *with* $\hat{y}<\tilde{y}$ *such that the employer arranges m+1 tournaments only if* $y\in[\hat{y},\tilde{y}]$.

(iii) $\partial\tilde{r}/\partial p < 0$, $\partial\tilde{r}/\partial q > 0$, $\partial\tilde{r}/\partial m > 0$, $\partial\hat{y}/\partial p < 0$, $\partial\hat{y}/\partial q > 0$, $\partial\hat{y}/\partial m > 0$, $\partial\tilde{y}/\partial p > 0$, $\partial\tilde{y}/\partial q < 0$, $\partial\tilde{y}/\partial m < 0$.

Proof: See Appendix.

In case of a small r, the employer assigns a high value to present payoffs, but not to future payoffs. So he will never arrange a further tournament, since the profits in period m+1 are too valuable for him.

For r higher than $\tilde{r}$, it is also worthwhile for the employer to care for future payoffs. In this case, it might be beneficial to arrange more than m tournaments in order to reduce uncertainty about the employees' abilities. As stated in proposition 2, the employer's decision in the case $r > \tilde{r}$ depends on the ratio $\Delta a/\sigma$.

For a small Δa (or a large σ), one could think that the employer decides to arrange a further tournament. The employees are very similar in their abilities and, hence, it is quite likely that arranging only m tournaments yields an incorrect promotion decision. Surprisingly, the employer does not so. The reason is as follows: Even if a further tournament was arranged, employee 1 is only little more likely to be promoted than employee 2. Due to the small ability difference (or the large impact of random components) a tournament in period m+1 would not entail very much new information about the two employees. Therefore, the disadvantage of lower profits prevails, and the employer decides not to extend the tournament. For intermediate values of Δa and σ, the argumentation is contrary. In this case the use of another tournament leads to much more information about the employees' abilities. Hence, it is beneficial to extend the promotion tournament to accumulate more data about the employees' abilities. Finally, for a large Δa (or a small σ), the high-ability employee is very likely to be the leader after the first m tournaments. Arranging a further tournament would only yield little new information and, therefore, the employer decides not do so.

The effects of p and q are intuitive. If p increases, the employer will be more likely to arrange a further tournament as it becomes more important to identify the agent types. Similarly, an increase in q makes the employer less likely to arrange another tournament, as an incorrect promotion decision has less severe impacts on expected profit.

Finally, the amount of new information when arranging a further tournament is decreasing in m. That is, if the employer has already observed many tournament outcomes, he will not benefit very strongly from arranging another one. Therefore, there are less combinations of parameter values (r,y), for which the employer decides to arrange a further tournament.

4. Extensions

In the previous analysis, a simple and widely used decision rule was considered. In this section, the model is extended in different ways. While in section 4.1 a general performance function is introduced, sections 4.2 to 4.4 deal with more sophisticated decision rules.

4.1 Consideration of general performance functions

Up to this point, performance of each employee was assumed to be additively separable in effort and ability. This assumption simplified the model a lot, as it led to a symmetric solution, in which both employees choose same effort. Let us now consider more general performance functions. Suppose that the performance of employee i in the tournament in period t is given by $y_{it} = f(e_{it}, a_i) + \varepsilon_{it}$, where $\partial f / \partial e_{it} > 0$, $\partial^2 f / \partial (e_{it})^2 \leq 0$, $\partial f / \partial a_i > 0$ and $\partial^2 f / \partial e_{it} \partial a_i \neq 0$. Consider the tournament in period k. Employee 1 maximises

$$\text{(8)} \qquad EU_{1k} = F_k\left(\sum_{t=1}^{k-1}(y_{1t} - y_{2t}) + f(e_{1k}, a_H) - f(e_{2k}, a_L)\right) \cdot (U(w)/1 - r) - C(e_{1k}).$$

Similarly, employee 2 maximises

$$\text{(9)} \qquad EU_{2k} = \left(1 - F_k\left(\sum_{t=1}^{k-1}(y_{1t} - y_{2t}) + f(e_{1k}, a_H) - f(e_{2k}, a_L)\right)\right) \cdot (U(w)/1 - r) - C(e_{2k}).$$

The resulting equilibrium is no longer symmetric. Solving for the first-order conditions, one can show that employee 1 will exert higher effort than employee 2, if and only if $\partial^2 f / \partial e_{it} \partial a_i > 0$, i.e., if effort and ability are complements in the performance function. This can be shown to hold in every period. Hence, if effort and ability are complements (substitutes) in the performance function, the high-ability employee will exert higher (lower) effort than the low-ability employee. As a result, the cut-off values $\hat{y}$ and $\tilde{y}$ should become relatively smaller (bigger) compared to the case where $\partial^2 f / \partial e_{it} \partial a_i = 0$, as the difference in effort increases (decreases) the effective ability gap.[10]

4.2 Handicapping of employees

[10] Interestingly, a similar result as in the case, where effort and ability are complements, should be obtained, if the employer, besides the tournament, makes use of test standards. Suppose that each employee would receive an additional payment of b units, if this employee's aggregate performance exceeded some threshold t. The employer would then determine t such that the more able employee is expected to exert higher effort than his opponent. As the setup of the model is different from the one in Clark and Riis (2001), introducing additional test standards does not solve the selection problem completely. As the difference in the random components follows some normal distribution, random effects may always outweigh the advantage of the high-ability employee resulting from the effort and ability difference.

Let us return to the assumption that employee performance is given by (1). As in Meyer (1991), the employer might be interested in biasing the tournament results. I consider this possibility in the simplest form, in which the employer has decided to arrange two tournaments. Suppose that, after the first round, the actual leader in the tournament receives, besides the performance difference $|z|$ from the first round, a further head start of $t(|z|)$ units. For $t(|z|) > 0$ ($t(|z|) < 0$), the loser (winner) of the first round is handicapped.[11] I first show that handicapping does not change the symmetry of efforts. From (2) and (3) it is clear that, in the second tournament, both employees choose same efforts, as $t(|z|)$ does not affect the marginal incentives of the two employees in a different way. Turn now to the first tournament. From the point of view of this tournament, the winning-probability of the high-ability employee equals

(10)

$$P_{12} = F_1(e_{11} - e_{21} + \Delta a) \cdot \int_0^{\infty} F_2(\Delta a + z + t(z)) \cdot \frac{f_1(z - e_{11} + e_{21} - \Delta a)}{F_1(e_{11} - e_{21} + \Delta a)} dz +$$

$$(1 - F_1(e_{11} - e_{21} + \Delta a)) \cdot \int_{-\infty}^{0} F_2(\Delta a + z - t(-z)) \cdot \frac{f_1(z - e_{11} + e_{21} - \Delta a)}{1 - F_1(e_{11} - e_{21} + \Delta a)} dz$$

$$= \int_0^{\infty} F_2(\Delta a + z + t(z)) \cdot f_1(z - e_{11} + e_{21} - \Delta a) dz + \int_{-\infty}^{0} F_2(\Delta a + z - t(-z)) \cdot f_1(z - e_{11} + e_{21} - \Delta a) dz.$$

It is given by his winning-probability in the first round times his conditional overall winning-probability given a victory in the first round plus the probability of losing the first round times conditional overall winning-probability given the first round was lost.

The employees determine e_{11} and e_{21} to maximise $EU_{11} = P_{12} \cdot (r \cdot U(w)/1 - r) - C(e_{11})$ and $EU_{21} = (1 - P_{12}) \cdot (r \cdot U(w)/1 - r) - C(e_{21})$, respectively, which can be shown to lead to a symmetric equilibrium with $e_{11} = e_{21} =: e_1$.

Next consider the employer's maximisation problem. The head start t is chosen such that

(11) $\quad \pi_2 = (r/1 - r) \cdot [(P_{12}(t(z)) \cdot p + (1 - P_{12}(t(z))) \cdot q) \cdot \pi_H]$

is maximised. This is equivalent to maximising P_{12}. The latter problem leads to the following first-order condition:[12]

[11] One could also think that the employer randomly handicaps the employees before the first tournament starts. As can be shown, such a change in the decision rule does not affect the symmetry of efforts to be derived subsequently. As a consequence, handicapping before the first tournament does not lead to new insights and is for simplicity not considered.

[12] The second-order condition $\int_0^{\infty} \partial f_2(\Delta a + z + t(z))/\partial t \cdot f_1(z - \Delta a) dz + \int_{-\infty}^{0} \partial f_2(\Delta a + z - t(-z))/\partial t \cdot f_1(z - \Delta a) dz < 0$ is assumed to be satisfied.

(12) $$\partial P_{12}(t(z))/\partial t(z)=\int_{0}^{\infty} f_2(\Delta a+z+t(z))\cdot f_1(z-\Delta a)dz-\int_{-\infty}^{0} f_2(\Delta a+z-t(-z))\cdot f_1(z-\Delta a)dz\overset{!}{=}0.$$

The employer optimally trades off the higher winning-probability of the able employee after winning the first tournament with the lower winning-probability after losing the first round. Note that, in the optimum, t need not necessarily be positive, as the principal also takes the employees' initial standings, i.e., the difference in first-round performance, which is transferred into the second one, into account. Finally, as optimally biasing the tournament results leads to a better selection decision and biasing is of no value in a one-period tournament[13], the employer is more likely to arrange a second round than he was in the model in section 3.

4.3 Decision based on the number of "sets" won

In sports like e.g. tennis or badminton a decision is based on the number of "sets" won by each player. To keep things as simple as possible, I compare the cases "best of one" and "best of three". The former case corresponds to the model in section 3, where k=1. In the latter case, the employee, who first wins two tournaments is declared the winner. Note that, in this case, the employer will not arrange a third round, if an employee wins the first two ones. Let us analyse, in which way the introduction of such a decision rule affects the results derived in section 3.

I start by assuming that the solution remained symmetric, i.e., that both employees still chose same efforts. In this case, the high-ability employee's winning-probability is $P_{11}=F(\Delta a)$ in the "best of one" case and $P_{13}=(F(\Delta a))^2+2\cdot(F(\Delta a))^2\cdot(1-F(\Delta a))$ otherwise.[14] One can easily verify that $P_{13}>P_{11}$ always holds. That is, if the solution remained symmetric, arranging more tournaments again yields a more accurate employee selection. However, contrary to our initial assumption, in the "best of three" case, the solution becomes asymmetric in tournaments one and two. To demonstrate this, let us derive the optimal efforts. If it comes to a third round, the solution will be the same as under "best of one", i.e., both employees choose same efforts. In the second round, we have to distinguish between the case, where employee 1 won the first round and the case, where he lost it. In case he has won the first round, employee 1 maximises

(13)

$$EU_{12}=F(\Delta a+e_{12}-e_{22})\cdot(U(w)/1-r)+[(1-F(\Delta a+e_{12}-e_{22}))\cdot F(\Delta a)]\cdot(r\cdot U(w)/1-r)-C(e_{12}),$$

while employee 2 maximises

(14) $$EU_{22}=[(1-F(\Delta a+e_{12}-e_{22}))\cdot(1-F(\Delta a))]\cdot(r\cdot U(w)/1-r)-C(e_{22}).$$

From the first-order conditions, one can derive the following condition:

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[13] A proof of this statement is available from the author upon request.

[14] Note that $f_i(\cdot)=:f(\cdot)$, for i=1, 2, 3, as the respective composite random variables follow the same distribution.

(15) $C'(e_{12}) - C'(e_{22}) = f(\Delta a + e_{12} - e_{22}) \cdot U(w)$.

It can be seen that employee 1 exerts higher effort than employee 2. Contrary to employee 2, employee 1 has the chance to be promoted after the second round and, therefore, to receive the higher wage one period before employee 2 may receive it. This has the effect that the two employees evaluate the winner prize in a different way and hence choose different efforts. Denote the effort difference by $\Delta\hat{e}$. Similarly, if employee 2 was the winner of the first round, the solution becomes asymmetric with efforts satisfying condition (16):

(16) $C'(e_{22}) - C'(e_{12}) = f(\Delta a + e_{12} - e_{22}) \cdot U(w)$.

Here, the low-ability employee chooses higher effort as he has the chance to be promoted earlier than his opponent. The effort difference in this case is denoted by $\Delta\tilde{e}$. Let us now proceed to the first tournament. The two employees then maximise (17) and (18), respectively:

(17)

$$
\begin{aligned}
EU_{11} &= F(\Delta a + e_{11} - e_{21}) \cdot F(\Delta a + \Delta\hat{e}) \cdot (r \cdot U(w)/1 - r) \\
&+ [(1 - F(\Delta a + e_{11} - e_{21})) \cdot F(\Delta a - \Delta\tilde{e}) + F(\Delta a + e_{11} - e_{21})(1 - F(\Delta a + \Delta\hat{e}))] \cdot F(\Delta a) \cdot (r^2 \cdot U(w)/1 - r) \\
&- C(e_{11}),
\end{aligned}
$$

(18)

$$
\begin{aligned}
EU_{21} &= (1 - F(\Delta a + e_{11} - e_{21})) \cdot (1 - F(\Delta a - \Delta\tilde{e})) \cdot (r \cdot U(w)/1 - r) \\
&+ [(1 - F(\Delta a + e_{11} - e_{21})) \cdot F(\Delta a - \Delta\tilde{e}) + F(\Delta a + e_{11} - e_{21})(1 - F(\Delta a + \Delta\hat{e}))] \cdot (1 - F(\Delta a)) \cdot (r^2 \cdot U(w)/1 - r) \\
&- C(e_{21}).
\end{aligned}
$$

Subtracting the first-order conditions, yields

(19) $C'(e_{11}) - C'(e_{21}) = f(\Delta a + e_{11} - e_{21}) \cdot U(w) \cdot r \cdot [F(\Delta a + \Delta\hat{e}) + F(\Delta a - \Delta\tilde{e}) - 1]$.

From (19), it can be seen that $e_{11} > e_{21} \Leftrightarrow \Delta\hat{e} + 2 \cdot \Delta a > \Delta\tilde{e}$. Unfortunately, in this general form one cannot unambiguously say, whether or not this condition is satisfied so that either employee might choose higher effort in the first tournament. Summarizing, introducing a "best of three" decision rule leads to asymmetric efforts of the employees. In the first round, it is not clear, which employee exerts higher effort, while in the second round, it is always the actual leader in the tournament. Consequently, the model does not clearly predict, whether or not extending the number of rounds leads to a more precise selection decision. It seems yet likely that, even with asymmetric efforts, arranging more tournaments increases selection accuracy.

4.4 Aborting the tournament before the pre-specified number of rounds

Until now, it was assumed that the employer is not allowed to abort the tournament before the pre-specified number of rounds. However, to save on vacancy costs, the employer might decide to do so, if, before the final tournament is reached, the employees' performances would differ significantly. This might induce a change in employee behaviour, so, the model results might also change. To keep the analysis tractable, suppose again that the principal decides to arrange two tournaments and announces to stop after the first round, if one player leads by at least $x \geq 0$ units. Let us solve the model by backward induction. If the second tournament is reached, efforts remain symmetric and satisfy $e_{12} = e_{22} =: e_2$. In the first tournament, employee 1 maximises

$$
\begin{aligned}
(20)\qquad & EU_{11} = F_1(\Delta a + e_{11} - e_{21} - x) \cdot (U(w)/1 - r) \\
& + \int_{\Delta a + e_{11} - e_{21} - x}^{\Delta a + e_{11} - e_{21} + x} F_2(2 \cdot \Delta a + e_{11} - e_{21} - y) \cdot f_1(y) dy \cdot (r \cdot U(w)/1 - r) - C(e_{11}).
\end{aligned}
$$

A similar expression can be given for employee 2. An employee's expected utility in the first round is given by the payment he will receive, if beating his opponent by more than x plus his payment, if winning the tournament after two rounds minus costs entailed by effort. Using Leibniz's rule, one can determine the first-order condition to the above maximisation problem. It is given by

$$
\begin{aligned}
& \frac{\partial EU_{11}}{\partial e_{11}} = f_1(\Delta a + e_{11} - e_{21} - x) \cdot (U(w)/1 - r) \\
(21)\qquad & + [(F_2(\Delta a - x) \cdot f_1(\Delta a + e_{11} - e_{21} + x) - F_2(\Delta a + x) \cdot f_1(\Delta a + e_{11} - e_{21} - x) + \\
& \int_{\Delta a + e_{11} - e_{21} - x}^{\Delta a + e_{11} - e_{21} + x} f_2(2 \cdot \Delta a + e_{11} - e_{21} - y) \cdot f_1(y) dy)] \cdot (r \cdot U(w)/1 - r) - C'(e_{11}) \overset{!}{=} 0.
\end{aligned}
$$

Again, one can derive a similar expression for employee 2. As $\varepsilon_{22} - \varepsilon_{12}$ and $\varepsilon_{21} - \varepsilon_{11}$ follow the same distribution (that is, $f_1(\cdot) = f_2(\cdot) =: f(\cdot)$), the difference of the two first-order conditions can be written as

$$
(22)\qquad C'(e_{11}) - C'(e_{21}) = U(w) \cdot [f(\Delta a + e_{11} - e_{21} - x) - f(\Delta a + e_{11} - e_{21} + x)].
$$

Condition (22) does not yield unambiguous results concerning the employees' efforts. Both, $e_{11} > e_{21}$ or $e_{21} > e_{11}$ may hold. The asymmetry results from a change in the incentive structure, for the employees now not only want to win the tournament, but want to win by at least x to become promoted earlier. Note that the employer determines x such that his expected profit is maximised. He therefore considers all the effects that a change in x has. A change in x affects the probability of aborting the tournament (and so of realizing potential profit one period earlier) and the promotion probability of the high-ability employee. The latter effect is twofold. On the one hand, the winning-probability of the high-ability employee is directly affected by x, as an increase in x decreases his probability of winning the tournament after one round, but increases his winning-probability after two rounds. On the other hand, an increase in x affects the two employees' efforts (and the effort difference) in the first round. Unfortunately, in this general form, the model does not unambiguously predict, how the effort difference changes with x.

5. Concluding remarks

This paper addressed the selection problem in promotion tournaments. It was analysed, whether an employer might prefer to arrange a series of promotion tournaments in order to improve selection accuracy. While extending the number of tournaments always leads to more detailed information about the employees' abilities, this information advantage may be outbalanced by vacancy costs that arise when the number of tournaments is increased.

Comparing the employer's expected utilities of arranging m or m+1 tournaments, offers further interesting results. When the employer is quite impatient, he always decides to arrange only m tournaments. When he is rather patient, his decision depends on the amount of new information another tournament entails. This new information depends non-monotonously on the ratio of the two agents' ability difference and the error term's standard deviation. For a very small or a very large ratio, the amount of new information is rather small, for intermediate values it is more significant. Hence, the tournament will only be extended if the ratio adopts an intermediate value.

Appendix

In this appendix, proposition 2 is proved.

Transforming condition (7) and using $y := \Delta a/\sigma$ yields:

$$\pi_{m+1} > \pi_m \Leftrightarrow \left(\Phi\left(\sqrt{(m+1)/2}\cdot y\right)\cdot r - \Phi\left(\sqrt{m/2}\cdot y\right)\right) > \left(q\cdot(1-r)/(p-q)\right).$$

The derivative of the function $H(y) := \left(\Phi\left(\sqrt{(m+1)/2}\cdot y\right)\cdot r - \Phi\left(\sqrt{m/2}\cdot y\right)\right) - \left(q\cdot(1-r)/(p-q)\right).$ with respect to y is $\partial H(y)/\partial y = \left(\Phi'\left(\sqrt{(m+1)/2}\cdot y\right)\cdot\sqrt{(m+1)/2}\cdot r - \Phi'\left(\sqrt{m/2}\cdot y\right)\cdot\sqrt{m/2}\right)$, or, using $\Phi'(y) = \left(1/\sqrt{2\cdot\pi}\right)\cdot e^{-0.5\cdot y^2}$,

$$\partial H(y)/\partial y = \left(\left(1/\sqrt{2\cdot\pi}\right)\cdot e^{-\frac{m+1}{4}\cdot(y)^2}\cdot\sqrt{(m+1)/2}\cdot r - \left(1/\sqrt{2\cdot\pi}\right)\cdot e^{-\frac{m}{4}\cdot(y)^2}\cdot\sqrt{m/2}\right).$$ This derivative is positive if the following condition holds:

$$e^{-0.25\cdot(y)^2}\cdot\sqrt{m+1}\cdot r > \sqrt{m} \Leftrightarrow \ln\left(\sqrt{m+1/m}\cdot r\right) > 0.25\cdot(y)^2 \Leftrightarrow y < \left(\ln\left(\sqrt{m+1/m}\cdot r\right)\right)^{0.5}\cdot 2.$$

We see that $y^* = \left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\cdot 2$ is the maximum of H. Note that $H(y) < 0$ for $y = 0$ and $y \to \infty$. As $H(y)$ is a continuous function, it must have exactly two nulls $\hat{y}$ and $\tilde{y}$, if $H(y^*) > 0$. Otherwise, that is, if $H(y^*) < 0$, it is always negative.

Inserting y^* into H yields:

$$H(y^*) = \left(\Phi\left(2\cdot\sqrt{\frac{m+1}{2}}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)\cdot r - \Phi\left(2\cdot\sqrt{\frac{m}{2}}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)\right) - \frac{q\cdot(1-r)}{(p-q)}.$$

This maximum is strictly negative for $r = ((m+1)/m)^{-0.5}$, but strictly positive for r=1. The derivative of $H(y^*)$ with respect to r is given by:

$$\frac{\partial(H(y^*))}{\partial r} = \Phi\left(2\cdot\sqrt{(m+1)/2}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)$$
$$+\Phi'\left(2\cdot\sqrt{(m+1)/2}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)\cdot\sqrt{(m+1)/2}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{-0.5}$$
$$-\Phi'\left(2\cdot\sqrt{m/2}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)\cdot\sqrt{m/2}\cdot\ln\left(\sqrt{(m+1)/m}\cdot r\right)^{-0.5}\cdot 1/r + q/(p-q).$$

This derivative is positive if the difference between the second and the third term is non-negative, i.e., if the subsequent condition holds:

$$\Phi'\left(2\cdot\sqrt{(m+1)/2}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)\cdot\sqrt{(m+1)/2}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{-0.5}$$
$$-\Phi'\left(2\cdot\sqrt{m/2}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)\cdot\sqrt{m/2}\cdot\ln\left(\sqrt{(m+1)/m}\cdot r\right)^{-0.5}\cdot 1/r \geq 0$$
$$\Leftrightarrow \Phi'\left(2\cdot\sqrt{(m+1)/2}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)\cdot\sqrt{m+1} - \Phi'\left(2\cdot\sqrt{m/2}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)\cdot\sqrt{m}\cdot 1/r \geq 0$$
$$\Leftrightarrow \left(1/\sqrt{2\cdot\pi}\right)\cdot e^{-0.5\cdot\left(2\cdot\sqrt{\frac{m+1}{2}}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)^2}\cdot\sqrt{m+1}\cdot r - \left(1/\sqrt{2\cdot\pi}\right)\cdot e^{-0.5\cdot\left(2\cdot\sqrt{\frac{m}{2}}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)^2}\cdot\sqrt{m} \geq 0$$
$$\Leftrightarrow e^{-0.5\cdot\left(2\cdot\sqrt{0.5}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)^2}\cdot\sqrt{m+1}\cdot r \geq \sqrt{m}$$
$$\Leftrightarrow \sqrt{(m+1)/m}\cdot r \geq \sqrt{(m+1)/m}\cdot r.$$

Hence, we have shown that $\partial H(y^*)/\partial r$ is positive. Since $H(y^*)$ is positive for r=1, there must be some cut-off value $\tilde{r}$, at which $H(y^*)$ becomes positive. This proves parts (i) and (ii) of proposition 2. Consider now part (iii). Using the method of implicit differentiation together with condition

$$\left(\Phi\left(2\cdot\sqrt{\frac{m+1}{2}}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot\tilde{r}\right)\right)^{0.5}\right)\cdot\tilde{r} - \Phi\left(2\cdot\sqrt{\frac{m}{2}}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot\tilde{r}\right)\right)^{0.5}\right)\right) - \frac{q\cdot(1-\tilde{r})}{(p-q)} = 0,$$

one can immediately show that $\partial\tilde{r}/\partial p < 0$ and $\partial\tilde{r}/\partial q > 0$, since $\partial H(y)/\partial p > 0$ and $\partial H(y)/\partial q < 0$. As the derivative $\partial H(y)/\partial y$ is independent of p and q, an increase in p (q) only shifts the function $H(y)$ upwards (downwards) and so yields a decrease (increase) in $\hat{y}$ and an increase (decrease) in $\tilde{y}$. Finally, consider the derivative of $\Phi\left(\sqrt{(m+1)/2}\cdot y\right)\cdot r - \Phi\left(\sqrt{m/2}\cdot y\right)$ with respect to m. One can show that this derivative is negative, if and only if

$\sqrt{(m+1)/m} > r \cdot e^{-0.5 \cdot y^2}$, which is always fulfilled. Hence, if m increases, the function $H(y)$ will decrease. As a consequence, $\tilde{r}$ must be increasing in m. Further, we know that $H(\hat{y}) = 0$. Therefore $\partial\hat{y}/\partial m = -(\partial H(\hat{y})/\partial m)/(\partial H(\hat{y})/\partial\hat{y})$, which is positive as $\hat{y} < y^*$. Analogously, one can prove that $\partial\tilde{y}/\partial m < 0$. This proves part (iii) of proposition 2.

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