A Conversational War of Attrition

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Abstract

We explore a model of costly dynamic deliberation in which two partially informed jurors with common interests aggregate their information by coarse binary communication to arrive at a common verdict. The model yields three key predictions on equilibrium information aggregation.

(1) The longest possible conversation is the most efficient, and is of unbounded length. Bounded conversations are not stable, since jurors must dismiss off-equilibrium communication as mistakes.

(2) The longer is the conversation, the more likely the aggregated information is moot. Thus investment into information aggregation correlates negatively with the value of the aggregated information.

(3) With a small marginal communication cost, information aggregation is approximately instantaneous and perfect, and delay costs are approximately twice the expected loss from taking the wrong verdict.
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1 Introduction

Better decisions via like minded committees

In many circumstances groups of like-minded individuals are believed to arrive at better decisions than any one of the individuals by herself, essentially because the group as a whole is better informed than any of its members. Examples are plentiful and include juries, panels of doctors, groups of economic advisors, hiring committees, etc.

The problem of information aggregation

Implicit in this argument is that the individuals' information is adequately aggregated for the decision. Absent a conflict of interest this aggregation poses no problem if the group members can just "put all their pertinent information on the table". This in turn seems unproblematic for hard facts that can be described in a common terminology such as, say, the forensic doctor's estimate of the time of death in a murder trial. In contrast, much of the pertinent information to determine the guilt of a defendant "beyond a reasonable doubt" may consist of subjective assessments, say, on the credibility of a testimony, or on information that is difficult to express because of differing backgrounds or terminologies. As it is of course not feasible to share this information completely, a natural way to aggregate it partially is a dynamic arguing process.

Our question: How much information will be shared if this is costly

This paper introduces a highly stylized model of this arguing process and asks how much information will be aggregated in equilibrium when the group members face explicit delay costs. We find that maximal delay is not only to be expected but is optimal, and yet surprisingly, we deduce that greater delay is associated with less informed decisions.

Frictions imposed by game form

The frictions of communication are modeled by restricting each of the two jurors to argue for one of two possible verdicts, Acquit or Convict, in any round. To convey compelling information for one's verdict a juror needs to argue for it repeatedly.

Literally this modeling assumption means that the jurors can only express which verdict they favor but cannot give an argument why they do so. Equivalently, and maybe more plausibly, it can be interpreted as saying that while the jurors can express more sophisticated arguments they are unable to understand each other's arguments beyond a statement which verdict the other favors.

To simplify the analysis further we assume that the arguing process stops, and a verdict is taken as soon as the two jurors argue for the same verdict. This assumption makes the game a "conversational war of attrition". Each juror has a fixed role in the game to argue for one of the verdicts and a decision is taken as soon as one of the jurors gives in to her colleague by arguing for the colleague's verdict. One can alternatively interpret the game form as a dynamic voting procedure with a unanimity rule in the sense that the voting process is repeated until the vote is unanimous.

We do not claim that this is an optimal communication protocol. In fact, we discuss in the conclusion that a game form that disentangles communication and decision would allow jurors to parse their information sets more efficiently by switching roles in the course of the argument. However, we do argue that it is not common for parties to switch sides in such a way. Consider the international discussion on whether global warming is happening. Some countries insist it is, others say the evidence is not clear enough, some are getting convinced of the case made by others but the roles of the main actors do not switch over the course of the debate.

Equilibrium Communication / Information Aggregation can break down

The game turns out to have multiple equilibria and so how much information will be aggregated depends on the choice of an equilibrium. We show that equilibria are of two kinds. In a *communicative* equilibrium any type of either juror interprets persistence of her colleague as increasingly strong evidence for the verdict favored by the colleague and will give in accordingly. In an *insistent* equilibrium communication breaks down at some point. All remaining types of one juror will give in at that point and her colleague will insist on her verdict forever, interpreting potential deviations from her opponent's strategy as mistakes rather than signals of informational strength. Thus, insistent equilibria aggregate less information than communicative equilibria.

Our first main result (Proposition 1) is that only communicative equilibria satisfy forward induction and that the slowest equilibrium is communicative and Pareto-dominant. Thus, the jurors should argue as long as possible, and they will do so if they are willing to always interpret each others' utterings as signals rather than uninformative mistakes.

The longer we argue, the more likely the information is moot

Next, we investigate for any fixed equilibrium how the length of the argument correlates with the joint information of the jurors. Our main result (Proposition 2) shows that as the conversation transpires the probability increases that the jurors' joint information is moot and does not clearly support either one of the two verdicts. Thus, ex-post, investment into information aggregation is negatively correlated with the value of the aggregated information. In other words it exhibits diminishing total returns, rather than just diminishing marginal returns.

Asymptotic Results

While communication costs are necessary to keep the jurors from sharing their information perfectly by talking forever, we think of the cost of listening to one more argument as small when compared to the cost of concluding the wrong verdict. Therefore we lastly consider equilibrium information aggregation as the time per round κ approaches 0. In other words talk here is actually cheap but not free (as it is in the "cheap talk" literature).

We find that asymptotically information is aggregated instantaneously and, in a communicative equilibrium, perfectly. More precisely we show (Proposition 3) that in an efficient equilibrium both waiting costs, i.e. the investment into information aggregation, and decision costs, i.e. the costs of taking the ex-post wrong decision because of insufficient information aggregation, are approaching 0 at rate $\kappa^{\frac{2}{3}}$. We also show that the ratio of waiting costs to decision costs converges to two.

The fast agreement is in sharp contrast to a "regular" war of attrition in which players must incur real waiting costs to "win the game". This difference is driven by the fact that the jurors' preferences are assumed to be identical under symmetric information and only disagree as long as they have not fully shared their information. Thus, when a juror is almost convinced that her colleague's opposing information is stronger than her own information she will want to give in, both to save on waiting costs and to ensure a good verdict. This leads to faster agreement than a war of attrition, in which the players always prefer different outcomes ex-post.

The asymptotic results are facilitated by the fact that an efficient equilibrium coincides with the solution of the game by a planner who, like the jurors, minimizes the sum of waiting costs and decision costs. The reason that the numbers "$\frac{2}{3}$" and "2" in our results do not depend on any parametric assumptions on the distributions is that waiting costs are linear in the expected number of rounds $N(\kappa)$ while decision costs are linear in $N(\kappa)^{-2}$. Thus the former are half as elastic in $N(\kappa)$ as the latter and are thus twice as high in the planner's solution. As κ decreases, the optimal number of round of arguing $N^*(\kappa)$ increases at rate $\kappa^{-\frac{1}{3}}$ to make both types of costs decrease at rate $\kappa^{\frac{2}{3}}$.

Relation to Literature

A previous title of the paper "We can't argue forever" refers to (Geanakoplos and Polemarchakis 1982) "We can't disagree forever" which shows that jurors who iteratively communicate to each other their respective posteriors of some event will eventually agree on a common posterior through Bayesian updating. In a vague sense one can interpret our contribution as an extension of this result: Even though 1) communication is costly and restricted to binary signals rather than a real number, 2) information can be unboundedly strong and 3) jurors are strategic, we find that "we can't argue forever" in expectation as the frictions vanish.

Crucial for fast and precise communication is the assumption of common interests, which is also implicit in (Geanakoplos and Polemarchakis 1982). If the jurors anticipated a conflict of interest they would partially conceal their private information to nudge the verdict in their favor. This observation goes back to (Crawford and Sobel 1982) and has more recently been analyzed in a dynamic cheap talk setting by (Aumann and Hart 2003) and a static committee decision problem by (Li, Rosen, and Suen 2001)[1]. Thus, partially conflicting interest in these papers, just like costly communication in this paper, predicts that optimal communication will be coarse rather than fully revealing.

Another motivation for this paper is the "swing voter's curse" investigated in the literature on strategic voting, (Austen-Smith and Banks 1996) and (Feddersen and Pesendorfer 1998). The idea is that asymmetric voting rules can lead like-minded, strategic voters to vote insincerely against the decision indicated by their private information when they take into account the additional information inferred from the condition that their vote is pivotal. Of course it is crucial for this argument that the group members cannot communicate and infer each other's information solely from anticipated voting behavior. This assumption seems to be satisfied mostly by groups of people that are too large to allow for a meaningful discussion. From this angle our paper can be seen as introducing a role for communication before a committee decision, while preventing perfect communication which would render a consecutive vote unnecessary.

The communication role of repeated voting has previously been studied by (Piketty 2000) who focuses on large electorates signalling information about third alternatives in the first of two voting rounds. He finds that multi-round elections can be more efficient than single round elections, which is broadly resonant with our results.

More closely related to our paper is (Damiano, Li, and Suen 2008) studying how repeated voting allows jurors with partially conflicting interests to share more of their information than a single vote. Just like in our model juror A infers strong opposing information from the insistence of her opponent B and will eventually give in unless she is certain that A is the better verdict. Because of the conflicting interests the costs of information aggregation are bounded away from zero even as the frictions vanish (unlike in our model) but for moderate levels of interest conflict the benefits of dynamic information aggregation still outweigh its costs (unlike in a war or attrition with purely conflicting interests or as in (Gul and Lundholm 1995)).

Outline of Paper

We procede as follows. The next section introduces the formal model and shows that all equilibria are represented in cut-off strategies where jurors with weak information give in earlier and jurors with strong information hold out longer. In Section 3 we derive the indifference condition of the equilibrium cut-off types, which implies a "zig-zag" property and a recursive monotone structure of all equilibria, and show that our game admits an efficient equilibrium. Section 4 introduces the notions of insistent and communicative equilibria and shows that the former do not satisfy forward induction and that the slowest communicative equilibrium is Pareto-dominant. Section 5 derives the negative correlation between investment into information aggregation and the value of the aggregated information. Section 6 develops the asymptotic properties of the

[1] Note that communication in these models is not just "cheap" (as it is in our model) but actually free.

efficient equilibrium as κ approaches zero. Section 7 concludes. Appendix 8.1 extends the results to a larger game including an initial vote and Appendix 8.2 contains technical proofs that would disrupt the flow of the main text.

2 Model

A. Actions and Timing

The game is a discrete time stopping game where two jurors alternate in arguing to convict (vote $\mathcal{C}$) or acquit (vote $\mathcal{A}$) the defendant of some crime. The game ends once agreement is reached, i.e. once both jurors argue for the same verdict. We assume that the initial argument has been in favor of $\mathcal{A}$, and focus on the subsequent sub-game calling the first player in the sub-game $\mathcal{C}$-juror and the other one the $\mathcal{A}$-juror.

This identification of a juror with a fixed role simplifies the subsequent notation. Of course, the analysis would apply equally to the sub-game following an initial $\mathcal{C}$-vote. However, the focus on the sub-game is not completely innocuous as it prevents us from studying the equilibrium action at the first node. We will comment on how our arguments and results extend to the larger game in Appendix 8.1.

B. Information

There are two states of the world $\theta = \mathcal{G}, \mathcal{I}$ (Guilt and Innocence) on which jurors $\mathcal{C}$ and $\mathcal{A}$ share a common prior. Jurors receive conditionally independent signals $\tilde{\lambda}, \tilde{\mu}$ of the state of the world, possibly from different distributions. Whereas the signals are private information, everything else is assumed to be common knowledge between the jurors.

To simplify matters, we represent these signals in a non-standard fashion, as log-likelihood-ratios, and choose different reference states for the different jurors. More precisely, we assume that the probability measures $d\tilde{F}^{\mathcal{G}}(\tilde{\lambda}), d\tilde{F}^{\mathcal{I}}(\tilde{\lambda})$ are mutually continuous such that $\frac{d\tilde{F}^{\mathcal{G}}(\tilde{\lambda})}{d\tilde{F}^{\mathcal{I}}(\tilde{\lambda})} \in (0, \infty)$, and analogously $\frac{d\tilde{G}^{\mathcal{I}}(\tilde{\mu})}{d\tilde{G}^{\mathcal{G}}(\tilde{\mu})} \in (0, \infty)$ for the $\mathcal{A}$-juror. We then let

- $\lambda := \log \frac{d\tilde{F}^{\mathcal{G}}(\tilde{\lambda})}{d\tilde{F}^{\mathcal{I}}(\tilde{\lambda})} \in (-\infty, \infty)$ be the log-likelihood-ratio of $\mathcal{G}$ (versus $\mathcal{I}$), and
- $\mu := \log \frac{d\tilde{G}^{\mathcal{I}}(\tilde{\mu})}{d\tilde{G}^{\mathcal{G}}(\tilde{\mu})} \in (\mu_0, \infty)$ be the log-likelihood-ratio of $\mathcal{I}$ (versus $\mathcal{G}$).

Given signal λ, resp. signal profile (λ, μ), the probability of $\mathcal{G}$ is given by $\pi(\lambda) = \frac{e^{\lambda}}{e^{\lambda}+1}$, resp. $\pi(\lambda, \mu) = \frac{e^{\lambda-\mu}}{e^{\lambda-\mu}+1} = \frac{e^{\lambda}}{e^{\lambda}+e^{\mu}}$. We call higher signals $\lambda' > \lambda, \mu' > \mu$ "stronger", since jurors with such signals feel more strongly in favor of the verdict they are arguing for. We will often refer to (λ, μ) as the *types* of the jurors.

To further streamline the exposition we assume the following on the distributions of λ and μ:

- *Regular distribution*: The distributions of λ and μ admit weakly log-concave pdfs f and g [2].

[2]Even if we allowed for arbitrary distributions with atoms we could still prove versions of our three main results at the cost of a more cumbersome notation. Log-concavity could be weakened to allow for densities that are not too log-convex, but some version is needed to derive the negative correlation between the length of the conversation and the value of the aggregated information.

- *Unbounded signal strength*: The support of f is unbounded below and above $(-\infty, \infty)$ and the support of g is bounded below and unbounded above $[\mu_0, \infty)$ [3].

In the analysis we need to keep track of the unconditional correlation of λ and μ. Denoting by $h(\lambda, \mu)$ the joint unconditional pdf we obtain the following explicit expression for the "local" correlation $r(\lambda, \mu) := \frac{h(\lambda,\mu)}{f(\lambda)g(\mu)}$ of λ and μ:

Lemma 1 (Unconditional Correlation)

$$r(\lambda, \mu) = \frac{2(e^\lambda + e^\mu)}{(1 + e^\lambda)(1 + e^\mu)} \in (0, 2)$$

Proof Denote by $f^\theta(\lambda), g^\theta(\mu)$ and $h^\theta(\lambda, \mu)$ the pdfs in state $\theta \in \{\mathcal{I}, \mathcal{G}\}$. The likelihood ratios satisfy the no-introspection condition as in (Smith and Sorensen 2000) $\frac{f^{\mathcal{G}}(\lambda)}{f^{\mathcal{I}}(\lambda)} = e^\lambda$, $\frac{g^{\mathcal{G}}(\mu)}{g^{\mathcal{I}}(\mu)} = e^{-\mu}$ and $\frac{h^{\mathcal{G}}(\lambda,\mu)}{h^{\mathcal{I}}(\lambda,\mu)} = e^{\lambda-\mu}$. Therefore:

$$\begin{aligned} h(\lambda, \mu) &= \frac{1}{2}h^{\mathcal{I}}(\lambda, \mu) + \frac{1}{2}h^{\mathcal{G}}(\lambda, \mu) \\ &= \frac{1 + e^{\lambda-\mu}}{2}h^{\mathcal{I}}(\lambda, \mu) = \frac{1 + e^{\lambda-\mu}}{2}f^{\mathcal{I}}(\lambda)g^{\mathcal{I}}(\mu) \\ &= \frac{1 + e^{\lambda-\mu}}{2}\frac{2}{1 + e^\lambda}f(\lambda)\frac{2}{1 + e^{-\mu}}g(\mu) \\ &= \frac{2(e^\lambda + e^\mu)}{(1 + e^\lambda)(1 + e^\mu)}f(\lambda)g(\mu) \end{aligned}$$

C. Preferences

The jurors have identical cardinal preferences, as both wish to make *the right decision*: convict the guilty and acquit the innocent. We express all payoffs as costs.

Waiting costs are explicit, equal to κ per argument. We assume throughout that $\kappa < 1$ and will often be interested in the case when κ tends to 0. Thus, total waiting costs equal $(2n - 1)\kappa$ if the $\mathcal{C}$-juror gives in by voting $\mathcal{A}$ in round n, and $2n\kappa$ when $\mathcal{A}$-juror votes $\mathcal{C}$ in round n.

Decision costs are zero for the right decision, and mistakes are symmetrically penalized: They equal 1 of verdict $\mathcal{C}$ in state $\mathcal{I}$ and verdict $\mathcal{A}$ in state $\mathcal{G}$. Also, jurors are risk neutral and do not discount future payoffs. Thus, for signals (λ, μ) the ex-post optimal verdict entails statistical decision costs of $\min\{\pi(\lambda, \mu), 1 - \pi(\lambda, \mu)\}$ and the ex-post preference of $\mathcal{C}$ over $\mathcal{A}$ is given by $2\pi(\lambda, \mu) - 1$. Given that the statistical decision costs cannot be influenced by the jurors, their goal is to minimize the integral of $|2\pi(\lambda, \mu) - 1|$ over all (λ, μ) where the ex-post inferior verdict is chosen. By virtue

[3] Remember that we are considering here the sub-game after an initial $\mathcal{A}$-vote by the $\mathcal{A}$-juror. We assume that the initial vote was straight-forward in the sense that types with a sufficiently low $\mu < \mu_0$ would have argued $\mathcal{C}$ instead of $\mathcal{A}$, leading to the alternative sub-game instead of the one studied here.

The unboundedness assumption is not needed for any of the main results and made purely for the convenience of defining communicative equilibria as equilibria where time to agreement is unbounded which would not be possible if there was a highest signal.

Intriguingly, the assumption of not-perfectly revealing signals $\lambda, \mu \neq -\infty, \infty$ is dispensable as well. Like bounded distributions, perfectly revealing signals imply a uniform bound of the length of argument.

of the log-likelihood transformation $\mathcal{C}$onviction is *ex-post preferred* over $\mathcal{A}$cquittal iff $\pi(\lambda,\mu) = \frac{e^{\lambda-\mu}}{e^{\lambda-\mu}+1} > \frac{1}{2}$ which is the case iff $\lambda > \mu$. Thus, under perfect information sharing the juror with the stronger type should get her way.

Enter figure "Ex-post preferable verdict"

D. Strategies and Equilibrium

As in any stopping game, the strategy of the $\mathcal{C}$-juror is given by a distribution over stopping times $\Phi_\lambda \in \Delta(\mathbb{N} \cup \{\infty\})$: With probability $\Phi_\lambda(n)$ type λ votes $\mathcal{C}$ until round $n-1$ and votes $\mathcal{A}$ ("gives in", "quits") in round n. Analogously define $\Gamma_\mu \in \Delta(\mathbb{N} \cup \{\infty\})$ for the $\mathcal{A}$-juror. We will be interested in sequential equilibria and refer to equilibrium paths as *conversations*.

Before turning to the main equilibrium analysis in the next section we will first show that equilibrium strategies are monotone in types and can therefore be represented by sequences of cut-off values.

In a *monotone strategy*, more extreme types of the $\mathcal{C}$-juror with $\lambda' > \lambda$ hold out weakly longer than type λ, i.e. for $n' > n \in \mathbb{N}$ it cannot be the case that $\Phi_\lambda(n') > 0$ and $\Phi_{\lambda'}(n) > 0$.

Lemma 2 (Monotone Best Responses) *Every best response strategy of one juror to some strategy of her colleague is monotone.*

Thus, any equilibrium has increasingly strong types quit in every round until, potentially, in some round communication terminates.

The proof of this lemma is complicated by the fact that a change in own signal, say λ, has an indirect effect on the conditionally expected distribution of the opponent's signal $h(\lambda,\mu)g(\mu)$. The idea of the proof is to compare the costs of different stopping times conditionally on the true state of the world $\mathcal{G}, \mathcal{I}$ and to show that preferences exhibit increasing differences in own stopping time and own type.

Proof We fix strategy Γ of the $\mathcal{A}$-juror and will derive a single-crossing property for the stopping problem of the $\mathcal{C}$-juror. Denote by $\Pi(n', n, \Gamma, \theta)$ the probability that the $\mathcal{A}$-juror will give in between rounds n and $n' > n$ according to strategy Γ conditional on state θ, and by $w(n', n, \Gamma, \theta)$ the expected incremental waiting cost from holding out until n' rather than quitting in round n.

Consider the total incremental cost from holding out until round n' vs. quitting in round n:

$$\pi(\lambda)(-\Pi(n', n, \Gamma, \mathcal{G}) + w(n', n, \Gamma, \mathcal{G})) + (1-\pi(\lambda))(\Pi(n', n, \Gamma, \mathcal{I}) + w(n', n, \Gamma, \mathcal{I})) \quad (*)$$

Incremental waiting costs w are always positive while incremental decision costs are negative (resp. positive) when the persistence of holding out until n' changes the verdict to $\mathcal{C}$ if the state is $\mathcal{G}$ (resp. $\mathcal{I}$).

By the single-crossing property it suffices to show that (*) is strictly decreasing in λ if it is non-positive for some λ. This implies that if some type λ weakly prefers to hold out until round n' then every more extreme type $\lambda' > \lambda$ does so strictly.

The second term in (*) is strictly positive. So for (*) to be non-positive for some λ it needs to be the case that the first term in (*) is strictly negative. In words, holding out until round n' can only be profitable for any type λ if it is profitable conditional on the $\mathcal{G}$uilt of the defendant. Also $\frac{d}{d\lambda}\pi(\lambda) > 0, \frac{d}{d\lambda}(1-\pi(\lambda)) < 0$ and the terms $\Pi(n', n, \Gamma, \theta) + w(n', n, \Gamma, \theta)$ do not depend on λ as they are conditional on the true state θ. This implies that (*) is decreasing in λ.

Any best-response strategy is therefore characterized by an increasing sequence of cut-off values $\ell = (\lambda_n)_{n\in\mathbb{N}}$ such that type λ quits in round n if $\lambda \in (\lambda_{n-1}, \lambda_n)$, where $\lambda_0 = -\infty$. Let $m = (\mu_n)_{n\in\mathbb{N}}$ be the analogue cut-off profile for the $\mathcal{A}$-juror. The equilibrium decision profile is illustrated in figure 1.

Insert figure "Monotonicity"

Comparing monotone strategy profiles, we call (ℓ, m) *longer*, or slower, than (ℓ', m') if for all type combinations (λ, μ) the decision takes longer in the former than in the latter. This translates into the condition that in each round n the cut-offs satisfy $\lambda_n \leq \lambda'_n$, resp. $\mu_n \leq \mu'_n$, as long as round n is reached on both equilibrium paths, i.e. $\mu'_{n-1} < \infty$, resp. $\lambda'_n < \infty$.

We will see below that under the log-concavity assumption, this partial order is actually complete on the subset of equilibria.

3 Auxiliary Equilibrium Analysis

In this section we will derive an explicit formula for equilibrium cut-off types λ_n, μ_n. This formula implies a "zig-zag" property of equilibria. We then prove as another auxiliary result that a planner's solution to the game exists and constitutes an efficient equilibrium of the game.

3.1 Equilibrium Cut-off Types

Given cut-offs $m = (\mu_n)_{n\in\mathbb{N}}$ of the $\mathcal{A}$-juror, the $\mathcal{C}$-juror's problem is to minimize her cost by choice of the optimal stopping time $C(\lambda) = \min_{n\in\mathbb{N}\cup\{\infty\}} C_n(\lambda)$.

$$C_n(\lambda) := \sum_{i=1}^{n-1} \left(\begin{array}{c} \int_{\mu_{i-1}}^{\mu_i} (2i\kappa + (1-\pi(\lambda,\mu)))r(\lambda,\mu)g(\mu)d\mu + \\ \int_{\mu_{n-1}}^{\infty} ((2n-1)\kappa + \pi(\lambda,\mu))r(\lambda,\mu)g(\mu)d\mu \end{array} \right) \tag{1}$$

When the $\mathcal{A}$-juror quits in round $i < n$ the waiting costs equal $2i\kappa$ and the decision costs of convicting the innocent equal the probability of innocence, $1 - \pi(\lambda, \mu)$. When the $\mathcal{C}$-juror quits in round n, the waiting costs equal $(2n-1)\kappa$ and the decision costs of acquitting the guilty equal the probability of guilt, $\pi(\lambda, \mu)$.

Thus, the $\mathcal{C}$-juror is indifferent between quitting in round n and holding out one more round iff

$$\begin{aligned} 0 &= C_n(\lambda) - C_{n+1}(\lambda) \\ &= -\int_{\mu_{n-1}}^{\mu_n} (2n\kappa + (1-\pi))rg - \int_{\mu_n}^{\infty} 2\kappa rg + \int_{\mu_{n-1}}^{\mu_n} ((2n-1)\kappa + \pi)rg \\ &= \int_{\mu_{n-1}}^{\mu_n} (2\pi - 1 - \kappa)rg - \int_{\mu_n}^{\infty} 2\kappa rg \end{aligned}$$

Lemma 3 (Equilibrium Cut-offs) *Equilibrium cut-off types $\ell = (\lambda_n)$ and $m = (\mu_n)$ are characterized as follows: If some types of the $\mathcal{A}$-juror hold out in round $n-1$, i.e. $\mu_{n-1} < \infty$, then the cut-off type λ_n of the $\mathcal{C}$-juror is either characterized by the indifference condition:*

$$\int_{\mu_{n-1}}^{\mu_n} (2\mu(\lambda_n, \mu) - 1) r(\lambda_n, \mu) g(\mu) d\mu = \kappa \left(\int_{\mu_{n-1}}^{\infty} r(\lambda_n, \mu) g(\mu) d\mu + \int_{\mu_n}^{\infty} r(\lambda_n, \mu) g(\mu) d\mu \right) \tag{2}$$

or all types of the $\mathcal{C}$-juror quit in round n, i.e. $\lambda_n = \infty$.

The analogue condition must hold for the cut-off types μ_n of the $\mathcal{A}$-juror.

Proof Fix $m = (\mu_n)$ with $\mu_{n-1} < \infty$. If there exist types λ and λ' who optimally quit in round n and $n+1$ respectively then the cut-off type λ_n is characterized uniquely by the condition that $C_{n+1}(\lambda) = C_n(\lambda)$, i.e. by equation (2). To complete the proof we will show that $\lambda_n = \infty$ if either λ or λ' do not exist.

First, we can focus on the case that the n^{th} round of the $\mathcal{C}$-juror is on the equilibrium path, i.e. $\lambda_{n-1} < \infty$ as otherwise we trivially have $\lambda_n = \infty$.

Now, if there is no type λ who optimally quits in round n, i.e. $\lambda_{n-1} = \lambda_n$, then any type μ of the $\mathcal{A}$-juror would prefer quitting in round $n-1$ to quitting in round n as this entails the same decision at lower waiting costs. This implies that no type μ will quit in round n and thus $\mu_{n-1} = \mu_n$. Iterating this argument we get that $\mu_{n-1} = \mu_n = \mu_{n+1} = ... < \infty$ and $\lambda_n = \lambda_{n+1} = ... < \infty$. This strategy profile cannot constitute an equilibrium as any remaining type profile (λ, μ) with $\lambda > \lambda_{n-1}$ and $\mu > \mu_{n-1}$ incurs infinite waiting costs and would thus prefer quitting in round n.

Finally, assume that not all types of the $\mathcal{C}$-juror have quit by round n, i.e. $\lambda_n < \infty$ but that there is no type λ' who optimally quits in round $n+1$, i.e. $\lambda_n = \lambda_{n+1} < \infty$. We will derive a contradiction from this assumption. If all types μ of the $\mathcal{A}$-juror quit in round n, i.e. $\mu_n = \infty$, then contrary to the assumption any type $\lambda > \lambda_n$ would find it weakly optimal to quit, off the equilibrium path, in round $n+1$. If on the other hand $\mu_n < \infty$, then by the same argument as above, every type μ would rather quit in round n than in $n+1$, implying $\mu_n = \mu_{n+1}$. Iterating this argument we get that $\mu_n = \mu_{n+1} = ... < \infty$ and $\lambda_n = \lambda_{n+1} = ... < \infty$ and the contradiction as above.

As long as communication has not terminated, the indifference condition of the cut-off type equates the net benefit from enforcing $\mathcal{C}$ by holding out in period n, LHS of (2), with the extra waiting cost from doing so, RHS of (2). The latter is positive and thus the benefit from enforcing $\mathcal{C}$ under the integral on the LHS must be positive and greater than zero.

3.2 Zig-zag

Actually the ex-post preference for $\mathcal{C}$onviction $2\pi(\lambda_n, \mu) - 1$ must greater than the cost of waiting one more round κ, at least in the most optimistic case $\mu = \mu_{n-1}$. Denote by $\varkappa \in \mathbb{R}^+$ the log-likelihood-ratio of $\mathcal{G}$ that makes the jurors exactly indifferent between taking the wrong decision, $\mathcal{A}$, immediately, and taking the correct decision, $\mathcal{C}$, after one more step, i.e. $\varkappa = \log \frac{\kappa}{1-\kappa}$ so that $\kappa = \frac{e^{\varkappa}}{1+e^{\varkappa}}$.

Definition The monotone strategy profile (ℓ, m) satisfies the zig-zag property if for all n with $\lambda_n, \mu_n < \infty$ the cut-off values obey the inequalities

$$\mu_{n-1} + \varkappa \quad < \quad \lambda_n \quad < \quad \mu_n - \varkappa$$

Lemma 4 (Zig-Zag) *Any equilibrium (ℓ, m) satisfies the zig-zag property.*

Thus, in every equilibrium the public posterior on the correct verdict $2\pi(\lambda_n, \mu_n) - 1$ changes sign after every move. This property is illustrated in figure xxx.

Insert figure "Zig-zag"

This lemma has two interesting implications. First, it shows that the size $(\lambda_n - \lambda_{n-1})$ of the set of types who quit in any round n is bounded below by $2\varkappa$. Thus, no single equilibrium can have one type profile (λ, μ) arguing forever and actually the time to agreement for any fixed type profile (λ, μ) is bounded across all equilibria.

Second, the lemma highlights that type λ_n who is indifferent between quitting and holding out in round n is almost convinced that her colleague is right and $\mathcal{A}$cquittal is the ex-post preferable verdict, because the colleague's signal $\mu \in [\mu_{n-1}, \infty)$ is likely stronger than her own λ_n. If the $\mathcal{A}$-juror holds out one more round the preferability of $\mathcal{A}$ becomes a certainty. Thus, for small step-length $(\lambda_n - \mu_{n-1})$ a juror who is giving in is actually convinced of the other's case, rather than just tired of the argument as is the case for a regular war of attrition. We extend this argument in Section 6 to show that a continuous time version of the game does not admit an informative equilibrium as both jurors would try to quit first.

Proof The first inequality, which is equivalent to $2\pi(\lambda_n, \mu_{n-1}) - 1 > \kappa$, follows from (2) because

$$0 < \int_{\mu_{n-1}}^{\mu_n} (2\pi - 1 - \kappa) rg < (2\pi(\lambda_n, \mu_{n-1}) - 1 - \kappa) \int_{\mu_{n-1}}^{\mu_n} rg$$

The second inequality owes to the analogous indifference condition of μ_n.

Many of the following results can be conveniently stated in terms of the following *propensity to hold out*

$$P(\underline{\mu}, \lambda, \bar{\mu}) := \int_{\underline{\mu}}^{\bar{\mu}} (2\pi - 1 - \kappa) rg - 2\kappa \int_{\bar{\mu}}^{\infty} rg \tag{3}$$

which of course does depend on λ as an (omitted) argument of π and r under the integral. By Lemma 3 we have $P(\mu_{n-1}, \lambda_n, \mu_n) = 0$ if these cut-off values are part of an equilibrium.

It is easy to see that P is decreasing in $\underline{\mu}$ and $\bar{\mu}$ as long as the arguments satisfy the zig-zag property $\underline{\mu} + \varkappa < \lambda_n < \bar{\mu} - \varkappa$. Intuitively, holding out and reaching a $\mathcal{C}$onvict verdict against types $\mu \in [\underline{\mu}, \bar{\mu}]$ of the colleague becomes less attractive as the strength of these types increases. Lemma 2 on the other hand implies that P is increasing in its second argument as stronger types have more to win from holding out. We will also utilize the fact that $\lim_{\lambda \to \infty} P(\underline{\mu}, \lambda, \infty) > 0$, i.e. if all types of one's opponent are about to quit, a sufficiently strong type λ will prefer to hold out. This follows easily from the fact that $2\pi(\lambda, \mu) > 1 + \kappa$ for $\lambda > \mu + \varkappa$ and that the "density function" rg is putting all of its weight on such μ as $\lambda \to \infty$.

3.3 Slower and Faster Equilibria

The equilibrium condition (2) of $\lambda_n < \infty$ allows us to write each cut-off μ_n as a function of the two preceding cut-off values μ_{n-1} and λ_n. Thus, an equilibrium (ℓ, m) is uniquely determined by its first cut-off value λ_1 (as μ_0 is given exogenously). We will now use this fact together with the log-concavity condition on the pdfs f, g to show that:

Lemma 5 (Ordered Equilibria) *If (ℓ, m) and (ℓ', m') are equilibria and $\lambda_1 < \lambda_1'$ then (ℓ, m) is longer than (ℓ', m'). Thus the set of equilibria is ordered according to their length.*

Proof We need to show that for all rounds n we have $\lambda_n < \lambda_n'$ as long as $\mu_{n-1}' < \infty$ and $\mu_n < \mu_n'$ as long as $\lambda_n' < \infty$. We proceed by induction, using as a basis that $\mu_0 = \mu_0'$ and $\lambda_1 < \lambda_1'$. The induction step will be that $\mu_{n-1} \leq \mu_{n-1}'$ and $\lambda_n - \mu_{n-1} < \lambda_n' - \mu_{n-1}'$ implies that $\lambda_n < \lambda_n'$ and $\mu_n - \lambda_n < \mu_n' - \lambda_n'$ (as long as $\lambda_n' < \infty$) [4].

The first part, that $\lambda_n < \lambda_n'$, follows by adding the two inequalities of the induction hypothesis. The second part follows by defining $\delta := \lambda_n' - \lambda_n > 0$ and then comparing

$$\begin{aligned} 0 &= P(\mu_{n-1}, \lambda_n, \mu_n) \\ &< P(\mu_{n-1} + \delta, \lambda_n + \delta, \mu_n + \delta) \\ &< P(\mu_{n-1}', \lambda_n', \mu_n + \delta). \end{aligned}$$

As P is decreasing in its last argument[5] and $P(\mu_{n-1}', \lambda_n', \mu_n') = 0$, this implies that $\mu_n + \delta < \mu_n'$ and after rearranging that $\mu_n - \lambda_n < \mu_n' - \lambda_n'$ as desired.

The second inequality above follows by the induction hypothesis $\lambda_n - \mu_{n-1} < \lambda_n' - \mu_{n-1}'$, implying $\mu_{n-1}' < \mu_{n-1} + \delta$, and the fact that P is decreasing in its first argument[6].

The first inequality above follows by the argument that (1) the "density function" rg in the integral $P(\mu_{n-1}, \lambda_n, \mu_n)$ is dominated in the MLRP sense by the "density function" rg in the integral with shifted domain $P(\mu_{n-1} + \delta, \lambda_n + \delta, \mu_n + \delta)$ and (2) the integrand $2\pi - 1 - \kappa$ is decreasing in μ. Due to the complication that the integrand is only decreasing on $[\mu_{n-1}, \mu_n]$ and then jumps up to -2κ, the formal proof of the first inequality is relegated to Lemma 10 in the Appendix.

3.4 Efficient Equilibria

A salient feature of common interest games is that a planner's solution for the game is also an equilibrium as any deviation from an efficient strategy profile raises joint costs, and thus own costs. Thus we have that

Lemma 6 (Efficiency and Equilibrium)
(a) Any cost-minimizing strategy profile is an equilibrium.
(b) A cost-minimizing strategy profile exists.

[4] Of course, there is another half of the induction step, to show that $\lambda_n \leq \lambda_n'$ and $\mu_n - \lambda_n < \mu_n' - \lambda_n'$ implies that $\mu_n < \mu_n'$ and $\lambda_{n+1} - \mu_n < \lambda_{n+1}' - \mu_n'$ (as long as $\mu_n' < \infty$, which follows by the analogue arguments.

[5] As (ℓ, m) is an equilibrium, the zig-zag property is satisfied at and above $\mu_n + \delta > \lambda_n + \delta + \varkappa = \lambda_n' + \varkappa$.

[6] Again, as (ℓ', m') is an equilibrium, the zig-zag property is satisfied at and below $\mu_{n-1}' < \lambda_n' - \varkappa$.

Part (a) follows from the above argument. To prove (b) note that by Lemma 2 we can restrict attention to monotone strategy profiles that can be expressed as a pair of countable vectors of cut-off points. In Appendix 8.2 we use a standard diagonal argument to prove that any sequence of strategy profiles that approaches a cost-infimum has an accumulation point that attains the cost-minimum.

4 Equilibrium Communication Break-down

How much information is aggregated in equilibrium depends on the choice of an equilibrium. We will show now that there are infinitely many equilibria in which communication between the jurors breaks down and less information is aggregated than would be Pareto-optimal.

The easiest such equilibrium corresponds to an asymmetric outcome of the war of attrition, in which one juror gives in immediately, e.g. $\mu_1 = \infty$, knowing that the other juror will forever insist, $\lambda_1 = \lambda_2 = ... < \infty$. Here, however, this insistent behavior need not start in period 0, since individuals have different types.

We will thus call an equilibrium $(\mathcal{C}, n)$-*insistent* if all types of the $\mathcal{A}$ juror quit after the n^{th} argument of the $\mathcal{C}$-juror, i.e. if the cut-offs satisfy $\mu_{n-1}, \lambda_n < \infty$ but $\mu_n = \infty$. Similarly, we call an equilibrium $(\mathcal{A}, n)$-*insistent*, if the cut-offs satisfy $\lambda_n, \mu_n < \infty$ but $\lambda_{n+1} = \infty$. Finally an equilibrium is called *insistent* if it is $(\mathcal{C}, n)$- or $(\mathcal{A}, n)$-insistent for some n. On its game path, a $(\mathcal{C}, n)$-insistent equilibrium is equivalent to the strategy profile (ℓ, m) with

- $\lambda_n = \lambda_{n+1} = \cdots < \infty$, i.e. the $\mathcal{C}$-juror never quits starting in round n, and
- $\mu_n = \mu_{n+1} = \cdots = \infty$, i.e. the $\mathcal{A}$-juror quits in round n.

Lemma 7 (Insistent Equilibria) *For each n there is a $(\mathcal{C}, n)$- and a $(\mathcal{A}, n)$-insistent equilibrium. These are unique up to changes off the equilibrium path.*

Proof Consider the $(\mathcal{C}, n)$-insistent equilibrium as defined above. Starting at the $\mathcal{A}$-juror's stage n move, the strategies defined above are easily seen to be mutual best-responses.

Consider the best such profile (ℓ, m) — i.e. the sequences $\lambda_1 \leq \cdots \leq \lambda_n$ and $\mu_1 \leq \cdots \leq \mu_{n-1}$ with the lowest expected cost. By Lemma 6, (ℓ, m) is an equilibrium.

To show that the n^{th} argument of the $\mathcal{C}$-juror is actually reached on the equilibrium path, i.e. $\mu_{n-1}, \lambda_n < \infty$, assume to the contrary that the equilibrium path terminates at an earlier round, say in round $n' < n$ with the move of the $\mathcal{C}$-juror, i.e. $\mu_{n'-1} < \lambda_{n'} = \infty$. As the n'^{th} argument of the $\mathcal{A}$-juror is off the equilibrium path, this strategy profile remains optimal, and thus an equilibrium, after setting $\mu_{n'} = \infty$. However, we noted above that sufficiently strong types λ will hold out if all opponent types are about to quit, i.e. $\lim_{\lambda \to \infty} P(\mu_{n'-1}, \lambda, \infty) > 0$, which contradicts the optimality of $\lambda_n = \infty$. This argument already implies that longer insistent equilibria (strictly) Pareto dominate shorter insistent equilibria.

To prove that this equilibrium is sequential, we need to consider the off-equilibrium action of the $\mathcal{C}$-juror in round $n + 1$ if, unexpectedly, the $\mathcal{A}$-juror holds out in round n. Sequential equilibrium allows her to interpret this is as a mistake made by some arbitrary type, say μ_0. This belief, together with the expectation that the $\mathcal{A}$-juror will now revert to equilibrium play, justifies her holding out, $\lambda_{n+1} = \lambda_n$.

Uniqueness follows by Lemma 5.

Insistent equilibria represent a communication failure. A strong types μ of $\mathcal{A}$ quits in round n not because she is convinced of her colleague's opinion but because she sees no way to convince the colleague of her own opinion, which may be based on superior information. However, the insistence of $\mathcal{C}$ in round $n+1$ can only be justified by the unreasonable belief outlined above, namely that any holding out of $\mathcal{A}$ in round n is interpreted as a mistake by some low type rather than a deliberate signal from some high type. We will see below that such beliefs violate the forward induction refinement, as defined in (Cho 1987).

The essence of an insistent equilibrium is not so much the fact that no types λ quit after some round n, but rather that so few of them $[\lambda_n, \lambda_{n+1}]$ quit that waiting one more round is not worthwhile for the $\mathcal{A}$-juror, implying $\mu_n = \infty$. Communication break-down is avoided precisely if an equilibrium is non-terminal, i.e. $\lambda_n, \mu_n < \infty$ for all n. Therefore, we will call such an equilibrium *communicative.*

Proposition 1 (Longer is Better) *// (a) A communicative equilibrium exists and is slower than any insistent equilibrium.*
(b) Slower equilibria ex ante Pareto dominate faster equilibria.
(c) A communicative equilibrium satisfies forward induction, whereas insistent equilibria do not.

Thus, maximal delay is optimal and to be expected whereas communication break-down is both unstable and unfavorable to both jurors.

Proof (a) The proof of Lemma 7 showed that every insistent equilibrium is strictly Pareto dominated. Thus an efficient equilibrium, which exists by Lemma 6, must be communicative. By Lemma 5 it is slower than any insistent equilibrium.

(b) If the communicative equilibrium is unique, the above argument already imply this claim. However, it is not clear whether or not this assumption is satisfied. Fortunately, we can show that if there are multiple communicative equilibria then they all share the same expected total costs and are thus all ex-ante Pareto optimal. While not difficult, the proof of this claim is not insightful and thus relegated to Lemma 10 in the Appendix.

(c) In a communicative equilibrium every action node is reached on the equilibrium path, so forward induction imposes no further restrictions on the equilibrium.

Now consider an insistent equilibrium, say the $(\mathcal{A}, n)$-insistent equilibrium. As argued above, there are types $\lambda > \mu_{n-1}$ that have something to gain from holding out if the $\mathcal{A}$-juror was to change strategy by increasing her next cut-off μ_n sufficiently. Denote the smallest such type by $\tilde{\lambda}$.

Assume, contrary to the Proposition, that this equilibrium satisfied forward induction. This restricts $\mathcal{A}$'s beliefs in round n to the interval $[\tilde{\lambda}, \infty)$, which in turn restricts her best responses to $\mu_n > \tilde{\lambda} + \varkappa$. This argument already shows that an off-equilibrium threat of $\mu_n = \mu_{n-1}$ can never be part of an equilibrium that satisfies forward induction.

We will now show that $\mu_n > \tilde{\lambda} + \varkappa$ implies that that any type of the $\mathcal{C}$-juror with $\lambda \geq \mu_n - \varkappa$ will prefer to hold out in round n and thus $\lambda_n = \infty$ is not optimal. To do so, consider

$$\begin{aligned}0 &\leq P(\mu_{n-1}, \tilde{\lambda}, \tilde{\lambda} + \varkappa) \\ &< P(\mu_{n-1}, \mu_n - \varkappa, \tilde{\lambda} + \varkappa) \\ &< P(\mu_{n-1}, \mu_n - \varkappa, \mu_n).\end{aligned}$$

$\tilde{\lambda}$ was defined as the smallest λ such that $P(\mu_{n-1}, \lambda, \mu) \geq 0$ for some μ. Now, $P(\mu_{n-1}, \tilde{\lambda}, \cdot)$ is maximized by $\tilde{\lambda} + \varkappa$, implying line one. Line two follows by the assumption that $\mu_n > \tilde{\lambda} + \varkappa$ and the fact that P is strictly increasing in its second argument. Line three follows again by the argument that $P(\mu_{n-1}, \mu_n - \varkappa, \cdot)$ is maximized by μ_n.

5 Long conversations - moot information

We will now come to our second main result, that long equilibrium paths are associated with moot information.

Definition Let $E, F \subset \{(\lambda, \mu) \in \mathbb{R}^2 : \lambda \geq \mu\}$ be measurable events. The jurors' information is *more moot* in E than in F if the distribution of $\lambda - \mu$ in E is first-order stochastically dominated by the distribution of $(\lambda - \mu)$ in F.

The analogue definition applies to sets $E, F \subset \{(\lambda, \mu) \in \mathbb{R}^2 : \mu \leq \lambda\}$ with respect to the distribution of $\mu - \lambda$.

By the definition of first-order stochastic dominance the information is more moot in E than in F if the probability that the $\mathcal{G}$uilt of the defendant is proven "beyond a reasonable doubt", i.e. $2\pi(\lambda, \mu)) - 1 \geq x$, is lower in E than it is in F for any threshold $x \in [0, 1]$. Moot information is of little value to the jurors. In the extreme case where $\lambda - \mu = 0$ the ex-post preferable verdict is a random coin-toss and any information aggregation has been futile.

Proposition 2 (Law of Diminishing Total Returns) *Fix any equilibrium (ℓ, m), not necessarily the efficient one. Conditional on $\mathcal{C}$onviction being the ex-post preferable verdict, the jurors' joint information is more moot when either of the jurors quits in a later round n' than when either of the jurors quits in an earlier round $n < n'$.*

Analogously, conditional on $\mathcal{A}$cquittal being the ex-post preferable verdict, the jurors' joint information is more moot when the $\mathcal{A}$-juror quits in round n' or the $\mathcal{C}$-juror quits in round $n' + 1$ than when they do so in earlier rounds $n, n + 1 < n', n' + 1$.

It is necessary for this result to condition on the ex-post preferred verdict, i.e. on $\lambda - \mu \gtreqless 0$, for several reasons. First, the way we introduced the "mootness order" this would capture a shift in beliefs towards $\mathcal{G}$uilt rather than an increase in the decisiveness $|\lambda - \mu|$ of the signal profile if applied unconditionally. Second, if we were to adopt an alternative definition relying on the distribution of $|\lambda - \mu|$, the proposition as stated would still not need to hold unconditionally. This is because an increase of all conditional probabilities does not imply an increase of the unconditional probability. It is possible to establish an unconditional version of this result but to do so it would be necessary to "symmetrize" the problem by assuming $f = g$, $\mu_0 = 0$ and to focus on the efficient equilibrium as $\kappa \to 0$.

Proof We will actually show the slightly stronger statement that the densities are ranked in the MLRP sense, i.e. we will show that the likelihood ratio $\frac{\Pr(\lambda - \mu = \delta, (\lambda, \mu) \in E_n)}{\Pr(\lambda - \mu = \delta', (\lambda, \mu) \in E_n)}$

of more moot information $\delta \leq \delta'$ is increasing in time n, where $E_n := \{(\lambda, \mu) : \mu \in [\mu_{n-1}, \mu_n], \lambda \geq \mu\}$ is the event that the conversation terminates in round n.

After some rearranging, this condition translates into

$$\frac{\int_{\mu_n}^{\mu_{n+1}} f(\mu+\delta)g(\mu)r(\mu+\delta,\mu)d\mu}{\int_{\mu_{n-1}}^{\mu_n} f(\mu+\delta)g(\mu)r(\mu+\delta,\mu)d\mu} \geq \frac{\int_{\mu_n}^{\mu_{n+1}} f(\mu+\delta')g(\mu)r(\mu+\delta',\mu)d\mu}{\int_{\mu_{n-1}}^{\mu_n} f(\mu+\delta')g(\mu)r(\mu+\delta',\mu)d\mu}.$$

The proof now follows by the log-concavity of f, i.e. $\frac{f(\mu'+\delta)}{f(\mu+\delta)} \geq \frac{f(\mu'+\delta')}{f(\mu+\delta')}$ for $\mu' \geq \mu$ and $\delta' \geq \delta$, and the log-concavity of r, i.e. $\frac{r(\mu'+\delta,\mu')}{r(\mu+\delta,\mu)} \geq \frac{r(\mu'+\delta',\mu')}{r(\mu+\delta',\mu)}$ which is derived in Lemma 9 in the Appendix.

If we were to replace the integrals over μ in the desired inequality above with the values of the integrand at some intermediate points $\mu \in [\mu_{n-1}, \mu_n]$ and $\mu' \in [\mu_n, \mu_{n+1}]$, the g-terms would cancel and the inequality would obtain because of the two log-concavity conditions. For the integrals, we use an argument from (Milgrom and Weber 1982) to obtain

$$\begin{aligned}
\frac{\int_{\mu_n}^{\mu_{n+1}} f(\mu+\delta)g(\mu)r(\mu+\delta,\mu)d\mu}{\int_{\mu_{n-1}}^{\mu_n} f(\mu+\delta)g(\mu)r(\mu+\delta,\mu)d\mu} &= \frac{\int_{\mu_n}^{\mu_{n+1}} \frac{f(\mu+\delta)g(\mu)r(\mu+\delta,\mu)}{f(\mu_n+\delta)g(\mu_n)r(\mu_n+\delta,\mu_n)}d\mu}{\int_{\mu_{n-1}}^{\mu_n} \frac{f(\mu+\delta)g(\mu)r(\mu+\delta,\mu)}{f(\mu_n+\delta)g(\mu_n)r(\mu_n+\delta,\mu_n)}d\mu} \geq \\
\geq \frac{\int_{\mu_n}^{\mu_{n+1}} \frac{f(\mu+\delta')g(\mu)r(\mu+\delta',\mu)}{f(\mu_n+\delta')g(\mu_n)r(\mu_n+\delta',\mu_n)}d\mu}{\int_{\mu_{n-1}}^{\mu_n} \frac{f(\mu+\delta')g(\mu)r(\mu+\delta',\mu)}{f(\mu_n+\delta')g(\mu_n)r(\mu_n+\delta',\mu_n)}d\mu} &= \frac{\int_{\mu_n}^{\mu_{n+1}} f(\mu+\delta')g(\mu)r(\mu+\delta',\mu)d\mu}{\int_{\mu_{n-1}}^{\mu_n} f(\mu+\delta')g(\mu)r(\mu+\delta',\mu)d\mu}.
\end{aligned}$$

Our result that higher total investment correlates with a lower value of the aggregate information seems at odds with the intuition that higher investment should lead to lower marginal but higher total returns. To reconcile these two views let us take a closer look at the timing of our result. The negative correlation is established ex post. When we consider the jurors' ex ante problem of searching (λ, μ) in the joint type space, the search exhibits diminishing marginal returns. Both the probability of terminating the argument[7] and the expected value of the true information is decreasing in the number of rounds. As the benefit of investment into information aggregation is realized only when the information is utilized and the decision is taken, the total return to the investment ex post is actually the marginal return to the investment ex ante. This argument indicates that the properties of investment into information differ from investment in, say, capital assets.

To see that our result is actually a robust feature of investment into information rather than a quirk of our specific model, consider for a moment a single agent dynamic experimentation / information acquisition problem. The agent will stop experimenting and take the decision early when these early experiments have yielded valuable information that clearly indicates one possible decision as superior. When the agent has experimented for a long time to the contrary, this is likely due to the fact that the received information has little value for the decision. A late stopping time may then rather indicate that the agent has run out of promising experiments, i.e. she concludes that she just does not know, and takes an uninformed decision. Again, investment into information in this setting is negatively correlated with the value of the information for the decision.

[7] not the hazard rate, but the probability as seen from the start of the conversation

6 Small Marginal Communication Cost

So far we have taken the time per round κ as fixed. If we think of κ as very small, i.e. in the limit as $\kappa \to 0$, the question arises whether equilibrium outcomes converge to instantaneous and perfect information aggregation, i.e. every juror putting all her information on the table at once and then deciding on the ex-post preferable verdict. For any fixed insistent equilibrium, say the $(\mathcal{C}, n)$-insistent equilibrium, the verdict is asymptotically instantaneous but information is not aggregated perfectly even in the limit. We thus focus on the asymptotic behavior of a communicative, efficient equilibrium.

The problem with showing asymptotically instantaneous agreement is of course that the direct effect of decreasing κ is counteracted by an increase of the expected number of (the cheaper) arguments to agreement $N(\kappa)$. Thus, it is not clear whether the expected time to agreement $N(\kappa)\kappa$ converges to 0, if so at which rate, and at the same time how decision costs from taking the wrong verdict are decreasing in κ.

Insert figure “Reduction of κ”

Instead of taking $\kappa \to 0$, an alternative and more direct way of modeling fast communication would be a continuous time game where jurors choose a time $t \in \mathbb{R}$ rather than a round $n \in \mathbb{N}$ when to quit. Unfortunately, this model offers no satisfactory results on information aggregation because any tentative informative equilibrium collapses, i.e. does not exist. To see this, consider any weakly monotone stopping function $t_{\mathcal{A}}(\mu) \in \mathbb{R}$ and the best response to it by the $\mathcal{C}$-juror. Type λ has no incentive to hold out once every weaker type $\mu < \lambda$ has quit, yielding $t_{\mathcal{C}} \leq t_{\mathcal{A}}$. Similarly $t_{\mathcal{A}} \leq t_{\mathcal{C}}$. To conclude, note that $t_{\mathcal{C}} = t_{\mathcal{A}}$ cannot be an equilibrium. Quitting slightly earlier than prescribed, at $t_{\mathcal{C}}(\lambda) - \varepsilon$ rather than at $t_{\mathcal{C}}(\lambda)$, decreases waiting costs by ε while raising decision costs by ε^2, a loss of size ε that occurs with probability ε. Thus, any tentative equilibrium unravels to an immediate, uninformative agreement, analogue to the $(\mathcal{C}, 1)$-insistent equilibrium.

To state our main result about vanishing communication costs, denote by $c(\kappa)$, $d(\kappa)$, $N(\kappa)$ the expected values of total cost, decision cost, and number of rounds to agreement in an efficient equilibrium. By definition $c(\kappa) = d(\kappa) + N(\kappa)\kappa$. Three remarks are in order.

First, the efficient equilibrium may not be unique. While $c(\kappa)$ is by definition independent of the choice of an efficient equilibrium, $d(\kappa)$ and $N(\kappa)$ generally do depend on such an arbitrary choice. The asymptotic results below however hold independently of this choice.

Second, $d(\kappa)$ only includes the *excess* expected cost of deciding on a different verdict than the ex-post prefered verdict. That is, if $\lambda - \mu > 0$ and $\mathcal{C}$onviction is the ex-post prefered verdict whereas the jurors reach an $\mathcal{A}$cquittal verdict the contribution towards $d(\kappa)$ is not the probability that the acquitted is guilty, $\pi(\lambda, \mu) > \frac{1}{2}$, but only the amount $2\pi(\lambda, \mu) - 1 > 0$ that could have been avoided in knowledge of (λ, μ). If the juror reach the ex-post prefered $\mathcal{C}$onvict verdict the contribution to $d(\kappa)$ is not the probability of innocence, $1 - \pi(\lambda, \mu) < \frac{1}{2}$, but 0. In other words $d(\kappa)$ is the expected value of the information that is not aggregated in an efficient equilibrium.

Related to this, the result should not be interpreted as saying that less sophisticated jurors, who may have lower opportunity costs, will reach better verdicts than sophisticated jurors with higher opportunity costs. It is true that κ could alternatively be interpreted as the unit cost of time rather then the length of the round of arguing, and that according to the proposition total costs would be decreasing in the opportunity costs κ. However, this argument assumes as fixed the distributions of λ and μ. If more sophisticated jurors have more precise signals, i.e. λ and μ have a higher variance, it may well be the case that they have a higher value of $d(\kappa)$ but still a lower probability of convicting an innocent or acquitting a guilty defendent as their expected statistical error $\mathbb{E}[\min\{\pi(\lambda,\mu), 1-\pi(\lambda,\mu)\}]$ is smaller.

Proposition 3 *For any selection of efficient equilibria as $\kappa \to 0$:*
(a) Total costs $c(\kappa)$ vanish at rate $\kappa^{\frac{2}{3}}$, i.e. $\theta\kappa^{\frac{2}{3}} \leq c(\kappa) \leq \Theta\kappa^{\frac{2}{3}}$, and
(b) Investment in information aggregation is twice the value of information not aggregated, i.e. $\lim_{\kappa\to 0} \frac{N(\kappa)\kappa}{d(\kappa)} = 2$.

This implies that the time to agreement $N(\kappa)\kappa$ vanishes at rate at most $\kappa^{\frac{2}{3}}$ for any selection of equilibria as the longest equilibrium is efficient by Proposition 1.

The proof of this proposition is facilitated by the fact that the efficient equilibrium is a planner's solution, reducing the proof to an exercise in dynamic optimization.

To build some intuition for the result consider the situation of a planner who does not know λ or μ, who has already decided the cut-off values up to μ_{n-1} and who now is trying to choose λ_n so as to minimize waiting plus decision costs. The total cost from choosing λ_n consist of (1) the incremental waiting cost κ, (2) the cost of taking the wrong verdict, i.e. giving in to $\mathcal{A}$cquittal although $\mu < \lambda$ where $\mu_{n-1} < \mu$ and $\lambda < \lambda_n$, and (3) the continuation costs if $\lambda > \lambda_n$.

Instead of maximizing over λ_n directly, we approximate the step length $(\lambda_n - \lambda_{n-1})$ to be proportional to the inverse $\frac{1}{N(\kappa)}$ of the expected number of arguments in the conversation. The joint density function h of (λ,μ) is continuous and can thus be approximated as constant on the small triangle with $\mu_{n-1} < \mu < \lambda < \lambda_n$. With this approximation the expected decision costs in round n are proportional to $\frac{1}{N(\kappa)^3}$ as the ex-post inferior verdict is chosen with probability $\sim \frac{1}{N(\kappa)^2}$ and the ex-post mistake is of size $\sim \frac{1}{N(\kappa)}$.

Insert figure "Triangle"

As the dynamic problem is approximately stationary, the continuation cost equals $c(\kappa)$ and the probability of not reaching the continuation game is proportional to $\frac{1}{N(\kappa)}$. This line of argument suggests the following approximate "Bellman equation of the planner"

$$c(\kappa) \approx \kappa + \frac{x}{N(\kappa)^3} + \left(1 - \frac{y}{N(\kappa)}\right) c(\kappa)$$

After some rearranging total costs are given by $c(\kappa) \approx \frac{1}{y}\left(N(\kappa)\kappa + \frac{x}{N(\kappa)^2}\right)$ implying $N^*(\kappa) \approx \sqrt[3]{2x}\kappa^{-\frac{1}{3}}$ for the optimal expected number of rounds. So, as discussed above, the expected number of rounds to agreement $N^*(\kappa)$ is indeed increasing as κ is decreasing. However, this countervailing indirect effect offsets only one third of the size of the direct effect. Plugging $N^*(\kappa)$ back into the cost functions we get that

- $N^*(\kappa)\kappa \approx \sqrt[3]{2x}\kappa^{\frac{2}{3}} = \frac{2x}{\sqrt[3]{2x}^2}\kappa^{\frac{2}{3}}$, and

- $d(\kappa) \approx N^*(\kappa)\frac{x}{N^*(\kappa)^3} = \frac{x}{\sqrt[3]{2x}^2}\kappa^{\frac{2}{3}}$

Thus, both waiting and decision costs in the efficient equilibrium vanish at rate $\kappa^{\frac{2}{3}}$ and the former exceed the latter by a factor 2.

How can we understand the factor "2"? As an optimization problem over the number of rounds N, decision costs x/N^2 are twice as sensitive to N than waiting costs κN:

$$\left|\frac{d'(N)}{d(N)}\right| = \left|\frac{-2x/N^3}{x/N^2}\right| = \frac{2}{N} \text{ whereas } \left|\frac{(\kappa N)'}{\kappa N}\right| = \frac{1}{N}$$

Therefore, the optimal trade-off between waiting costs and decision costs calls for twice as much of the former than of the latter.

The actual proof of Proposition 3 does not follow the above dynamic optimization argument. Instead the proof proceeds by defining strategy profiles $(\ell, m)_\kappa$ for a decreasing sequence of κ and proving that the total cost of these profiles $\overline{c}(\kappa) \geq c(\kappa)$ diminishes at rate $\kappa^{\frac{2}{3}}$. It is heavy in notation and little insightful. We therefore relegate it to the appendix.

7 Conclusion

We have introduced a common interest model of information aggregation via coarse and costly arguing. Within our model, we showed that it is optimal to argue as long as possible, although long arguments are associated with more moot information. When marginal arguing costs are small information aggregation is almost perfect and instantaneous.

The model of the arguing process is highly stylized in (1) allowing only two possible arguments, $\mathcal{A}$ or $\mathcal{C}$, at every node and (2) entangling the arguing with the decision taking by assuming that a verdict is taken as soon as both jurors argue for the same verdict. Let us reemphasize that this communication protocol is not optimal in, say, the class of communication protocols where each binary information has a certain transmission cost. As κ decreases, so does the set of types $[\lambda_{n-1}, \lambda_n]$ who quit in any round n in an efficient equilibrium. This forces a strong type λ to wait many rounds to reveal her strength of belief.

A communication protocol that disentangles arguing from decision making can avoid this problem. Consider a game where each juror can either argue for $\mathcal{A}$, or argue for $\mathcal{C}$, or concede in any round n. After one juror concedes the other juror chooses the verdict; if both concede simultaneously, juror 1 chooses the verdict. Without a formal proof it seems pretty clear that the following strategy profile has total cost of order $\kappa \log \kappa$ which is lower than the $\kappa^{\frac{2}{3}}$ in the optimal protocol in our model: In every round each juror bisects her type space in every round, arguing for $\mathcal{A}$ if the type is one half and for $\mathcal{C}$ if it is in the other half. A juror concedes if either both argued for the same verdict in the previous round or if the ex-post preferred verdict is reasonably clear otherwise. As κ decreases we can alter this strategy profile by adding one more argument to those conversations that have not yet revealed the ex-post preferred verdict with certainty. Thus the number of rounds grows linearly while the cost per round and the precision of the decision improve exponentially, yielding the order $\kappa \log \kappa$.

While the spirit of all our main results carries over to this alternative game, let us make the case for our model by pointing out two shortcomings of this alternative.

First, it can easily be the case in this alternative that more controversial information (λ', μ') is aggregated quicker than less controversial information (λ, μ) where $\lambda < \lambda'$ and $\mu < \mu'$. This effect does not seem natural when compared to our prediction in Lemma 2 that stronger types will argue longer. Second, forcing jurors to argue repeatedly for their preferred verdict and incur real cost to convey a strong signal can be seen as a robustness device to ensure incentive compatibility. If, contrary to the assumption in our model, the jurors disagree about the relative cost of convicting the innocent and acquitting the guilty as is the case in (Li, Rosen, and Suen 2001), a protocol in which a juror can just claim to have very strong information seems less robust to manipulation than a model in which a juror has to incur costs arguing for her preferred verdict.

Going beyond the current analysis we see multiple potentially interesting extensions.

- **Testable implications:** The inverse relationship between the length of the argument and the decisiveness of the aggregated information is in principle testable empirically. The problem is how to measure the decisiveness $(\lambda - \mu)$. One approach would be to argue that verdicts based on moot information are more likely to be overturned in the future and thus the length of a jury trial should correlate positively with the probability of the verdict being overturned. This argument assumes that across the different observations, jurors play the same equilibrium and attach the same relative value to the opportunity cost of their time κ and the importance of the verdict.

- **Conflicting interests:** The current analysis focuses on a technological communication constraint by putting a unit cost on each bit of transmitted information. Alternatively, communication can be obstructed by strategic considerations of the jurors as is the case in (Crawford and Sobel 1982) and (Li, Rosen, and Suen 2001), who find that ex-post dominated verdicts are unavoidable in equilibrium. This result breaks down when we model the decision taking process as dynamic and costly as in our model. An interesting question may be whether the waiting costs necessarily outweigh the improvement in the decision as is the case in a (Gul and Lundholm 1995) or whether repeated costly communication can enhance welfare as in (Damiano, Li, and Suen 2008).

8 Appendix

8.1 Game Including Initial Vote

The analysis in the paper assumes that the $\mathcal{A}$-juror has voted for $\mathcal{A}$ in an initial round. This round can be endogenized without conceptual difficulty. A strategy for juror 1 defines for each type μ an initial vote $\iota_\mu \in \{\mathcal{A}, \mathcal{C}\}$, a stopping rule $\Gamma_{\mathcal{A},\mu} \in \mathbb{N} \cup \{\infty\}$ conditional on having voted $\mathcal{A}$ initially, and a stopping rule $\Gamma_{\mathcal{C},\mu} \in \mathbb{R} \cup \{\infty\}$ conditional on having voted $\mathcal{C}$ initially. Similarly a strategy for juror 2 is given by two stopping rules $(\Phi_{\mathcal{C},\lambda}, \Phi_{\mathcal{A},\lambda})$, contingent on the initial vote.

We know from Lemma 2 that the strategies following an initial $\mathcal{A}$-vote are described by increasing sequences of cut-off values (ℓ, m) (and similarly by decreasing sequences of cut-offs after an initial $\mathcal{C}$-vote). We will now show that while juror 1 will also optimally apply a cut-off rule in the initial round, it is not clear that high types μ will vote $\mathcal{A}$ and low types will vote $\mathcal{C}$.

Lemma 8 *Fix any strategy* $\Phi : (-\infty, \infty) \to (\mathbb{N} \cup \{\infty\})^2$ *of juror* 2. *The best response of juror* 1 *is to pick a cut-off value* μ_0 *and either*
(*a*) *vote straight-forwardly, i.e. vote* $\mathcal{A}$ *if* $\mu > \mu_0$ *and vote* $\mathcal{C}$ *if* $\mu < \mu_0$, *or*
(*b*) *vote counter-intuitively, i.e. vote vote* $\mathcal{A}$ *if* $\mu < \mu_0$ *and vote* $\mathcal{C}$ *if* $\mu > \mu_0$

Proof Denote by $c(\mathcal{A}, \theta)$ the expected cost of initially voting $\mathcal{A}$ given Φ and best responses $\Gamma_{\mathcal{A},\mu} \in \mathbb{N} \cup \{\infty\}$ and conditional on the true state $\theta \in \mathcal{G}, \mathcal{I}$ (and similarly for $\mathcal{C}$). Thus, the benefit of voting $\mathcal{A}$ rather than $\mathcal{C}$ is given by

$$\pi(\mu)(c(\mathcal{C}, \mathcal{G}) - c(\mathcal{A}, \mathcal{G})) + (1 - \pi(\mu))(c(\mathcal{C}, \mathcal{I}) - c(\mathcal{A}, \mathcal{I})).$$

The terms in the parentheses do not depend on μ, and as π is decreasing in μ the whole expression is increasing in μ iff $(c(\mathcal{C}, \mathcal{I}) - c(\mathcal{A}, \mathcal{I})) - (c(\mathcal{C}, \mathcal{G}) - c(\mathcal{A}, \mathcal{G}))$ is greater than 0 and decreasing in μ if the term is less than 0. In either case, the optimal initial voting rule is described by some cut-off point μ_0.

It is tempting to think that $(c(\mathcal{C}, \mathcal{I}) - c(\mathcal{A}, \mathcal{I})) - (c(\mathcal{C}, \mathcal{G}) - c(\mathcal{A}, \mathcal{G}))$ is always greater than 0, as conditional on $\mathcal{I}$ndependence the cost of initially voting to $\mathcal{C}$onvict should be greater than to $\mathcal{A}$cquit. However, there are strategies Φ of juror 1 that support counter-intuitive voting in equilibrium, namely voting for the opposite verdict than juror 1 for all but those types, who very much agree with the initial vote. We implicitly ruled these equilibria out by assuming that $\mu_0 > -\infty$ in Section 2. While logically consistent these equilibria do not seem to resemble any real behavior.

The main results of the paper carry over to the larger game.

1. While equilibria in the larger game are no longer ranked according to their length there are still communicative and insistent equilibria and it is still true that the latter do not satisfy forward induction and are Pareto dominated.

2. The finding that long conversations are correlated with moot informaiton remains valid. However, in the larger game we have to condition the result not only on the ex-post preferable verdict but additionally on the initial vote by juror 1.

3. The asymptotic results in Proposition 3 remain valid as stated. Actually it can be extended to show that even equilibria with a counter-intuitive initial vote can aggregate the information perfectly and instantaneously as $\kappa \to 0$.

8.2 Omitted Proofs

Lemma 9 (Log-supermodular Correlation) *The correlation function r is log-supermodular in the sense that*

$$\frac{r(\lambda' + \delta, \lambda')}{r(\lambda + \delta, \lambda)} > \frac{r(\lambda' + \delta', \lambda')}{r(\lambda + \delta', \lambda)}$$

for all $\lambda' > \lambda$ *and* $\delta' > \delta$.

Insert figure "log-concavity of r"

Proof By Lemma 1 we have

$$r(\lambda, \lambda + \delta) = \frac{2e^{\lambda}\left(1 + e^{\delta}\right)}{\left(1 + e^{\lambda}\right)\left(1 + e^{\lambda+\delta}\right)},$$

so all but the $1+e^{\lambda+\delta}$ term cancel in $\frac{r(\lambda'+\delta,\lambda')}{r(\lambda+\delta,\lambda)} > \frac{r(\lambda'+\delta',\lambda')}{r(\lambda+\delta',\lambda)}$. Thus, it suffices to show that

$$\frac{1+e^{\lambda+\delta}}{1+e^{\lambda'+\delta}} > \frac{1+e^{\lambda+\delta'}}{1+e^{\lambda'+\delta'}}$$

which follows from the fact that $e^{\lambda+\delta}$ is strictly supermodular in λ and δ.

Lemma 10 (Increasing Propensity to Hold Out) *If (ℓ, m) is an equilibrium, then the propensity to hold out increases as the cut-off values are increased by the same constant $\delta > 0$:*

$$0 = P^{\mathcal{C}}\left(\mu_{n-1}, \lambda_n, \mu_n\right) < P^{\mathcal{C}}\left(\mu_{n-1}+\delta, \lambda_n+\delta, \mu_n+\delta\right)$$

Proof By definition of the propensity to hold out we need to show that

$$\begin{aligned} & \int_{\mu_{n-1}}^{\mu_n} (2\pi\left(\lambda_n, \mu\right) - 1 - \kappa) r\left(\lambda_n, \mu\right) g\left(\mu\right) d\mu - 2\kappa \int_{\mu_n}^{\infty} r\left(\lambda_n, \mu\right) g\left(\mu\right) d\mu \\ \leq \; & \int_{\mu_{n-1}}^{\mu_n} (2\pi\left(\lambda_n+\delta, \mu+\delta\right) - 1 - \kappa) r\left(\lambda_n+\delta, \mu+\delta\right) g\left(\mu+\delta\right) d\mu - \\ & -2\kappa \int_{\mu_n}^{\infty} r\left(\lambda_n+\delta, \mu+\delta\right) g\left(\mu+\delta\right) d\mu \end{aligned}$$

By the log-concavity of g and Lemma 9 the density $r\left(\lambda_n, \mu\right) g\left(\mu\right)$ MLRP dominates $r\left(\lambda_n+\delta, \mu+\delta\right) g\left(\mu+\delta\right)$. The integrand is the same on both sides of the inequality as $\pi\left(\lambda_n, \mu\right) = \pi\left(\lambda_n+\delta, \mu+\delta\right)$. So if the integrand was a decreasing function in μ the lemma would be established. But while $2\pi\left(\lambda_n, \mu\right) - 1 - \kappa$ is decreasing, the integrand jumps up at μ_n because $2\pi\left(\lambda_n, \mu_n\right) - 1 - \kappa < -2\kappa$.

We abbreviate notation by setting $\phi\left(\mu\right) = 2\pi\left(\lambda_n, \mu\right) - 1 - \kappa$, $\psi\left(\mu\right) = \frac{r(\lambda_n,\mu)g(\mu)}{\int_{\mu_{n-1}}^{\mu_n} r(\lambda_n,\mu)g(\mu)d\mu}$, $\psi^{\delta}\left(\mu\right) = \frac{r(\lambda_n+\delta,\mu+\delta)g(\mu+\delta)}{\int_{\mu_{n-1}}^{\mu_n} r(\lambda_n+\delta,\mu+\delta)g(\mu+\delta)d\mu}$. In these terms the claim of the lemma translates to

$$0 = \int_{\mu_{n-1}}^{\mu_n} \phi\left(\mu\right)\psi\left(\mu\right) d\mu - 2\kappa \int_{\mu_n}^{\infty} \psi\left(\mu\right) d\mu < \int_{\mu_{n-1}}^{\mu_n} \phi\left(\mu\right)\psi^{\delta}\left(\mu\right) d\mu - 2\kappa \int_{\mu_n}^{\infty} \psi^{\delta}\left(\mu\right) d\mu$$

Now first, $\psi\left(\mu\right)$ MLRP dominates $\psi^{\delta}\left(\mu\right)$, both are probability densities on $[\mu_{n-1}, \mu_n]$ and ϕ is decreasing on this interval. This implies

$$\int_{\mu_{n-1}}^{\mu_n} \phi\left(\mu\right)\psi\left(\mu\right) d\mu < \int_{\mu_{n-1}}^{\mu_n} \phi\left(\mu\right)\psi^{\delta}\left(\mu\right) d\mu.$$

Second, we know that

$$\begin{aligned} \int_{\mu_n}^{\infty} \psi\left(\mu\right) d\mu &= \frac{\int_{\mu_n}^{\infty} r\left(\lambda_n, \mu\right) g\left(\mu\right) d\mu}{\int_{\mu_{n-1}}^{\mu_n} r\left(\lambda_n, \mu\right) g\left(\mu\right) d\mu} > \\ &> \frac{\int_{\mu_{n-1}}^{\mu_n} r\left(\lambda_n+\delta, \mu+\delta\right) g\left(\mu+\delta\right) d\mu}{\int_{\mu_{n-1}}^{\mu_n} r\left(\lambda_n+\delta, \mu+\delta\right) g\left(\mu+\delta\right) d\mu} = \int_{\mu_n}^{\infty} \psi^{\delta}\left(\mu\right) d\mu \end{aligned}$$

as $r\left(\lambda_n, \mu\right) g\left(\mu\right)$ MLRP dominates $r\left(\lambda_n+\delta, \mu+\delta\right) g\left(\mu+\delta\right)$. This completes the proof.

Proof of Lemma 6 In looking for the optimal strategy profile we can restrict ourselves to monotone strategies represented by vectors of cut-off values (ℓ, m) because any best-response is in monotone strategies by Lemma 2. Rather than considering the cut-offs in log-likelihood notation $\lambda_n, \mu_n \in (-\infty, \infty)$ it will prove advantageous to consider the respective values of the cdfs $\alpha_n := \int_{-\infty}^{\lambda_n} f, \beta_n := \int_{\mu_0}^{\mu_n} g \in [0,1]$ and vectors $a = (\alpha_n)_n, b = (\beta_n)_n$ thereof. Moreover, note that we can restrict ourselves to strategy profiles (a, b) with either $\lim \alpha_n = 1$ or $\lim \beta_n = 1$ (profiles that do not satisfy this property have expected total costs of ∞).

So consider any sequence $(a^\nu, b^\nu)_\nu$ of strategy profiles that approaches the lower cost limit $\lim_{\nu\to\infty} c_\kappa(a^\nu, b^\nu) = c(\kappa)$ [8]. We will show that there exists a subsequence $(a^{\nu_\iota}, b^{\nu_\iota})_\iota$ that converges point-wise to a cost-minimizing strategy profile (a^*, b^*), i.e. with $c(a^*, b^*) = c(\kappa)$.

The proof is a typical "diagonal argument". Consider the sequence of the first cut-off points $(\alpha_1^\nu)_\nu$. As a sequence in the compact space $[0,1]$ it has a convergent subsequence which we denote by $(\alpha_1^{\nu_\iota^1})_\iota$. Similarly, considering the first $n \in \mathbb{N}$ cut-off points we can find a subsequence $(\nu_\iota^n)_\iota$ such that $(\alpha_1^{\nu_\iota^n}, \beta_1^{\nu_\iota^n}, \ldots, \alpha_n^{\nu_\iota^n}, \beta_n^{\nu_\iota^n})_\iota$ converges in $[0,1]^{2n}$. Thus the diagonal sub-sequence $(\alpha^{\nu_\iota^\iota}, \beta^{\nu_\iota^\iota})_\iota$ converges point-wise against some sequence of cut-offs (a^*, b^*).

It remains to be shown that $c(a^*, b^*) = \lim_{\iota\to\infty} c_\kappa(a^{\nu_\iota^\iota}, b^{\nu_\iota^\iota}) = c(\kappa)$. We will show for any ε that $c(a^*, b^*) \le c_\kappa(a^{\nu_\iota^\iota}, b^{\nu_\iota^\iota}) + \varepsilon$ for all sufficiently large ι. First note that for sufficiently large n and ι we have that $\alpha_n^{\nu_\iota^\iota}$ and α_n^* (or $\beta_n^{\nu_\iota^\iota}$ and β_n^*) are arbitrarily close to 1 and therefore the probability of changing the verdict after round n by switching from strategy profile $(a^{\nu_\iota^\iota}, b^{\nu_\iota^\iota})$ to (a^*, b^*) is going to zero. Also, the loss in case of a wrong verdict is bounded by 1, and so the expected loss from switching is bounded by $\frac{\varepsilon}{2}$.

As for the cut-off types 1 through n, there exists ι above which these first $2n$ cut-off points of $(a^{\nu_\iota^\iota}, b^{\nu_\iota^\iota})$ are uniformly so close to those of (a^*, b^*), that the expected loss of inducing the wrong verdict before round n by changing strategy profile $(a^{\nu_\iota^\iota}, b^{\nu_\iota^\iota})$ to (a^*, b^*) is bounded by $\frac{\varepsilon}{2}$, proving that $c_\kappa(a^*, b^*) \le c_\kappa(a^{\nu_\iota^\iota}, b^{\nu_\iota^\iota}) + \varepsilon$.

Lemma 11 *If there are multiple communicative equilibria (ℓ, m) and (ℓ', m') then for every intermediate value $\lambda \in [\lambda_1, \lambda_1']$ there is a communicative equilibrium (ℓ'', m'') with $\lambda_1'' = \lambda$ and all of these equilibria share the same ex-ante expected total costs.*

Proof Remember that the equilibria (ℓ, m) and (ℓ', m') are uniquely determined by their initial cut-off values λ_1 and λ_1'. Any value $\lambda_1'' \in [\lambda_1, \lambda_1']$ defines a sequence $\mu_1'', \lambda_2'', \mu_2'', \ldots$ and by Lemma 5 we have $\lambda_n'' \in [\lambda_n, \lambda_n'] \subset \mathbb{R}$ and $\mu_n'' \in [\mu_n, \mu_n'] \subset \mathbb{R}$, implying that the cut-off profile (ℓ'', m'') is actually an equilibrium.

Denote by $(\ell, m)(\lambda)$ the equilibrium defined by $\lambda \in [\lambda_1, \lambda_1']$ and by $\lambda_n(\lambda), \mu_n(\lambda)$ the respective cut-off values. As $\lambda_n(\lambda), \mu_n(\lambda)$ are finite and the signal distributions are continuous and bounded on any compact set, $\lambda_n(\lambda), \mu_n(\lambda)$ are differentiable as functions of λ. We denote by $C(\lambda) = C(\ell, m)_\lambda$ the total cost (waiting cost plus decision cost) of the equilibrium $(\ell, m)_\lambda$ and will now show that $C'(\lambda) = 0$ on $[\lambda_1, \lambda_1']$.

For any ε there exists $n \in \mathbb{N}$ such that $C(\ell, m)_\lambda$ decreases by at most ε^2 if we set the cut-offs $\lambda_{n'}, \mu_{n'}$ equal to ∞ for rounds $n' \ge n$. Now $C(\lambda + \varepsilon) - C(\lambda)$ is a sum

[8] For a formal definition of $\underline{c}(\kappa)$, compare Section 6

of terms that are either of the kind $\frac{\partial}{\partial\lambda_n} c(\ell, m)_\lambda \frac{d\lambda_n}{d\lambda}(\lambda)\varepsilon$ or of degree ε^2. As (ℓ, m) is an equilibrium $\frac{\partial}{\partial\lambda_n} c(\ell, m)_\lambda = 0$ and the first class of terms vanishes, implying that $C'(\lambda) = \lim_{\varepsilon\to 0} \frac{1}{\varepsilon}(C(\lambda+\varepsilon) - C(\lambda)) = 0$.

Proof of Proposition 3 (a): As indicated the proof does not directly use the equilibrium conditions but rather characterizes an optimal strategy profile by constructing as an upper bound $\bar{c}(\kappa)$ a sequence of strategy profiles (ℓ, m) with costs decreasing at rate at least $\kappa^{\frac{2}{3}}$ (and a lower bound $\underline{c}(\kappa)$ to the cost of an efficient equilibrium that decreases at rate at most $\kappa^{\frac{2}{3}}$).

To do so we need explicit expressions for expected duration $N(\ell, m)$ and decision cost $d(\ell, m)$ of any strategy profile (ℓ, m). For a concise expression of waiting costs we introduce the following measure of average correlation above (λ, μ)

$$\rho(\lambda, \mu) = \frac{\int_\lambda^\infty \int_\mu^\infty h}{\int_\lambda^\infty f \int_\mu^\infty g}$$

where $h = rfg$ is the joint probability density function. With this the expected duration of strategy profile (ℓ, m) is given by[9]

$$\begin{aligned} N(\ell, m) \quad : &= \sum_{n\in\mathbb{N}} \int_{\lambda_n}^\infty \int_{\mu_{n-1}}^\infty h + \int_{\lambda_n}^\infty \int_{\mu_{n-1}}^\infty h \\ &= \sum_{n\in\mathbb{N}} \rho(\lambda_n, \mu_{n-1}) \int_{\lambda_n}^\infty f \int_{\mu_{n-1}}^\infty g + \rho(\lambda_n, \mu_n) \int_{\lambda_n}^\infty f \int_{\mu_n}^\infty g \end{aligned}$$

and waiting costs are given by $w_\kappa(\ell, m) = \kappa N(\ell, m)$.

As for decision costs, remember our earlier discussion around taking the statistical decision error $\min\{\pi(\lambda, \mu), 1-\pi(\lambda, \mu)\}$ as given and focusing on the avoidable part of the costs. These equal $1-2\pi(\lambda, \mu)$ for a conviction if $\lambda < \mu$, they are equal to $2\pi(\lambda, \mu)-1$ for an acquittal if $\lambda > \mu$, and they are equal to 0 whenever the ex-post preferred verdict is taken. Given some strategy profile (ℓ, m), satisfying $\mu_{n-1} \le \lambda_n \le \mu_n$, ex-post decision costs are thus equal to

$$\begin{array}{lll} 2\pi(\lambda, \mu) - 1 & \text{for} & (\lambda, \mu) \in \Delta(\lambda_n, \mu_{n-1}) := \{(\lambda, \mu) : \mu_{n-1} \le \mu \le \lambda \le \lambda_n\}, \\ 1 - 2\pi(\lambda, \mu) & \text{for} & (\lambda, \mu) \in \Delta(\lambda_n, \mu_n) := \{(\lambda, \mu) : \mu_n \ge \mu \ge \lambda \ge \lambda_n\}, \\ 0 & \text{else.} & \end{array}$$

Thus, expected decision costs of strategy profile (ℓ, m) are given explicitly by

$$d(\ell, m) := \sum_n \int_{\Delta(\lambda_n, \mu_{n-1})} (2\pi - 1) h + \int_{\Delta(\lambda_n, \mu_n)} (1 - 2\pi) h.$$

Total costs are given by $c_\kappa(\ell, m) = \kappa N(\ell, m) + d(\ell, m)$.

[9] Strictly speaking, this is leaving out the very first round of the $\mathcal{C}$-juror. Omitting this summand of 1 innocuous to the proof.

The plan of action now is to define a tractable upper bound $\bar{c}(\kappa)$ for $c(\kappa) := \min_{(\ell,m)} c_\kappa(\ell, m)$

$$\begin{aligned} c_\kappa(\ell,m) &= \kappa \sum_{n\in\mathbb{N}} \int_{\lambda_n}^{\infty}\int_{\mu_{n-1}}^{\infty} h + \int_{\lambda_n}^{\infty}\int_{\mu_n}^{\infty} h + \\ &\quad + \sum_{n\in\mathbb{N}} \int_{\Delta(\lambda_n,\mu_{n-1})} (2\pi - 1)\, h + \int_{\Delta(\lambda_n,\mu_n)} (1-2\pi)\, h \end{aligned}$$

and to show that $\bar{c}(\kappa) \in \mathcal{O}(\kappa^{2/3})$.

As a preparation we derive upper bounds for r, ρ and $|\zeta|$

- $r(\lambda,\mu) = \frac{2(e^\lambda + e^\mu)}{(1+e^\lambda)(1+e^\mu)} \leq 4e^{-\min\{\lambda,\mu\}}$
- $\rho(\lambda,\mu) < r(\lambda,\mu) \leq 4e^{-\min\{\lambda,\mu\}}$ for $\lambda,\mu > 0$ and otherwise $\rho(\lambda,\mu) < 4 \leq 4e^{-\min\{\lambda,\mu\}}$
- $|2\pi(\lambda,\mu) - 1| = |\frac{e^{\lambda-\mu}-1}{e^{\lambda-\mu}+1}| \leq \frac{1}{2}|\lambda-\mu| < |\lambda-\mu|$.

Next, we restrict attention to strategy profiles (ℓ,ℓ) with $\lambda_n = \mu_n$. These strategy profiles cannot be equilibria (the $\mathcal{A}$-juror never risks an ex-post wrongful $\mathcal{C}$onviction) but are easier to handle technically, essentially because $\Delta(\lambda_n,\mu_n)$ is empty for such a profile.

The components of the waiting costs for strategy profile (ℓ,ℓ) satisfy

$$\int_{\lambda_n}^{\infty}\int_{\mu_n}^{\infty} h = \rho(\lambda_n,\mu_n)\int_{\lambda_n}^{\infty} f \int_{\mu_n}^{\infty} g \leq 4e^{-\lambda_n}\int_{\lambda_n}^{\infty} f \int_{\lambda_n}^{\infty} g$$

and the decision cost satisfy

$$\int_{\Delta(\lambda_n,\mu_n)} (2\pi-1)\, h \leq 4e^{-\lambda_n}|\lambda_{n+1}-\lambda_n| \int_{\lambda_n}^{\lambda_{n+1}} f \int_{\lambda_n}^{\lambda_{n+1}} g.$$

Using these bounds, the cost $c_\kappa(\ell,\ell)$ of strategy profile (ℓ,ℓ) is bounded above by

$$\bar{c}_\kappa(\ell,\ell) := \sum_{n=0}^{\infty} 4e^{-\lambda_n}\left(2\kappa\int_{\lambda_n}^{\infty} f \int_{\lambda_n}^{\infty} g + (\lambda_{n+1}-\lambda_n)\int_{\lambda_n}^{\lambda_{n+1}} f \int_{\lambda_n}^{\lambda_{n+1}} g\right) \tag{4}$$

Minimizing costs over this subset of strategy profiles $\bar{c}(\kappa) := \min_\ell \bar{c}_\kappa(\ell,\ell)$ gives thus an upper bound for the minimum over all strategy profiles $c(\kappa)$.

We need to verify that $\bar{c}(\kappa) < \infty$ because $|\lambda-\mu|$ is a very coarse upper bound for $|2\pi(\lambda,\mu)-1|$. To do so, note that if we choose $\lambda_n = n$, say, the term in the parenthesis in equation (4) is uniformly bounded, whereas $\sum 4e^{-\lambda_n}$ converges.

We will now show that $\bar{c}(\frac{\kappa}{8}) \leq \frac{1}{4}\bar{c}(\kappa)$. This implies $\bar{c}(\kappa) \in O\left(\kappa^{\frac{2}{3}}\right)$ after noting that for any $\kappa \in [8^{-(i+1)}, 8^{-i}]$ we have $\bar{c}(\kappa) \leq \bar{c}(8^{-i}) \leq 4^{-i}\bar{c}(1) \leq 4(8^{-(i+1)})^{\frac{2}{3}}\bar{c}(1) \leq \Theta\kappa^{\frac{2}{3}}$ where $\Theta = 4\bar{c}(1)$.

Consider ℓ^* that minimizes $\bar{c}_\kappa(\ell,\ell)$. We will now define a "twice as slow" strategy profile ℓ' that satisfies $\bar{c}_{\frac{\kappa}{8}}(\ell',\ell') \leq \frac{1}{4}c_\kappa(\ell^*,\ell^*) = \frac{1}{4}\bar{c}(\kappa)$ by adding intermediate steps:

$$\lambda'_{2n} := \lambda^*_n, \lambda'_{2n+1} \in [\lambda^*_n, \lambda^*_{n+1}], \lambda'_{2n+2} = \lambda^*_{n+1}$$

with the property that

$$
\begin{aligned}
& (\lambda'_{2n+2} - \lambda'_{2n+1}) \int_{\lambda'_{2n+1}}^{\lambda'_{2n+2}} f \int_{\lambda'_{2n+1}}^{\lambda'_{2n+2}} g + (\lambda'_{2n+1} - \lambda'_{2n}) \int_{\lambda'_{2n}}^{\lambda'_{2n+1}} f \int_{\lambda'_{2n}}^{\lambda'_{2n+1}} g \\
\leq \quad & \frac{1}{4}(\lambda^*_{n+1} - \lambda^*_n) \int_{\lambda^*_n}^{\lambda^*_{n+1}} f \int_{\lambda^*_n}^{\lambda^*_{n+1}} g
\end{aligned}
$$

The existence of such a $\lambda'_{2n+1} \in [\lambda^*_n, \lambda^*_{n+1}]$ is guaranteed by Lemma 12 below.

The above shows that $d(\ell', \ell') \leq \frac{1}{4} d(\ell^*, \ell^*)$. Also, it follows immediately from the construction of ℓ' that $\frac{\kappa}{8} N(\ell', \ell') \leq \frac{1}{4}\kappa N(\ell^*, \ell^*)$. Thus, we have shown that $c_{\frac{\kappa}{8}}(\ell', \ell') \leq \frac{1}{4} c_\kappa(\ell^*, \ell^*)$, proving the upper bound of point (a) of the Proposition.

For the lower bound we will now conversely construct $\underline{c}(\kappa) \leq c(\kappa)$ and show that $\theta\kappa^{\frac{2}{3}} \leq \underline{c}(\kappa)$ for some constant $\theta \in \mathbb{R}$. To do so we

- only consider costs that are contingent on types less extreme than some $\omega < \infty$; thus waiting cost $\kappa N(\ell, m)$ of any strategy profile (ℓ, m) are bounded below by $\underline{N}(\ell, m)\theta_1$ where $\underline{N}(\ell, m) = 2\#\{n : \lambda_n, \mu_n \leq \omega\}$ is the number of rounds until agreement is reached for type profile (ω, ω) and $\theta_1 = \int_\omega^\infty \int_\omega^\infty h$ is a lower bound for the probability of reaching any round before $\underline{N}(\ell, m)$,
- bound the joint density function h below by its minimum $\theta_2 := \min_{\lambda,\mu \leq \omega} h(\lambda, \mu)$, and
- bound below the decision costs $|2\pi(\lambda, \mu) - 1| \geq \theta_3 |\lambda - \mu|$ for small values of $|\lambda - \mu|$.

It follows that

$$
\underline{c}(\kappa) := \min_{(\ell,m):\mu_{n(\ell,m)}=\omega} \left\{ \theta_1 N(\ell, m)\kappa + \theta_2\theta_3 \sum_{n:\lambda_n,\mu_n \leq \omega} \left(\frac{(\lambda_n - \mu_{n-1})^3}{6} + \frac{(\mu_n - \lambda_n)^3}{6} \right) \right\}
$$

is a lower bound for $c(\kappa)$. If (ℓ^*, m^*) minimizes $c(\kappa)$ a simple optimization argument shows that the steps $\lambda^*_n - \mu^*_{n-1}$ and $\mu^*_n - \lambda^*_n$ are equi-distant, i.e. $\lambda^*_n = \frac{2n-1}{\underline{n}(\ell^*, m^*)}\omega$ and $\mu^*_n = \frac{2n}{\underline{n}(\ell^*, m^*)}\omega$. Thus,

$$
\sum_{n:\lambda^*_n,\mu^*_n \leq \omega} (\lambda^*_n - \mu^*_{n-1})^3 + (\mu^*_n - \lambda^*_n)^3 = \underline{N}(\ell^*, m^*)(\frac{\omega}{\underline{N}(\ell^*, m^*)})^3 = \frac{\omega^3}{\underline{N}(\ell^*, m^*)^2}.
$$

With $\theta_4 := \theta_2\theta_3\frac{\omega^2}{6}$ we get that $\underline{c}(\kappa) = \theta_1 \underline{N}(\ell^*, m^*)\kappa + \frac{\theta_4}{\underline{N}(\ell^*, m^*)^2}$. Denote by $\underline{N}(\kappa)$ the optimal number of steps $\underline{N}(\ell^*, m^*)$. Analogue to above we have that $\underline{N}(\frac{\kappa}{8}) = \frac{1}{4}\underline{N}(\kappa)$ and thus $\underline{c}(\kappa) \geq \theta\kappa^{\frac{2}{3}}$ for some constant $\theta \in \mathbb{R}$.

Lemma 12 *Let $F, G : [0, 1] \to [0, 1]$ be continuous, increasing functions with $F(0) = G(0) = 0$ and $F(1) = G(1) = 1$. Then there exists $x \in [0, 1]$ such that $xF(x)G(x) + (1 - x)(1 - F(x))(1 - G(x)) \leq \frac{1}{4}$.*

Proof It suffices to prove existence of x with $F(x)G(x) \leq \frac{1}{4}$ and $(1 - F(x))(1 - G(x)) \leq \frac{1}{4}$. First, $F(x)G(x)$ and $(1 - F(x))(1 - G(x))$ cannot both be strictly greater than $\frac{1}{4}$. By continuity of FG there exists $x \in [0, 1]$ such that $F(x)G(x) \leq \frac{1}{4}$ and $(1 - F(x))(1 - G(x)) \leq \frac{1}{4}$.

Proof of Proposition 3 (b) : The proof formalizes the idea that decision costs are asymptotically twice as sensitive to the number of rounds as waiting costs and are therefore half as large in a planner's solution. The main argument relies on an approximation that the joint density h and the joint survivor function $\int_\lambda^\infty \int_\mu^\infty h$ are constant over a range of rounds n_i to n_{i+1}. It is therefore necessary to disaggregate the sum in the expression for total cost

$$\begin{aligned} c_\kappa(\ell, m) &= \kappa \sum_{n\in\mathbb{N}} \int_{\lambda_n}^\infty \int_{\mu_{n-1}}^\infty h + \int_{\lambda_n}^\infty \int_{\mu_n}^\infty h + \\ &\quad + \sum_{n\in\mathbb{N}} \int_{\Delta(\lambda_n, \mu_{n-1})} (2\pi - 1)\, h + \int_{\Delta(\lambda_n, \mu_n)} (1 - 2\pi)\, h \end{aligned}$$

into summands for which this approximation holds.

For any strictly increasing sequence $(n_i)_{i=1}^\nu$ together with $n_0 = 0$ and $n_{\nu+1} = \infty$ we can write expected waiting costs "from rounds $n_i + 1$ to n_{i+1}" as

$$\kappa N_i := \kappa \sum_{n=n_i+1}^{n_{i+1}} \int_{\lambda_n}^\infty \int_{\mu_{n-1}}^\infty h + \int_{\lambda_n}^\infty \int_{\mu_n}^\infty h$$

and expected decision costs "in rounds $n_i + 1$ to n_{i+1}" as

$$d_i := \sum_{n=n_i+1}^{n_{i+1}} \int_{\Delta(\lambda_n, \mu_{n-1})} (2\pi - 1)\, h + \int_{\Delta(\lambda_n, \mu_n)} (1 - 2\pi)\, h$$

to obtain

$$c_\kappa(\ell, m) = \sum_{i=0}^{\nu-1} (\kappa N_i + d_i) + \kappa N_\nu + d_\nu.$$

The proof now proceeds by showing that for any $\epsilon > 0$, we can choose $\kappa > 0$ small enough so that for any efficient equilibrium (ℓ, m) we can choose ν large enough and $(n_i)_{i=1}^\nu$ adequately spaced so that

1. $\left| \frac{\kappa N_i}{d_i} - 2 \right| \leq \epsilon$ for every i, i.e. the ratio of waiting costs to decision costs "in rounds $n_i + 1$ to n_{i+1}" is close to 2, and at the same time
2. $\frac{\kappa N_\nu + d_\nu}{c_\kappa(\ell, m)} \leq \epsilon$, i.e. waiting and decision costs incurred after round n_ν are negligible.

It follows from the proof of part (a) that for any $\lambda \in \mathbb{R}$ and any bound $\delta > 0$ there is κ small enough, such that the maximum of the step-lengths $\max_{\lambda_n < \lambda} (\lambda_n - \lambda_{n-1}) < \delta$ in an efficient equilibrium (ℓ, m). Therefore, given any $\lambda \in \mathbb{R}, \varepsilon > 0, \Omega \in N$ we can pick κ small enough so that we can partition any efficient equilibrium (ℓ, m) via $(n_i)_{i=1}^\nu$ so that

- there are at least $\Omega \leq \min_i |n_{i+1} - n_i|$ rounds in every element of the partition,
- type λ has given in by the last round of the last partition, $\lambda \leq \lambda_{n_\nu}$, and
- each element of the partition i only covers a small part of the type space $\left[\mu_{n_i}, \mu_{n_{i+1}}\right]$, so that
  - the joint pdf h is almost constant on $\left[\mu_{n_i}, \mu_{n_{i+1}}\right]^2$, i.e. $\frac{\overline{h}}{\underline{h}} \leq \sqrt{1+\varepsilon}$ where
    - $\overline{h} = \max\left\{h(\lambda,\mu) : \lambda,\mu \in \left[\mu_{n_i}, \mu_{n_{i+1}}\right]\right\}$ and
    - $\underline{h} = \min\left\{h(\lambda,\mu) : \lambda,\mu \in \left[\mu_{n_i}, \mu_{n_{i+1}}\right]\right\}$
  - the joint survivor function $\int_\lambda^\infty \int_\mu^\infty h$ is almost constant on $\left[\mu_{n_i}, \mu_{n_{i+1}}\right]^2$, i.e. $\frac{\int_{\mu_{n_i}}^\infty \int_{\mu_{n_i}}^\infty h}{\int_{\mu_{n_{i+1}}}^\infty \int_{\mu_{n_{i+1}}}^\infty h} \leq 1+\varepsilon$, and
  - the decision cost function $2\pi - 1$ is almost linear on $\left[\mu_{n_i}, \mu_{n_{i+1}}\right]^2$, i.e. $\frac{\overline{|2\pi-1|}}{\underline{|2\pi-1|}} \leq \sqrt{1+\delta}$ where
    - $\overline{|2\pi-1|} = \max\left\{|2\pi(\lambda,\mu)-1| \,/\, |\lambda-\mu| : \lambda,\mu \in \left[\mu_{n_i}, \mu_{n_{i+1}}\right]\right\}$ and
    - $\underline{|2\pi-1|} = \min\left\{|2\pi(\lambda,\mu)-1| \,/\, |\lambda-\mu| : \lambda,\mu \in \left[\mu_{n_i}, \mu_{n_{i+1}}\right]\right\}$.

Using the choice of small intervals $\left[\mu_{n_i}, \mu_{n_{i+1}}\right]$ with many rounds Ω we first show claim (1).
Waiting and decision costs "in the interval $\left[\mu_{n_i}, \mu_{n_{i+1}}\right]$" are bounded below by

- $\kappa N_i \geq \kappa 2\Omega \int_{\mu_{n_{i+1}}}^\infty \int_{\mu_{n_{i+1}}}^\infty h$, and
- $d_i \geq 2\Omega \int_\Delta (2\pi - 1)\, h \geq 2\Omega \underline{|2\pi-1|} \frac{1}{6}\left(\frac{\mu_{n_{i+1}} - \mu_{n_i}}{2\Omega}\right)^3 \underline{h}$

where Δ in the lower inequality is any triangle $\Delta_{(\lambda_n, \mu_{n-1})}$ with side-length $\lambda_n - \mu_{n-1} \geq \frac{\mu_{n_{i+1}} - \mu_{n_i}}{2\Omega}$.

To prove $\kappa N_i \geq 2 d_i$ we will now define a slower strategy profile $(\widetilde{\ell}, \widetilde{m})$ with one more step in $\left[\mu_{n_i}, \mu_{n_{i+1}}\right]$ and use $c_\kappa(\widetilde{\ell}, \widetilde{m}) \geq c_\kappa(\ell, m)$ to conclude.

- For $n \leq n_i$ the cut-offs are the same as in (ℓ, m), i.e. $\widetilde{\lambda}_n := \lambda_n$ and $\widetilde{\mu}_n = \mu_n$
- For $n = n_i + 1$ to $n_{i+1} + 1$: there are $2\,(\Omega + 1)$ cut-offs evenly spaced between μ_{n_i} and $\mu_{n_{i+1}}$, i.e. $\widetilde{\mu}_n := \frac{(\Omega+1-n)\mu_{n_i} + (n-n_i)\mu_{n_{i+1}}}{\Omega+1}$ and $\widetilde{\lambda}_n := \frac{\widetilde{\mu}_{n-1} + \widetilde{\mu}_n}{2}$
- For $n > n_{i+1} + 1$ the cut-offs are shifted by one round, i.e. $\widetilde{\lambda}_n = \lambda_{n-1}$ and $\widetilde{\mu}_n = \mu_{n-1}$

In analogy to the expected delay N_i and decision cost d_i for the profile (ℓ, m) we define $\widetilde{N}_i$ and $\widetilde{d}_i$ for the strategy profile $(\widetilde{\ell}, \widetilde{m})$. Similar to the above, waiting and decision costs of $(\widetilde{\ell}, \widetilde{m})$ "in the interval $\left[\mu_{n_i}, \mu_{n_{i+1}}\right]$" are bounded below by

- $\kappa \widetilde{N}_i \leq 2(\Omega+1) \int_{\mu_{n_i}}^\infty \int_{\mu_{n_i}}^\infty h$, and that

- $\widetilde{c} \le 2\left(\Omega+1\right)\overline{|2\pi-1|}\frac{1}{6}\left(\frac{\mu_{n_{i+1}}-\mu_{n_i}}{2(\Omega+1)}\right)^3\overline{h}$

Therefore

- $\frac{\kappa\widetilde{N}_i}{\kappa N_i} \le \frac{\Omega+1}{\Omega}(1+\varepsilon)$ and
- $\frac{\widetilde{d}_i}{d_i} \le (\frac{\Omega}{\Omega+1})^2(1+\varepsilon)$.

Now

$$\begin{aligned} \kappa\widetilde{N}_i + \widetilde{d}_i &\le \frac{\Omega+1}{\Omega}(1+\varepsilon)\kappa N_i + \left(\frac{\Omega}{\Omega+1}\right)^2(1+\varepsilon)d_i \\ &= \kappa N_i + d_i + \\ &\quad + \frac{1}{\Omega}\left(\kappa N_i - 2d_i\right) + \\ &\quad + O\left(\Omega^{-2}\right) + \varepsilon\left(\frac{\Omega+1}{\Omega}\kappa N_i + \left(\frac{\Omega}{\Omega+1}\right)^2 d_i\right) \end{aligned}$$

On the other hand we know by the optimality of (ℓ, m) that $\kappa\widetilde{N}_i + \widetilde{d}_i \le \kappa N_i + d_i$.

As Ω can be chose large and ε be chosen small we can neglect the terms in the last line, yielding $\kappa N_i \ge 2d_i$ in the limit. A similar argument, constructing a strategy profile with one step less than (ℓ, m), shows that in the limit $\kappa N_i \le 2d_i$.

To show part (2) of the proof we need to show that there is λ such that for all κ small enough we have $\frac{\kappa N_\nu + d_\nu}{c_\kappa(\ell,m)} \le \epsilon$ where

$$\begin{aligned} \kappa N_\nu + d_\nu &= \kappa \sum_{n=n_\nu+1}^{\infty}\int_{\lambda_n}^{\infty}\int_{\mu_{n-1}}^{\infty} h + \int_{\lambda_n}^{\infty}\int_{\mu_n}^{\infty} h + \\ &\quad \sum_{n=n_\nu+1}^{\infty}\int_{\Delta(\lambda_n,\mu_{n-1})}\left(2\pi-1\right)h + \int_{\Delta(\lambda_n,\mu_n)}\left(1-2\pi\right)h \end{aligned}$$

and $\lambda_{n_\nu} > \lambda$.

By part (a) of the Proposition we know that $c_\kappa(\ell,m) \ge \theta\kappa^{\frac{2}{3}}$ and a similar argument shows that $\kappa N_\nu + d_\nu \le \Theta' e^{-\lambda}\kappa^{\frac{2}{3}}$, so that $\frac{\kappa N_\nu + d_\nu}{c_\kappa(\ell,m)} \le \frac{\Theta'}{\theta}e^{-\lambda}$ independent of κ.

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