Lumpy Investment and State-Dependent Pricing in General Equilibrium^{*}

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Abstract

The lumpy nature of plant-level investment is generally not taken into account in the context of monetary theory (see, e.g., Christiano et al. 2005 and Woodford 2005). We propose a generalized (S,s) pricing and investment model which is empirically more plausible along that dimension. Surprisingly, our main result shows that a quantitatively relevant monetary transmission mechanism is hard to entertain in the presence of lumpy investment.

Keywords: Lumpy Investment, Sticky Prices.

JEL Classification: E22, E31, E32

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1 Introduction

Many microeconomic decisions are lumpy in nature. Caballero and Engel (2007) note that examples include not only infrequent price adjustment by firms but also investment decisions, durable purchases, hiring and firing decisions, inventory accumulation, and many other economic variables of interest. We develop a dynamic stochastic general equilibrium (DSGE) framework featuring state-dependent pricing combined with monopolistic competition and lumpy investment. In this way we integrate the New Keynesian (NK) framework with the dominant approach of the recent micro-founded investment literature. The combination of these two literatures allows us to address the following question. Do NK models, which are the workhorse of current monetary policy analysis, still deliver a quantitatively relevant monetary transmission mechanism¹ when they are augmented by a standard micro-founded investment model? Surprisingly, our answer is no. Let us put this result into perspective. Traditionally, capital accumulation has been ignored in NK theory.² Woodford (2003, p. 352) comments on this modeling choice: '[...] while this has kept the analysis of the effects of interest rates on aggregate demand quite simple, one may doubt the accuracy of the conclusions obtained, given the obvious importance of variations in investment spending both in business fluctuations generally and in the transmission mechanism for monetary policy in particular. By now, prominent treatments of the monetary transmission mechanism do feature endogenous capital accumulation. (See, e.g., Christiano et al. 2005 and Woodford 2005.) We observe, however, that those models simply brush away the lumpy nature of plant level investment. More importantly, our main result shows that this is of crucial importance for the ability of monetary DSGE models to generate a quantitatively relevant monetary transmisson mechanism. Let us now be more specific about our results.

¹The monetary transmission mechanism is generally viewed as being the hallmark of monetary economics. See, e.g., Walsh (2003), Woodford (2003) and Galí (2008).

 $^{{}^{2}}$ See, e.g., Clarida et al. (1999).

Under the baseline calibration, we find that the impact responses to monetary policy shocks are way too large and there is too little persistence. How does this result change in the presence of Calvo pricing? In that case there is not more persistence and the impact responses to monetary policy shocks become even larger. Taken together our main result therefore suggests that a quantitatively relevant monetary transmission mechanism is hard to entertain in the presence of an empirically plausible investment decision at the firm level.

Along the way, we also analyze the dynamic consequences of technology shocks. We find that equilibrium dynamics under our baseline calibration are reasonably similar to the ones implied by a flexible price version of our model. In this sense we generalize the Thomas (2002) irrelevance result. However, we obtain deviations from the RBC dynamics that are quantitatively more important if Calvo pricing is assumed instead.

The technical difficulties implied by simultaneous (S,s) decision making in the context of a general equilibrium model are quite substantial. This explains that most existing theoretical analyses in the related literature have focused on one particular lumpy decision at a time. For instance, Thomas (2002), Gourio and Kashyap (2007), Bachmann et al. (2008) and Khan and Thomas (2008) analyze aggregate consequences of lumpy investment in the context of RBC models, whereas Dotsey et al. (1999), Dotsey and King (2005), Gertler and Leahy (2006), Midrigan (2006), Bakhshi et al. (2007), Golosov and Lucas (2007), Dotsey et al. (2008) and Nakamura and Steinsson (2008) focus exclusively on the role of state-dependent pricing for aggregate dynamics. We overcome those difficulties by using the method developed in Reiter (2008). Another paper which integrates (S,s) pricing and investment decisions in general equilibrium is Johnston (2007).³ We regard his work as complementary to ours. He assumes a stationary process for the growth rate of real balances (combined with an interest rate inelastic demand for real balances), whereas we con-

 $^{^{3}}$ Kryvtsov and Midrigan (2008) integrate pricing and inventory decisions in the context of a menu cost model. They use their model to analyze the behavior of inventories in the aftermath of monetary policy shocks.

sider an interest rate rule for the conduct of monetary policy. More importantly, Johnston (2007) ensures tractability of his framework by making assumptions which limit the extent to which the timing of pricing decisions is chosen optimally. Our model is therefore not nested with his framework. Interestingly, however, we can recover results that ressemble the ones he shows if we analyze our model assuming Calvo pricing combined with Johnston's assumptions on the conduct of monetary policy and on the determination of real balances. We will come back to this.

The remainder of the paper is organized as follows. Section 2 outlines the model. Section 3 presents the results and Section 4 concludes.

2 The Model

2.1 Households

There is a continuum of households and they are assumed to have access to a complete set of financial markets. Each household has the following period utility function

$$U(C_t, L_t) = \ln C_t + \frac{\eta}{1-\phi} (1-L_t)^{1-\phi},$$

which is separable in its two arguments C_t and L_t . The former denotes a Dixit-Stiglitz consumption aggregate while the latter is meant to indicate hours worked. Our notation reflects that a household's time endowment is normalized to one per period and throughout the analysis the subscript t is used to indicate that a variable is dated as of that period. Parameter ϕ is used to calibrate the labor supply elasticity which is given by $\frac{\phi L}{1-L}$ and we adopt the convention that a variable without time subscript indicates its steady state value. Parameter η is a scaling parameter whose role will be discussed below. The consumption aggregate reads

$$C_t \equiv \left(\int_0^1 C_t\left(i\right)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}},\tag{1}$$

where ϵ is the elasticity of substitution between different varieties of goods $C_t(i)$. The associated price index is defined as follows

$$P_t \equiv \left(\int_0^1 P_t\left(i\right)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}},\tag{2}$$

where $P_t(i)$ is the price of good *i*. Requiring optimal allocation of any spending on the available goods implies that consumption expenditure can be written as P_tC_t . Households are assumed to maximize expected discounted utility

$$E_t \sum_{k=0}^{\infty} \beta^k U\left(C_{t+k}, L_{t+k}\right),$$

where β is the subjective discount factor. The maximizations is subject to a sequence of budget constraints of the form

$$P_t C_t + E_t \{ Q_{t,t+1} D_{t+1} \} \le D_t + P_t W_t L_t + T_t,$$
(3)

where $Q_{t,t+1}$ denotes the stochastic discount factor for random nominal payments and D_{t+1} gives the nominal payoff associated with the portfolio held at the end of period t. We have also used the notation W_t for the real wage and T_t is nominal dividend income resulting from ownership of firms.

The labor supply equation implied by this structure takes the standard form

$$\phi C_t \left(1 - N_t\right)^{-\phi} = W_t,\tag{4}$$

and the consumer Euler equation is given by

$$Q_{t,t+1}^R = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-1},\tag{5}$$

where where $Q_{t,t+1}^R \equiv Q_{t,t+1} \left(\frac{P_{t+1}}{P_t}\right)$ is the real stochastic discount factor. We also note that $E_t \{Q_{t,t+1}\} = R_t^{-1}$, where R_t is the gross risk free nominal interest rate.

2.2 Firms

There is a continuum of firms and each of them is the monopolistically competitive producer of a differentiated good. Each firm $i \in [0, 1]$ is assumed to maximize its market value subject to constraints implied by the demand for its good and the production technology it has access to. Moreover each firm faces random fixed costs of price and capital adjustment. This implies generalized (S, s) rules for price-setting and for investment. Productivity shocks and monetary policy shocks represent the sources of aggregate uncertainty. In each period the time line is as follows.

- 1. The cost of adjusting the price, c_p , realizes.
- 2. The firm changes its price (or not).
- 3. Production takes place.
- 4. The cost of adjusting the capital stock, c_k , realizes.
- 5. The firm invests (or not).

Let us now be more specific about the above mentioned constraints. Each firm i has access to the following Cobb-Douglas production function

$$Y_t(i) = Z_t L_t(i)^{1-\alpha} K_t(i)^{\alpha},$$
(6)

where α denotes the capital share in production. The aggregate level of technology, Z_t , is assumed to be given by the following process

$$\ln Z_t \equiv z_t = \rho_z z_{t-1} + e_{z,t},\tag{7}$$

where $e_{z,t}$ is i.i.d. with zero mean.

In order to invest or change its price the firm must pay a fixed cost. More precisely, we denote the cost functions for investment and for price-setting as $C_{p,t}(i)$ and $C_{k,t}(i)$, respectively. They are both measured in units of the aggregate good and are given by

$$C_{k,t}(K_{t}(i), K_{t+1}(i), c_{k}) = \begin{cases} \mu K_{t}(i) \text{ if } K_{t+1}(i) = (1-\delta) K_{t}(i), \\ K_{t+1}(i) - (1-\delta-\mu) K_{t}(i) + c_{k} \text{ otherwise,} \end{cases}$$
(8)
$$C_{p,t}(P_{t}(i), P_{t+1}(i), c_{p}) = \begin{cases} 0 \text{ if } P_{t+1}(i) = P_{t}(i), \\ c_{p} \text{ otherwise,} \end{cases}$$
(9)

where δ is the rate of depreciation net of maintenance, μ . The cost distribution functions are assumed to have the form $G(\xi) = c_1 + c_2 \tan(c_3\xi - c_4)$. For the price adjustment cost we follow Dotsey et al. (1999) in assuming an S-shaped distribution, whereas we assume a linear distribution function for capital adjustment costs, which is a conventional choice in that literature (see, e.g., Khan and Thomas 2008).

Cost-minimization on the part of households and firms implies that demand for good i is given by

$$Y_t^d(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t^d,\tag{10}$$

where aggregate demand is $Y_t^d = C_t + I_t + C_{p,t}$, which consists of consumption, aggregate investment, $I_t \equiv \int_0^1 C_{k,t}(i) di$, and aggregate price-setting costs, $C_{p,t} = \int_0^1 C_{p,t}(i) di$.

Each firm maximizes its market value

$$E_{t} \sum_{k=0}^{\infty} Q_{t,t+k}^{R} \left\{ \Phi_{t+k} \left(i \right) - C_{k,t+k} \left(i \right) - C_{p,t+k} \left(i \right) \right\},$$
(11)

where $\Phi_t(i) \equiv P_t(i) Y_t(i) - W_t L(i) - \zeta$ is the gross operating surplus net of a fixed cost ζ . The maximization is done subject to the constraints in equations (6), (8), (9), and (10).

2.3 Market Clearing and Monetary Policy

The goods market clearing condition reads

$$Y_t(i) = Y_t^d(i) \text{ for all } i.$$
(12)

Clearing of the labor market requires

$$\int_{0}^{1} L_t(i) \, di = L_t.$$
(13)

Last, we follow Walsh (2005) and let monetary policy take the form of a simple interest rate rule

$$R_{t} = R_{t-1}^{\phi_{r}} \left(\beta^{-1} \left(\frac{P_{t}}{P_{t-1}} \right)^{\phi_{\pi}} \right)^{1-\phi_{r}} e^{e_{r,t}},$$
(14)

where parameters ϕ_{π} and ϕ_{r} measure the responsiveness of the nominal interest rate in response to changes in current inflation and past nominal interest rates, respectively, and $e_{r,t}$ is i.i.d.

2.4 Baseline Calibration

We require that the steady state of our model is empirically plausible. The discount factor β is set to 0.99, which implies a steady state real interest rate of about 4 per cent. Steady state inflation is set to 0.005, i.e., about a 2 per cent annual growth rate of consumer prices. Parameter η is set to imply that households spend one-third of their available time working. We follow Golosov and Lucas (2007) in assuming $\epsilon = 7$, which implies a desired frictionless markup of about 20%. Technology is parametrized such that our model implies a labor share of 0.64 and a yearly capitalto-labor ratio of 2.352 (see, e.g., Khan and Thomas 2008). We therefore choose $\alpha = 0.3398$ and $\phi = 0.0139$. The rate of depreciation (gross of maintenance) is set to $\delta + \mu = 0.025$ which implies a steady state investment to capital ratio of 10% a year. We allow for 33% maintenance, i.e., we set μ to 0.025/3. This value is well in line with the empirical evidence reported in Bachmann et al. (2008) and the references therein. The upper bounds of the cost distribution functions are set such that our model is in line with the following micro evidence. Each quarter 25% of firms change their nominal price (Aucremanne and Dhyne 2004, Baudry et al. 2004, and Nakamura and Steinsson 2008) and each year about 18% of firms make lumpy investments (I/K > 20%) (see, e.g., Khan and Thomas 2008).⁴ The concavity of the profit function in a frictionless version of our model is 0.592 which is in line with the evidence in Cooper and Haltiwanger (2006). Finally, in calibrating the exogenous driving forces of our model we use standard values from the literature. As in Walsh (2005) we use $\phi_r = 0.9$ and $\phi_{\pi} = 1.1$. Finally, the autocorrelation in the technology process, ρ_z , is set to 0.95 (see, e.g., Erceg et al. 2000 and Walsh 2005). The description of our numerical method is provided in the Appendix.

3 Results

3.1 Steady State

[To be added]

3.2 The Monetary Transmission Mechanism with Sticky Prices

To fix ideas let us first assume that the capital stock is held constant at the firm level. Using this simplified version of our model allows us to highlight some typical differences between state-dependent and time-dependent pricing models.⁵

[Figure 1 about here]

 $^{^{4}}$ We target an average frequency of inversement spikes of 4.3 per cent. In addition lumpy investors make up 50.5 per cent of total investment.

 $^{{}^{5}}$ Both modeling choices are interesting to pursue. Woodford (2008) argues forcefully that the micro-foundations of time-dependent pricing might be at least as good as those of (S,s) decision making.

Figure 1 illustrates the dynamic consequences of a 100 basis point increase in the nominal interest rate.⁶ Those findings confirm standard results on the monetary transmission mechanism.⁷ The Calvo model predicts that monetary policy shocks have reasonaly strong and presistent consequences for real and nominal variables.⁸ To the extent that prices are set in an (S,s) fashion we obtain much less persistence in the dynamic consequences of monetary policy shocks. Moreover, the (S,s) modeling of price stickiness also implies oscillating dynamcis in the aftermath of a monetary policy shock. Intuitively, if some firms increase their prices this gives an incentive to other firms to increase their prices as well for otherwise their relative prices would decrease due to an increase in the aggregate price level. This front-loading of pricing decisions changes the distribution of relative prices in the economy in such a way that fewer firms are likely to change their price in the next period.

After those preparations we turn to the central question of the present paper. Do New Keynesian models, which are the workhorse of current monetary policy analysis, still deliver a quantitatively relevant monetary transmission mechanism when thy are augmented by a standard micro-founded investment model?

3.3 The Monetary Transmission Mechanism with Sticky Prices and Lumpy Investment

We analyze dynamic consequences of a one hundred basis points shock to the interest rate rule. Again, we compare Calvo pricing with (S,s) pricing, but this time the comparison is conducted in the context of the lumpy investment model which we have outlined above. The results are shown in Figure 2.

[Figure 2 about here]

⁶In order to see the precentage point change in the endogenous variables implied by a 100 basis point shock to the annualized nominal interest rate (in the absence of a further change induced by policy responses to inflation and past interest rates) the numbers shown in Figure 5 would need to be multipled by 100 and divided by 4.

⁷See, e.g., Galí (2008) for the Calvo model and Bakshi et al. (2007) for the (S,s) pricing model. ⁸However, additional real and nominal frictions are typically added to the standard Calvo model

in order to increase its empirical realism. See, e.g., Christiano et al. (2005).

Monetary policy shocks do not imply persistent effects on real variables and the impact responses are implausibly large. Especially in the Calvo model the impact responses are simply enormous. Clearly, those results cast doubt on the ability of conventional sticky price models to imply an empirically relevant monetary transmission mechanism.

3.4 Dynamic Consequences of Technology Shocks with Sticky Prices and Lumpy Investment

It is natural to use our model to study the dynamic consequences of technology shocks. This is illustrated in Figure 3.

[Figure 3 about here]

The last figure shows impulse responses to a one standard deviation shock to aggregate technology. To put those results into perspective we compare the impulse responses under our baseline calibration to a benchmark case with flexible prices. Figure 3 shows that there are quantitatively important differences in the impact effects between the two impulse responses. Interestingly, (S,s) pricing implies a negative comovement between labor input and productivity.⁹ Apart from those differences in the first periods after the shock the presence of state-dependent pricing does not imply quantitatively important deviations from the RBC dynamics. But how does this result depend on the state-dependent nature of price-setting?

[Figure 4 about here]

With Calvo pricing the deviations from the RBC dynamics are much more pronounced. Specifically, we find a negative comovement between output and productivity conditional on a productivity shock. This extends a result by Thomas (2002). She observes that lumpy investment is relevant for aggregate dynamics if prices are

 $^{^{9}}$ This is interesting because the presence of sticky prices is often used to explain the empirical effects of technology shocks which are generally estimated to differ from the predictions of a standard RBC model. See, e.g., Galí (1999) and Galí and Rabanal (2004) and the references therein.

held constant. We show that an empirically plausible degree of price stickiness can also alter the RBC dynamics.

It is natural to relate our last result to Johnston's (2007) analysis. We have already noted that he integrates (S,s) pricing and investment decisions in a way that limits the extent to which the timing of pricing decisions is chosen optimally.¹⁰ His model is therefore not nested with ours. Interestingly, he also obtains impulse responses to technology shocks that are different from the RBC benchmark in a way that is reminiscent of New Keynesian predictions. We show, however, that the extent to which technology shocks imply dynamcis that are diffrent from the RBC benchmark depends crucially on the restrictions on price adjustment that one assumes. In this regard it is interesting to note that we are able to obtain results that are similar to the ones he shows if we combine Calvo pricing with his assumptions on monetary policy and on the determination of real balances.¹¹

4 Conclusion

The lumpy nature of plant-level investment is generally not taken into account in the context of monetary theory (see, e.g., Christiano et al. 2005, Woodford 2005). We propose a generalized (S,s) pricing and investment model which is empirically more plausible along that dimension. Surprisingly, our first result shows that a quantitatively relevant monetary transmission mechanism is hard to entertain in the presence of lumpy investment. In fact, neither state-dependent pricing nor timedependent price-setting à la Calvo can generate dynamic consequences of monetary policy shocks that are consistent with their counterpart in the data. Along the way, we also analyze dynamic consequences of technology shocks. We find that the Thomas (2002) result is reasonably robust under our baseline calibration, but not if Calvo pricing is assumed instead.

¹⁰Specifically, he assumes that if a firm wants to adjust its capital, it must also adjust its price and he also assumes that capital is installed and productive immediately after purchase.

¹¹Those additional results are available upon request.

Does our first result mean that an explanation for the empirical effects of monetary policy shocks must be found elsewhere? Not necessarily. The results presented in the present paper hinge crucially on the (S,s) nature of the investment decisions under consideration. In fact, the monetary transmission mechanism is well and alive if pricing and investment decisions are modeled in a time-dependent fashion, as shown in Sveen and Weinke (2007).¹² Put into this perspective our results simply suggest that the feature of endogenous capital accumulation did not receive sufficiently much attention in the context of monetary models. Following up on the issues raised in the present paper will therefore be high on our research agenda. In particular, it would be interesting to see how the addition of other empirically plausible features of plant-level investment, such as time-to-build, would affect the results presented here.

¹²Specifically, Sveen and Weinke (2007) obtain the following equivalence result. If pricing and lumpy investment decisions are made in a time-dependent fashion then a convex capital adjustment cost at the firm-level à la Woodford (2005) is observationally equivalent to its counterpart featuring lumpy investment.

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Appendix: Numerical Method

Let $k(i) \equiv K(i)/K$ and $p(i) \equiv P(i)/P$ denote firm *i*'s relative to average capital stock and price. We choose a two-dimensional discrete rectangular grid in log k and log p, centered (roughly) around the average values of those variables.¹³ The distance between grid points in k-direction equals $m \log(1 - \delta)$ for some integer m, such that a firm which does not adjust its capital stock just moves m steps down the grid. The grid in p is not a multiple of the inflation rate. If a firm that starts at a point of the grid and does not adjust its price, then it moves down the grid by the equivalent of the inflation rate, and would therefore end up inbetween grid points. To stay on the discrete grid, we approximate this situation by assuming that the price jumps stochastically to one of the two neighboring grid points, such that the expected price does not change.

Solving for the steady state is a two-dimensional fixed point problem in aggregate demand Y and wage rate W. Given a guess of Y and W, we solve the firm's problem by the following iterative procedure:

- 1. Assume we have a guess of the firm value function V(k, p). The firm then maximizes its value, defined as current period profits plus the discounted continuation value V(k, p). Then we compute optimal choices, *conditional on adjusting*, as follows:
 - In the second part of each period, the firm chooses next period's k. Choices are discrete, restricted to the points on the discrete grid. Since adjustment costs are independent of adjustment size, the optimal capital is only a function of the price set by the firm, not its current k. The chosen capital stock enters into next period's production.
 - In the first part of each period, the firm chooses the price at which it sells its product in that same period. We first find the optimal p on the

¹³We center the grid around the frictionless steady state values of $\log k$ and $\log p$. Obviously, this is only "roughly" equal to their average values in our baseline model.

discrete grid; assume it is the *i*-th point p_i . Then we assume the firm chooses the price continuously in the range (p_{i-1}, p_{i+1}) . Call the optimal price p^* , which is a function of firm capital k, and will in general not be on the discrete grid. For the profit maximization, we assume that the firm sells at p^* this period, but next period the price jumps stochastically to neighbouring grid points, so as to leave the expected price unchanged. Given optimal choices, the adjustment probabilities are a function of the distribution of the adjustment costs.

2. Given a firm policy (i.e., optimal choices of k and p), we can compute a new guess of the value function V(k, p) under the assumption that the policy is played forever. This is just a linear equation system in V.

Iterate steps 1. and 2. until convergence; this is a standard iteration in policy space, for which convergence can be proven. Given equilibrium adjustment probabilities, we can compute the ergodic distribution of k and p, and see whether they are consistent with the guesses of Y and W. We solve for equilibrium Y and W by a quasi-Newton method.

Having computed the steady state, we compute the dynamics, assuming (infinitesimally) small shocks. We can restrict attention to the ergodic set of (k, p)-points in the steady state. With our choices for the dynamics of k and p, infinitesimally small shocks would not move the economy away from the ergodic set. Assume the ergodic set is given by n points x_1, \ldots, x_n , where each x is a (k, p)-pair from the grid. The state of the economy at each point in time is then given by the following variables:

$$V(x_i), \qquad i=1,\ldots,n$$

 $\Phi(x_i), \qquad i=1,\ldots,n$
 z

where $\Phi(x_i)$ is the mass of firms at point x_i , and z is the vector of exogenous shocks. We stack all the state variables plus aggregate jump variables of interest into the vector Θ_t . Finally, we compute an approximation of the dynamics of Θ_t about the steady state of those variables. This approximation is linear in the aggregate shocks and in Θ_t itself.



Figure 1: Monetary Policy Shocks with Fixed Capital



Figure 2: Monetary Policy Shocks with Sticky Prices and Lumpy Investments



Figure 3: Technology Shocks: Baseline vs. Flexible Prices



Figure 4: Technology Shocks: Baseline vs. Calvo Pricing